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LARGE SOLUTIONS OF QUASILINEAR ELLIPTIC EQUATIONS

SAEID SHOKOOH

Department of Mathematic
Faculty of Basic Sciences
University of Gonbad-E-Kavous
Gonbad
Iran
e-mail: sd.shokooh@gmail.com

Abstract

We consider the equation $\Delta_s u = p(x)u^{\alpha(s-1)} + q(x)u^{\beta(s-1)}$ on $R^N (N \geq 3)$, where p, q are nonnegative continuous functions and $0 < \alpha \leq \beta$. We establish conditions sufficient to ensure the existence and nonexistence of nonnegative entire large solutions of the equation.

1. Introduction

We consider the problem

$$\Delta_s u = p(x)u^{\alpha(s-1)} + q(x)u^{\beta(s-1)}, \quad x \in R^N (N \geq 3), \tag{1}$$

$$u(x) \to \infty, \quad |x| \to \infty, \tag{2}$$

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where the nonnegative functions p and q are locally Hölder continuous on R^N, have the property that $\min p(x), q(x)$ is c-positive R^N (i.e., if $\min p(x), q(x)$ vanishes at any point x_0, then there is an open set Ω in R^N containing x_0 such that it is positive for all x on the boundary of Ω), and $0 < \alpha \le \beta$. We call positive solutions to (1) that satisfy (2) positive entire large solutions (PELS) of (1). Such problems arise in Riemannian geometry when studying conformal deformation of a metric with prescribed scalar curvature [5] and in the study of large solutions of elliptic systems [6]. Our purpose here is to establish conditions on p and q, which ensure the existence or nonexistence of PELS of (1). The vast majority of papers studying nonnegative entire large solutions for quasilinear equations consider equations of the form

$$\Delta_s u = p(x) f(u), \tag{3}$$

where f is nondecreasing. (See, for example, [7, 8] and references therein.) The existence and nonexistence of PELS for (3) when $f(s) = a^s$ $(a > 0)$ can be fairly easily characterized. Indeed, for the quasilinear case $(a > 1)$, Equation (3) has a PELS [7] if p satisfies

$$\int_0^\infty r^{1/s-1} (M_p(r))^{1/s-1} dr < \infty, \tag{4}$$

where $M_p(r) \equiv \max_{|x|=r} p(x)$. On the other hand, it will not generally have a solution if

$$\int_0^\infty r^{1/s-1} (m_p(r))^{1/s-1} dr = \infty, \tag{5}$$

where $m_p(r) \equiv \min_{|x|=r} p(x)$. The purpose of this paper is to establish similar results for (1). Our primary interest is in the mixed case, which $0 < \alpha \le 1 < \beta$ with p satisfying (5), while q satisfies

$$\int_0^\infty r^{1/s-1} (M_q(r))^{1/s-1} dr < \infty, \tag{6}$$

where $M_q(r) \equiv \max_{|x|=r} q(x)$. In this case, there is no PELS in general, as we shown in the proposition in next section. In particular, we show that the equation

$$\Delta_s u = u + e^{-|x|}u^{\beta(s-1)}, \qquad (\beta > 2), \tag{7}$$

has no PELS. This is somewhat surprising since both $\Delta_s u = u$ and $\Delta_s u = e^{-|x|}u^{\beta(s-1)}$ have PELS. We then show (Theorem 1) that if the coefficient on $u^{\beta(s-1)}$ decays at a much faster rate as $|x| \to \infty$, then a nonnegative entire large solution does exist. Next, we show that if $\Delta_s u = e^{-|x|}u^{\beta(s-1)}$ $(\beta > 1)$ has no PELS.

2. Text

We begin by showing that no PELS exist for (7) demonstrating that the existence of a PELS for both $\Delta_s u = p(x)u^{\alpha(s-1)}$ and $\Delta_s u = q(x)u^{\beta(s-1)}$ does not necessarily mean that (1) will have one.

