Journal of Mathematical Sciences: Advances and Applications
Volume 1, Number 2, 2008, Pages 215-225

ON UNIFORM TRUNCATION ERROR BOUNDS FOR SAMPLING EXPANSION FROM LOCAL AVERAGES

SHOUZHEN ZENG

Institute of Mathematics
Zhejiang Wanli University
Ningbo 315101
P. R. China
e-mail: zszzxl@163.com

Abstract

In this paper,we study the truncation error bounds in a new method - local averages. The uniform bounds error by local average is estimated for $f \in B^{\infty}_{\pi\Omega}$ and satisfying the following decay condition:

$$|f(t)| \le \frac{A}{|t|^{1+\gamma}}, \ t \ne 0. \tag{1}$$

associated with the Shannon sampling representations.

1. Introduction and the Main Result

The Shannon sampling theorem plays an important role in signal analysis as it provides a foundation for digital signal processing. It says that any bandlimited function f, having its frequencies bounded by $\pi\Omega$, can be recovered from its sampled values taken at instances k/Ω, i.e.,

2000 Mathematics Subject Classification: 42C15, 94A20.

Keywords and phrases: truncation error, local averages, sampling theorem.

Supported by the Natural Science Foundation of China (60472042, 60572113) and the Liuhui Center for Applied Mathematics.

Received March 20, 2008

$$f(t) = \sum_{k=-\infty}^{+\infty} f\left(\frac{k}{\Omega}\right)\sin c(\Omega t - k), \tag{2}$$

where $\sin c(t) = \sin \pi t/(\pi t)$, $t \neq 0$, and $\sin c(0) = 1$.

Equation (2) requires values of a signal f that are measured on a discrete set. However, due to its physical limitation, say the inertia, a measuring apparatus may not be able to obtain exact values of f at epoch t_k for $k = 0, 1, 2, \cdots$. Instead, what a measuring apparatus often gives us is local averages of f near t_k for each k. The sampled values defined as local averages may be formulated by the following equation

$$\lambda_k(f) := \langle f, u_k \rangle = \int f(t)u_k(t)dt, \tag{3}$$

where u_k for each $k \in \mathbb{Z}$ is a weight function characterizing the inertia of measuring apparatus. Particularly, in an ideal case, the function is given by Dirac δ-function, $u_k = \delta(\cdot - t_k)$, because $\lambda_k(f) = \langle f, u_k \rangle = f(t_k)$ is the exact value of t_k. The local averaging method in sampling was studied by a number of papers [4, 6, 8, 11, 12, 13, 14, 15] from 1994 to 2007.

The associated truncation error of (2) is defined by

$$T_N(t) := f(t) - \sum_{k=-N}^{N} f\left(\frac{k}{\Omega}\right)\sin c(\Omega t - k). \tag{4}$$

At present, many authors are studying the truncation error bounds, which can be commonly seen in the literature, i.e., [1-3, 5, 7, 9, 16], but these error bounds are almost not uniform. In 1998 Xin Min Li [10] gave the following result.

Proposition 1.1 (Xin Min Li [10])**.** *Let* $f \in B_{\pi\Omega}^{\infty}$ *satisfy a decay condition of form*

$$|f(t)| \leq \frac{A}{|t|^{1+\gamma}}, \ t \neq 0 \tag{5}$$

for some choice of $A > 0$ *and* $\gamma > 0$, *then*

$$| T_N(t) | \le A \cdot (2\Omega)^{r+1} \cdot \frac{2\sqrt{2}e}{\pi} \cdot \frac{(\pi + \ln N)}{N^{r+1}}, \; N \ge 8. \tag{6}$$

For a continuous function f we denote by $\omega(f, \sigma)$ the modulus of continuity

$$\omega(f, \sigma) = \sup_{|h| \le \sigma} \| f(\cdot + h) - f(\cdot) \|_C, \tag{7}$$

where σ can be any positive number. We will say that a function f belongs to the class $B^{\infty}_{\pi\Omega}$ for some $\Omega > 0$, provided that f is an entire function (on C) of exponential type $\pi\Omega$ that belongs to $L_{\infty}(\mathbb{R})$ when restricted to $\mathbb{R}$.

