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THE EXTENDED TANH METHOD FOR SOLVING THE $(2+1)$-DIMENSION NONLINEAR DISPERSIVE LONG WAVE EQUATION

SHENGQIANG TANG, KELEI ZHANG and JIHONG RONG

School of Mathematics and Computing Science
Guilin University of Electronic Technology
Guilin, Guangxi 541004
P. R. China
e-mail: tangsq@guet.edu.cn

Abstract

This paper is devoted to studying the $(2+1)$-dimension nonlinear dispersive long wave equation. The extended tanh method is used to conduct the analysis. The $(2+1)$-dimension NDLW equation give rise to the compactons solutions: kink and periodic waves. The work confirms the power of the proposed method.

1. Introduction

In this paper, we consider the $(2+1)$-dimension nonlinear dispersive long wave equation which was introduced by Boiti et al. [1], as follow:

$$u_{ty} + v_{xx} + \frac{1}{2}(u^2)_{xy} = 0, \tag{1a}$$

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$$v_t + (uv + u + u_{xy})_x = 0. \tag{1b}$$

If we let x be equal to y, the $(2+1)$-dimension nonlinear dispersive long wave equation can be reduced to the $(1+1)$-dimension nonlinear dispersive long wave equation that describes the travel of the shallow water wave. [1] as a compatibility for a "weak" Lax pair. Recently, considerable effort has been devoted to the study of this system. In [4] Paquin and Winternitz showed that the symmetry algebra of $(2+1)$-dimensional dispersive long wave equation (1) is infinite-dimensional and possesses a Kac-Moody-Virasoro structure. Some special similarity solutions are also given in [4] by using symmetry algebra and the classical theoretical analysis. Lou [3] has given nine types of the two-dimensional partial differential equation reductions and 13 types of the ordinary differential equation reductions by means of the direct and nonclassical method. The system (1) have no Painlevé property though they are Lax or IST integrable [2]. More recently, Tang et al. [6], by means of the variable separation approach, the abundant localized coherent structures of the system (1) are given out. In [7] the possible chaotic and fractal localized structures are revealed for the system (1). [5] by using a new extended Riccati equation rational expansion method to constructing multiple exact solutions. In this paper, we will consider the kink and periodic waves for (1), by using the extended tanh method [8].

To find the travelling wave solutions of (1), we first consider the travelling wave solutions in the form

$$u(x, y, t) = u(\xi), v(x, y, t) = v(\xi), \xi = ax + by - ct, ab \neq 0, \tag{2}$$

where c denotes the wave speed. Therefore (1) reduces to be

$$-bcu'' + a^2v'' + ab(uu'' + (u')^2) = 0, \tag{3a}$$

$$-cv' + a(uv + u + abu'')' = 0, \tag{3b}$$

where "$'$" is the derivative with respect to ξ. Integrating (3b) once

$$-cv + a(uv + u + abu'') = g, \tag{4}$$

where g is integration constant. Now we consider the following transformations:

$$v = \alpha + \beta u + \gamma u^2, \tag{5}$$

where α, β, γ are constants to be determined later. Substituting (5) into (3a)

$$-bcu'' + a^2(\beta u'' + 2\gamma(uu'' + (u')^2)) + ab(uu'' + (u')^2) = 0, \tag{6}$$

Obviously, when

$$\alpha = \alpha,\ \beta = \frac{bc}{a^2},\ \gamma = -\frac{b}{2a}, \tag{7}$$

substituting (5) and (7) into (4), therefore (4) reduces to be

$$u'' = \frac{1}{a^2 b}\left[g + c\alpha + \left(\frac{bc^2}{a^2} - a - a\alpha\right)u - \frac{3bc}{2a}u^2 + \frac{b}{2}u^3\right]. \tag{8}$$

In this paper, we always assume that Eq. (1) satisfies (7). Therefore, we only consider the system (8).

2. Using the Extended Tanh Method

Applying the balancing technique of u'' with u^3 gives $M = 1$. The extended tanh method substitutes the finite expansion

$$u(x, y, t) = S(Y) = a_0 + a_1 Y + a_2 Y^{-1},\ Y = \tanh(\mu\xi) \tag{9}$$

into (8), collecting the coefficients of each power of Y, we obtain a system of algebraic equations for $a_0, a_1, a_2, c,$ and μ, and using any symbolic computation program such as Maple, solving this system gives the following four sets of solutions

(i) The first set:

$$a_0 = \frac{c}{a},\ a_1 = \pm\sqrt{\frac{bc^2 + 2a + 2a\alpha}{b}},\ a_2 = 0,$$

$$\mu = \pm\frac{1}{2a}\sqrt{\frac{bc^2 + 2a + 2a\alpha}{b}},\ g = c,\ b(bc^2 + 2a + 2a\alpha) > 0. \tag{10}$$