Theorem 1. *Equation* (7) *has no PELS.*

Proof. Suppose (7) does indeed have a solution v. In such case, it has a spherically symmetric (or radial) solution u, which will satisfy the integral equation

$$u(r) = u_0 + \int_0^r x^{\frac{1-N}{s-1}} \int_0^x y^{\frac{N-1}{s-1}} (u(y) + e^{-y}u(y)^{\beta(s-1)})dydx, \tag{8}$$

where $0 < u_0 < v(0)$. Indeed, if (8) did not have a positive solution u, since it is an increasing function, would blow up at some $R > 0$ so that u would be a positive radial large solution of (7) on the ball $|x| < R$ and would, of necessity, satisfy $v \le u$ on $|x| < R$, contradicting the fact that $u(0) < v(0)$. Therefore, since u satisfies (8), we have

$$u(r) \ge u_0 + \int_0^r x^{\frac{1-N}{s-1}} \int_0^x y^{\frac{N-1}{s-1}} u(y)dydx.$$

Substituting $u(r) \geq u_0$ into the right-hand side, we get

$$u(r) \geq u_0 + \int_0^r x^{\frac{1-N}{s-1}} \int_0^x y^{\frac{N-1}{s-1}} u_0 dy dx$$

$$= u_0 + u_0 \int_0^r x^{\frac{1-N}{s-1}} \left(\frac{y^{\frac{N-1}{s-1}+1}}{\frac{N-1}{s-1}+1} \right)_0^x dx$$

$$= u_0 \left(1 + \frac{s-1}{N+s-2} \frac{r^2}{2}\right)$$

$$\Rightarrow u(r) \geq u_0 \left(1 + \frac{r^2}{2\left(\frac{N-1}{s-1}+1\right)}\right).$$

We now substitute this expression into the right-hand side as before to get

$$u(r) \geq u_0 \left(1 + \frac{r^2}{2\left(\frac{N-1}{s-1}+1\right)} + \frac{r^4}{2^3\left(\frac{N-1}{s-1}+1\right)\left(\frac{N-1}{s-1}+3\right)}\right).$$

Continuing this process, we get

$$u(r) \geq u_0 \sum_{k=0}^{\infty} \frac{r^{2k}}{2^k k! \left(\frac{N-1}{s-1}+1\right)\left(\frac{N-1}{s-1}+3\right)\cdots\left(\frac{N-1}{s-1}+1+2k-2\right)}.$$

The expression on the right-hand side can be rewritten to produce

$$u(r) \geq u_0 \Gamma\left(\frac{\frac{N-1}{s-1}+1}{2}\right) \sum_{k=0}^{\infty} \frac{1}{k!\,\Gamma\left(\frac{\frac{N-1}{s-1}+1}{2}+k\right)} \left(\frac{r}{2}\right)^{2k} = c_0 r^{-\gamma} I_\gamma(r),$$

where $c_0 = u_0 \Gamma\left(\frac{\frac{N-1}{s-1}+1}{2}\right)$, $\gamma = \frac{\frac{N-1}{s-1}+1}{2} - 1$, and I_γ is the modified Bessel function with index γ. For large r, it is well known that there is a positive constant δ such that

$$I_\gamma(r) \geq \delta e^r / \sqrt{r},$$

so that $u(r) \geq c_\delta r^{-\gamma-1/2}e^r$ for large r, where $c_\delta = \delta c_0$. Thus, there exist $\epsilon > 0$ small such that $u(r) \geq \epsilon(1 + e^{(1-\epsilon)r})$ for all $r \geq 0$. We will assume that $\epsilon > 0$ so small that $\beta(s-1) - \frac{1}{1-\epsilon} > 1$. Now, let v_k be a nonnegative solution to the problem $(\zeta = \beta(s-1) - \frac{1}{1-\epsilon},\ a = \epsilon^{1/(1-\epsilon)})$

$$\Delta_s v = av^\zeta \text{ on } |x| < k,$$

which is an impossibility since this equation has no such solution [7, 8]. To show that $u \leq v$, we show that $u \leq v_k$ on $|x| \leq k$ for all k. Thus suppose there is a k, such that $\max_{|x|\leq k}(u(x) - v_k(x)) > 0$. Since this maximum cannot occur on $|x| = k$, there must be an x_0 such that $|x_0| < k$, where it does occur. At this point, we have

$$0 \geq \Delta_s(u - v_k) \geq u + e^{-|x_0|}u^{\beta(s-1)} - av_k^\zeta = u + e^{-|x_0|}u^{1/(1-\epsilon)}u^\zeta - av_k^\zeta$$

$$\geq u + e^{-|x_0|}(\epsilon(1 + e^{(1-\epsilon)|x_0|}))^{1/(1-\epsilon)}u_k^\zeta \geq u + au^\zeta - av_k^\zeta > 0,$$

which is a contradiction. Thus $u \leq v_k$ for all k and hence $u \leq v$ on R^N, so we obtain our contradiction. This proves the theorem.