Now we assume that the functionals λ_k are given by (3) in terms of the weight functions u_k, and u_k satisfy the following properties:

(i) $\sup u_k\left[\frac{k}{\Omega} - \sigma'_k, \frac{k}{\Omega} + \sigma''_k\right]$, where $\sigma/4 \le \sigma'_k, \sigma''_k \le \sigma/2$, σ are positive constants;

(ii) $u_k(t) \ge 0$, $\int u_k(t)dt = 1$;

(iii) $\alpha = \inf_{k \in Z}\{\alpha_k\}$, where $\alpha_k := \int_{k/\Omega - \sigma/4}^{k/\Omega + \sigma/4} u_k(t)dt$.

Our main result is the following theorem.

Theorem 1.2. *Let* $\Omega \ge 2$, $\sigma < 1/\Omega$, $\gamma \in (0, 1]$ *and* $f \in B^{\infty}_{\pi\Omega}$ *satisfy* (1). *Suppose that* $\sigma/4 \le \sigma'_k, \sigma''_k \le \sigma/2$, *then we have*

$$\left| f(\cdot) - \sum_{k=-N}^{N} \left[\int_{k/\Omega - \sigma'_k}^{k/\Omega + \sigma''_k} u_k(t) f(t) dt \right] \sin c(\Omega \cdot - k) \right|$$

$$\le K \cdot \Omega^{(1+\gamma)} \cdot \frac{2\sqrt{2}e}{\pi} \cdot \frac{(\pi + \ln N)}{N^{1+\gamma}}, \; N \ge 8, \tag{8}$$

where $K = \left[2A\left(\alpha\left(\frac{16}{15}\right)^{(1+\gamma)} + (1 - \alpha)\left(\frac{8}{7}\right)^{(1+\gamma)} \right) + \frac{3\sqrt{2}}{4}(2 - \sigma) \right]$.

2. Proof of the Main Result

Let us introduce two lemmas first.

Lemma 2.1. *For* $1 < q < 2$, $1/p + 1/q = 1$, *and* $\Omega > 0$, *we have*

$$\sum_{k=-\infty}^{\infty} |\sin c(\Omega t - k)|^q \le 2 + \frac{2}{\pi} + \frac{2}{\pi} \cdot \frac{1}{q-1}. \tag{9}$$

Proof. We use the periodicity of the sum to write

$$\begin{aligned}
\sum_{k=-\infty}^{\infty} |\sin c(\Omega t - k)|^q &\le \sup_{0\le t\le 1/\Omega} \sum_{k=-\infty}^{\infty} |\sin c(\Omega t - k)|^q \\
&\le \sup_{0\le t\le 1/\Omega} \Bigg(|\sin c(\Omega t)|^q + |\sin c(\Omega t - 1)|^q + \\
&\qquad \sum_{k\ne 0,1} \frac{|\sin \pi(t-k)|^q}{\pi^q |t-k|^q} \Bigg) \\
&\le 2 + \sup_{0\le t\le 1/\Omega} \sum_{k\ne 0,1} \frac{1}{\pi^q |t-k|^q} \\
&\le 2 + \frac{1}{\pi} \sup_{0\le t\le 1/\Omega} \left(\sum_{k=2}^{\infty} \frac{1}{(k-t)^q} + \sum_{k=-\infty}^{-1} \frac{1}{(t-k)^q} \right) \\
&\le 2 + \frac{1}{\pi} \left(2 + 2\sum_{k=2}^{\infty} \frac{1}{k^q} \right) \\
&\le 2 + \frac{2}{\pi} + \frac{2}{\pi} \cdot \frac{1}{q-1}.
\end{aligned}$$