(ii) The second set:

$$a_0 = \frac{c}{a},\ a_1 = \pm i\sqrt{\frac{bc^2 + 2a + 2a\alpha}{b}},\ a_2 = 0,$$

$$\mu = \pm i\,\frac{1}{2a}\sqrt{\frac{bc^2 + 2a + 2a\alpha}{-b}},\ g = c,\ b(bc^2 + 2a + 2a\alpha) < 0. \tag{11}$$

(iii) The third set:

$$a_0 = \pm\sqrt{\frac{c}{a}},\ a_1 = 0,\ a_2 = \pm\frac{1}{a}\sqrt{\frac{2\left(a^3 + a^3\alpha \pm 3abc\sqrt{\frac{c}{a}} - 2bc^2\right)}{b}},$$

$$\mu = \pm\frac{1}{2a^2}\sqrt{\frac{2\left(a^3 + a^3\alpha \pm 3abc\sqrt{\frac{c}{a}} - 2bc^2\right)}{b}},$$

$$\left(\frac{3bc^2}{2a^2} - g - c\alpha\right)^2 = \frac{c}{a}\left(\frac{bc^2}{a^2} - a - a\alpha + \frac{bc}{2a}\right)^2,$$

$$b\left(a^3 + a^3\alpha \pm 3abc\sqrt{\frac{c}{a}} - 2bc^2\right) > 0. \tag{12}$$

(iv) The fourth set:

$$a_0 = \pm\sqrt{\frac{c}{a}},\ a_1 = 0,\ a_2 = \pm i\,\frac{1}{a}\sqrt{\frac{2\left(a^3 + a^3\alpha \pm 3abc\sqrt{\frac{c}{a}} - 2bc^2\right)}{-b}},$$

$$\mu = \pm i\,\frac{1}{2a^2}\sqrt{\frac{2\left(a^3 + a^3\alpha \pm 3abc\sqrt{\frac{c}{a}} - 2bc^2\right)}{-b}},$$

$$\left(\frac{3bc^2}{2a^2} - g - c\alpha\right)^2 = \frac{c}{a}\left(\frac{bc^2}{a^2} - a - a\alpha + \frac{bc}{2a}\right)^2,$$

$$b\left(a^3 + a^3\alpha \pm 3abc\sqrt{\frac{c}{a}} - 2bc^2\right) < 0. \tag{13}$$

This in turn gives the following kink wave solutions

$$\begin{cases} u_1(x,\ y,\ t) = \dfrac{c}{a} \pm \sqrt{\dfrac{bc^2 + 2a + 2a\alpha}{b}} \tanh\left(\dfrac{1}{2a}\sqrt{\dfrac{bc^2 + 2a + 2a\alpha}{b}}(ax + by - ct)\right), \\ v_1(x,\ y,\ t) = \alpha + \beta u_1 + y u_1^2, \end{cases}$$

$$\begin{cases} u_2(x,\ y,\ t) = \pm\sqrt{\dfrac{c}{a}} \pm \dfrac{\overline{\mu}}{a} \coth \dfrac{\overline{\mu}}{2a^2}(ax + by - ct),\ \overline{\mu} = \sqrt{\dfrac{2(a^3 + a^3\alpha \pm 3abc\sqrt{\dfrac{c}{a}} - 2bc^2)}{-b}}, \\ v_2(x,\ y,\ t) = \alpha + \beta u_2 + \gamma u_2^2. \end{cases}$$

However, for $b(bc^2 + 2a + 2a\alpha) < 0,$ we obtain the periodic solutions

$$\begin{cases} u_3(x\ \ y\ \ t) = \dfrac{c}{a} \pm \sqrt{\dfrac{bc^2 + 2a + 2a\alpha}{-b}} \tan\left(\dfrac{1}{2a}\sqrt{\dfrac{bc^2 + 2a + 2a\alpha}{-b}}(ax + by - ct)\right), \\ v_3(x\ \ y\ \ t) = \alpha + \beta u_3 + \gamma u_3^2, \end{cases}$$

for $b(a^3 + a^3\alpha \pm 3abc\sqrt{\frac{c}{a}} - 2bc^2) < 0,$ we obtain the periodic solutions

$$\begin{cases} u_4(x,\ y,\ t) = \pm\sqrt{\dfrac{c}{a}} \pm \dfrac{\overline{\mu}}{a} \cot \dfrac{\overline{\mu}}{2a^2}(ax + by - ct), \\ v_4(x,\ y\ \ t) = \alpha + \beta u_4 + \gamma u_4^2. \end{cases}$$

3. Discussion

The (2+1)-dimension nonlinear dispersive long wave equation was investigated by using the extended tanh method. Several exact distinct travelling wave solutions were formally derived by using the method. The obtained results clearly demonstrate the efficiency of the method used in this work. Moreover, the method is capable of greatly minimizing the size of computational work compared to other existing techniques. Unlike what was thought before that the (2+1)-dimension nonlinear dispersive long wave equation cannot be solved by the extended tanh method, the introduced analysis confirm that the extended tanh method can be effectively used for these equations.

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