We now show more generally that if the coefficient on $u^{\beta(s-1)}$ in (7) were to decay much more rapidly to zero as $|x| \to \infty$, then a large solution would, in fact, exist.

Theorem 2. *Suppose* $0 < \alpha \leq 1 < \beta$ *and p satisfies*

$$\int_0^\infty r^{\frac{1}{s-1}} M_p^{\frac{1}{s-1}}(r)dr = \infty. \tag{9}$$

Suppose furthermore that q satisfies

$$\int_0^\infty r^{\frac{1}{s-1}} (g(r))^{\frac{1}{s-1}} dr < \infty, \tag{10}$$

where

$$g(r) = M_q(r)\exp\{\frac{(\beta-1)(s-1)}{N-2}\int_0^r tM_p(t)^{\frac{1}{s-1}}dt\}. \tag{11}$$

Then equation (1) *has a nonnegative entire large solution.*

Lemma 1. *Suppose* $\beta > 1$, q *satisfies* (10) *and* p *satisfies* (9). *Then the equation*

$$\Delta_s z = M_p(|x|)z + M_q(|x|)z^{\beta(s-1)}, \tag{12}$$

has a positive radial entire large solution.

Proof. Since p satisfies (9), we have that the equation

$$\Delta_s h = M_p(r)h,$$

has a positive radial entire large solution [8], h, which satisfies

$$h(r) = 1 + \int_0^r x^{\frac{1-N}{s-1}}\int_0^x y^{\frac{N-1}{s-1}}M_p(y)h(y)dydx.$$

From [2], it is clear that

$$h(r) \le 1 + \frac{s-1}{N-s}\int_0^r yM_p(y)h(y)dy.$$

Applying Gronwall's inequality and elementary analysis, we get

$$(k(y) \equiv \frac{yM_p(y)(s-1)}{N-s});$$

$$h(r) \le 1 + \int_0^r K(y)e^{\int_y^r k(\zeta)d\zeta}dy = 1 + \int_0^r -\frac{d}{dy}e^{\int_y^r k(\zeta)d\zeta}dy = e^{\int_0^r k(\zeta)d\zeta}. \tag{13}$$

Now, let w be a positive radial entire large solution of

$$\Delta_s w = g(|x|)w^{\beta(s-1)},$$

where g comes from (11). Thus w satisfies

$$w(r) = w_0 + \int_0^r x^{\frac{1-N}{s-1}} \int_0^x y^{\frac{N-1}{s-1}} g(y)(w(y))^{\beta(s-1)} dydx, \tag{14}$$

for some positive w_0. We note that w exists [7] since g satisfies (10). We now show that (12) has a PELS by proving that the integral equation

$$z(r) = z_0 + \int_0^r x^{\frac{1-N}{s-1}} \int_0^x y^{\frac{N-1}{s-1}} (M_p(y)z(y) + M_q(y)(z(y))^{\beta(s-1)}) dydx, \tag{15}$$

has for an appropriately chosen positive value for z_0, a positive solution valid for all nonnegative r. If we can do this, then the solution z will of necessity be large since [6]

$$z(r) \geq z_0 + \int_0^r x^{\frac{1-N}{s-1}} \int_0^x y^{\frac{N-1}{s-1}} (M_p(y)z(y)) dydx$$

$$\geq z_0 + z_0 \int_0^r x^{\frac{1-N}{s-1}} \int_0^x y^{\frac{N-1}{s-1}} M_p(y) dydx \to \infty,$$

as $r \to \infty$. To show that (15) has a solution valid for all $r \geq 0$, we let $z_0 \in (0, w_0)$, where w_0 comes from (14). Then, it is clear that a solution to (15) exist on some, perhaps quite small, interval. We show that this interval is, in fact, $[0, \infty)$. To do this, we shall show that $z(r) \leq h(r)w(r)$ for all $r \geq 0$. Thus, we define R as

$$R = \sup\{r_0 \in (0, \infty) | z(r) \leq h(r)w(r)\}, \tag{16}$$

for all $r \in [0, r_0]$. If $R = \infty$, we are done. Thus suppose $R < \infty$. Then

$$z(R) = z_0 + \int_0^R x^{\frac{1-N}{s-1}} \int_0^x y^{\frac{N-1}{s-1}} (M_p(y)z(y) + M_q(y)z^{\beta(s-1)}(y)) dydx$$

$$< w_0 + \int_0^R x^{\frac{1-N}{s-1}} \int_0^x y^{\frac{N-1}{s-1}} (w(y)M_p(y)h(y)$$

$$+ M_q(y)h^{\beta(s-1)}(y)w^{\beta(s-1)}(y)) dydx$$

$$= w_0 + \int_0^R x^{\frac{1-N}{s-1}} \int_0^x y^{\frac{N-1}{s-1}} (w(y)M_p(y)h(y)$$

$$+ h(y)M_q(y)h^{(\beta-1)(s-1)}(y)w^{\beta(s-1)}(y))dydx.$$

Using the estimate (13) in the term $h^{(\beta-1)(s-1)}$, we get

$$z(R) < w_0 + \int_0^R x^{\frac{1-N}{s-1}} \int_0^x y^{\frac{N-1}{s-1}} (w(y)M_p(y)h(y) + h(y)g(y)(w(y))^{\beta(s-1)})dydx. \tag{17}$$

However, we note that since the derivatives h' and w' are nonnegative, we get

$$\Delta_s(hw) \geq w\Delta_s h + h\Delta_s w + \frac{N-1}{r(s-1)} h'w'$$

$$\geq w\Delta_s h + h\Delta_s w = wM_p h + hgw^{\beta(s-1)},$$

which, in radial form, produces $(r^{N-1}(hw)')' \geq r^{N-1}(wM_p h + hgw^{\beta(s-1)})$. We can now integrate this using $h(0) = 1$, $h'(0) = w'(0) = 0$, and $w(0) = w_0$ to get (for all $r \geq 0$)

$$h(r)w(r) \geq w_0 + \int_0^r x^{\frac{1-N}{s-1}} (w(y)M_p(y)h(y) + h(y)g(y)(w(y))^{\beta(s-1)})dydx.$$

Combining this with (17), we get $z(R) < h(R)w(R)$. Thus, there exists $\epsilon > 0$ such that $z(r) < h(r)w(r)$ on $[0, R + \epsilon]$, contradicting definition R given in (16). Thus $z \leq hw$ for all r and hence (12) has a positive entire large solution. This completes the proof.

Proof of Theorem 2. From the preceding lemma, we get know that

$$\Delta_s z = M_p(r)z + M_q(r)z^{\beta(s-1)},$$

has a positive radial entire large solution. Furthermore, for each natural number n, the problem

$$\Delta_s u_n = p(x)u_n^{\alpha(s-1)} + q(x)u_n^{\beta(s-1)}, \qquad |x| < n,$$

$$\lim_{|x|\to n} u_n(x) = \infty,$$

has a nonnegative solution. We note that this is easily proven by using the upper/lower solution method since the methods of [1] can be used to show that the equations $\Delta_s w = (M_p + M_q)(w^{\alpha(s-1)} + w^{\beta(s-1)})$ and $\Delta_s v = m_q v^{\beta(s-1)}$ have nonnegative large solutions on $|x| \le n$ with $w \le v$. The desired solution u_n then lies between v and w. We now show that $z \le u_n + 1$ on $|x| \le n$. To do this, we use a typical maximum principle argument. We first note that clearly $z < u_n + 1$ for $|x| = n$, and suppose $\max_{|x|\le n}(z(x) - u_n(x) - 1) > 0$. This maximum will necessary occur at some point x_0 for which $|x_0| < n$ and at that point, we have (note that $z(x_0) > z^{\alpha}(x_0)$) since $z(x_0) > u_n(x_0) + 1 > 1$

$$0 \ge \Delta_s(z - u_n - 1) \ge (M_p z + M_q z^{\beta(s-1)}) - (pu_n^{\alpha(s-1)} + qu_n^{\beta(s-1)})$$

$$\ge M_p(z^{\alpha(s-1)} - u_n^{\alpha(s-1)}) + M_q(z^{\beta(s-1)} - u_n^{\beta(s-1)}) > 0,$$

a contradiction. Hence $z \le u_{n+1}$ on $|x| \le n$. Since u_n is a nonnegative decreasing sequence, it converges to a nonnegative function u and $z - 1 \le u$. Thus $\lim_{|x|\to\infty} u(x) = \infty$ since z is large. A standard regularity will show that u is a classical solution of (1). This completes the proof. To complete our work for the case $\beta > 1$, we give two nonexistence results.

Theorem 3. *Suppose the equation*

$$\Delta_s v = q(x)v^{\beta(s-1)}, \quad \beta > 1, \tag{18}$$

has no PELS. Then Equation (1) *has no PELS.*

Proof. Suppose (1) has a PELS, u. Let v_k be a nonnegative solution of

$$\Delta_s v_k = q(x)v_k^{\beta(s-1)}, \qquad \text{for } |x| < k, \quad \lim_{|x|\to k} v_k(x) = \infty.$$

Then the sequence v_k is decreasing and $u \le v_k$ on $|x| \le k$ for all $k \in N$. Thus v_k convergence to a function, v, on R^N and $u \le v$ on R^N. Since u is nonnegative and $u(x) \to \infty$ as $|x| \to \infty$, the function v has same properties. A standard regularity argument can be used to show that the function v is a nonnegative entire large solution of (18) which, by hypothesis, has none. This completes the proof.

Theorem 4. *Suppose* $0 < \alpha \le \beta \le 1$, $h(r) \equiv M_p(r) + M_q(r) - m_p(r) - m_q(r)$, *and*

$$\int_0^\infty r^{\frac{1}{s-1}} h(r)\psi(r)dr < \infty, \tag{19}$$

where $\psi(r) = \exp \frac{s-1}{N-1} \int_0^r y(m_p(y) + m_q(y))dy$. *Then Equation* (1) *has a PELS, if and only if*

$$\int_0^\infty r^{\frac{1}{s-1}} (m_p(r) + m_q(r))dr = \infty. \tag{20}$$

Proof. The proof is similar to that for $\Delta_s u = p(x)f(u)$, where $\sup_{s \ge 1} f(s)/s < \infty$ [5], and therefore, we merely highlight the differences in the two proofs. The proof hinges on an upper and lower solution argument and a key part of the proof is to show (see [6], Lemma 1) that the equations

$$\Delta_s v = M_p(|x|)v^{\alpha(s-1)} + M_q(|x|)v^{\beta(s-1)},$$

$$\Delta_s w = m_p(|x|)w^{\alpha(s-1)} + m_q(|x|)w^{\beta(s-1)}, \tag{21}$$

have PELS for which $v \le w$. Completion of the proof is then virtually identical to [4] and need not be repeated here. To show that Equations (21) have PELS for which $v \le w$, we show that there exists a number $b > 1$ such that the integral equations

$$v(r) = 1 + \int_0^r y^{\frac{1-N}{s-1}} \int_0^y x^{\frac{N-1}{s-1}} (M_p(x) v^{\alpha(s-1)}(x) + M_q(x) v^{\beta(s-1)}(x)) dx dy,$$

(22)

$$w(r) = b + \int_0^r y^{\frac{1-N}{s-1}} \int_0^y x^{\frac{N-1}{s-1}} (m_p(x) w^{\alpha(s-1)}(x) + m_q(x) w^{\beta(s-1)}(x)) dx dy,$$

(23)

have positive solutions valid for all $r \geq 0$ with $v \leq w$. We note that the condition (20) means that these entire solutions will be PELS. To establish that both integral equations have positive solutions for all $r \geq 0$, we note that the proof is again quite similar to that of [5]. Indeed, we let $v_0 = 1$ and define the sequence v_n iteratively by

$$v_{k+1}(r) = 1 + \int_0^r y^{\frac{1-N}{s-1}} \int_0^y x^{\frac{N-1}{s-1}} (M_p(x) v^{\alpha(s-1)}(x) + M_q(x) v^{\beta(s-1)}(x)) dx dy.$$

(24)

The sequence v_k is monotonically increasing and $v_k \geq 1$ for all k so that, we have

$$v_{k+1}(r) \leq 1 + \int_0^r y^{\frac{1-N}{s-1}} \int_0^y x^{\frac{N-1}{s-1}} (M_p(x) + M_q(x)) v_k(x) dx dy. \qquad (25)$$

As in [8], it is now easy to show that

$$v_k(r) \leq e^{Mr}, \qquad \text{for } 0 \leq r \leq R, \quad k \geq 1,$$

where $M \equiv \frac{s-1}{N-1} \max_{0 \leq r \leq R} r[M_p(r) + M_q(r)]$. Thus, the sequence v_k is increasing and locally bounded and therefore converges R^N. Furthermore, its limit v is a solution to (22). A similar argument shows that (23) has a solution, w. The only thing left is to show the existence of $b > 1$ such that $v \leq w$ on R^N. In fact, we choose

$$b > 1 + \frac{k}{N-1}\int_0^{\infty} y^{\frac{1}{s-1}} h(y)\psi(y),$$

where

$$k = \exp\frac{s-1}{N-1}\int_0^{\infty} x^{\frac{1}{s-1}} h(x)dx,$$

and show that this works. To do this, we define

$$R_{\infty} = \sup R > 0 | v(r) < w(r), \quad \text{for all } r \in [0, R],$$

and show that $R_{\infty} = \infty$. Thus suppose $R_{\infty} < \infty$ and note that since $v \le w$ on $[0, R_{\infty}]$, we use the fact that $v \ge 1$ and the definition of h to get

$$\begin{aligned}
v(R_{\infty}) &= 1 + \int_0^{R_{\infty}} y^{\frac{1-N}{s-1}} \int_0^{y} x^{\frac{N-1}{s-1}} [(M_p(x) - m_p(x))v^{\alpha(s-1)}(x) \\
&\quad + (M_q(x) - m_q(x))v^{\beta(s-1)}(x)]dxdy \\
&\quad + \int_0^{R_{\infty}} y^{\frac{1-N}{s-1}} \int_0^{y} x^{\frac{N-1}{s-1}} [m_p(x)v^{\alpha(s-1)}(x) \\
&\quad + m_q(x)v^{\beta(s-1)}(x)]dxdy \\
&\le 1 + \int_0^{R_{\infty}} y^{\frac{1-N}{s-1}} \int_0^{y} x^{\frac{N-1}{s-1}} h(x)v(x)dxdy \\
&\quad + \int_0^{R_{\infty}} y^{\frac{1-N}{s-1}} \int_0^{y} x^{\frac{N-1}{s-1}} [m_p(x)v^{\alpha(s-1)}(x) \\
&\quad + m_q(x)v^{\beta(s-1)}(x)]dxdy \\
&\le 1 + \int_0^{R_{\infty}} y^{\frac{1-N}{s-1}} \int_0^{y} x^{\frac{N-1}{s-1}} h(x)v(x)dxdy \\
&\quad + \int_0^{R_{\infty}} y^{\frac{1-N}{s-1}} \int_0^{y} x^{\frac{N-1}{s-1}} [m_p(x)w^{\alpha(s-1)}(x)
\end{aligned}$$

$$+ m_q(x)w^{\beta(s-1)}(x)]dxdy. \tag{26}$$

Following the proof in [5], it can be shown that the inequality

$$v(r) = \int_0^r y^{\frac{1-N}{s-1}} \int_0^y x^{\frac{N-1}{s-1}} [M_p(x)v^{\alpha(s-1)}(x) + M_q(x)v^{\beta(s-1)}(x)]dxdy$$

$$\leq 1 + \frac{s-1}{N-1} \int_0^r x[M_p(x) + M_q(x)]v(x)dx,$$

produces the estimate

$$v(r) \leq k\psi(r).$$

Replacing v in (26) with this expression and using inequality (13) from [6] (valid for arbitrary ψ and g) gives

$$v(R_\infty) \leq 1 + \frac{k}{N-1} \int_0^{R_\infty} xh(x)\psi(x)dx$$

$$+ \int_0^{R_\infty} y^{\frac{1-N}{s-1}} \int_0^y x^{\frac{N-1}{s-1}} [m_p(x)w^{\alpha(s-1)}(x)$$

$$+ m_q(x)w^{\beta(s-1)}(x)]dxdy;$$

$$b + \int_0^{R_\infty} y^{\frac{1-N}{s-1}} \int_0^y x^{\frac{N-1}{s-1}} [m_p(x)w^{\alpha(s-1)}(x)$$

$$+ m_q(x)w^{\beta(s-1)}(x)]dxdy = w(R_\infty).$$

Thus, there must exist $R > R_\infty$ such that $v < w$ on $[0, R]$, contradicting of R_∞. Therefore, we must have $R_\infty = \infty$ and hence $v \leq w$ on R^N. This completes the proof.

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