Lemma 2.2. *For* $f \in C(R)$, $|f(t)| \le \dfrac{A}{|t|^{1+\gamma}}$, $\gamma \in (0, 1]$, $p \ge 2/\gamma$ and $N \ge 4$, *we have*

$$\left(\sum_{|k|>N} |f(k/\Omega)|^p \right)^{1/p} \le 2^{1/p} A \Omega^{(1+\gamma)} \left(\frac{N^{[1-p(1+\gamma)]}}{p(1+\gamma)-1} \right)^{1/p}$$

$$\leq 2^{1/p} A\Omega^{(1+\gamma)} N^{[1-p(1+\gamma)/p]}. \tag{10}$$

Proof. In fact, we have

$$\left(\sum_{|k|>N} |f(k/\Omega)|^p \right)^{1/p} \leq A\left(2 \sum_{k=N+1}^{\infty} \left(\frac{k}{\Omega} \right)^{-(1+\gamma)p} \right)^{1/p}$$

$$\leq 2^{1/p} A\left(\int_N^{\infty} \left(\frac{x}{\Omega} \right)^{-(1+\gamma)p} dx \right)^{1/p}$$

$$\leq 2^{1/p} A\Omega^{(1+\gamma)} \left(\frac{N^{[1-(1+\gamma)p]}}{p(1+\gamma)-1} \right)^{1/p}$$

$$\leq 2^{1/p} A\Omega^{(1+\gamma)} N^{[1-(1+\gamma)p]/p}.$$

Proof of Theorem 1.2. From (2), we have

$$\left| f(t) - \sum_{k=-N}^{N} \left[\int_{k/\Omega-\sigma'_k}^{k/\Omega+\sigma''_k} u_k(t) f(t) dt \right] \sin c(\Omega t - k) \right|$$

$$= \left| f(t) - \sum_{k=-\infty}^{\infty} \left[\int_{k/\Omega-\sigma'_k}^{k/\Omega+\sigma''_k} u_k(t) f(t) dt \right] \sin c(\Omega t - k) \right.$$

$$+ \sum_{k=-\infty}^{\infty} \left[\int_{k/\Omega-\sigma'_k}^{k/\Omega+\sigma''_k} u_k(t) f(t) dt \right] \sin c(\Omega t - k)$$

$$\left. - \sum_{k=-N}^{N} \left[\int_{k/\Omega-\sigma'_k}^{k/\Omega+\sigma''_k} u_k(t) f(t) dt \right] \sin c(\Omega t - k) \right|$$

$$\leq \left| f(t) - \sum_{k=-\infty}^{\infty} \left[\int_{k/\Omega-\sigma'_k}^{k/\Omega+\sigma''_k} u_k(t) f(t) dt \right] \sin c(\Omega t - k) \right|$$

$$+ \left| \sum_{k \geq |N|} \left[\int_{k/\Omega-\sigma'_k}^{k/\Omega+\sigma''_k} u_k(t) f(t) dt \right] \sin c(\Omega t - k) \right|. \tag{11}$$

Using the Hölder's inequality, we obtain

$$\left| f(t) - \sum_{k=-\infty}^{\infty} \left[\int_{k/\Omega-\sigma'_k}^{k/\Omega+\sigma''_k} u_k(t) f(t) dt \right] \sin c(\Omega t - k) \right|$$

$$= \left| \sum_{k=-\infty}^{\infty} \left[f(k/\Omega) - \int_{k/\Omega-\sigma'_k}^{k/\Omega+\sigma''_k} u_k(t) f(t) dt \right] \sin c(\Omega t - k) \right|$$

$$\le \left(\sum_{k=-\infty}^{\infty} \left| f(k/\Omega) - \int_{k/\Omega-\sigma'_k}^{k/\Omega+\sigma''_k} u_k(t) f(t) dt \right|^p \right)^{1/p} \left(\sum_{k=-\infty}^{\infty} |\sin c(\Omega t - k)|^q \right)^{1/q}$$

$$\le \left(\sum_{k=-N}^{N} \left| f(k/\Omega) - \int_{k/\Omega-\sigma'_k}^{k/\Omega+\sigma''_k} u_k(t) f(t) dt \right|^p \right)^{1/p} \left(\sum_{k=-\infty}^{\infty} |\sin c(\Omega t - k)|^q \right)^{1/q}$$

$$+ \left(\sum_{|k|>N} \left| \int_{k/\Omega-\sigma'_k}^{k/\Omega+\sigma''_k} u_k(t) f(t) dt \right|^p \right)^{1/p} \left(\sum_{k=-\infty}^{\infty} |\sin c(\Omega t - k)|^q \right)^{1/q} \tag{12}$$

and

$$\left| \sum_{k\ge|N|} \left[\int_{k/\Omega-\sigma'_k}^{k/\Omega+\sigma''_k} u_k(t) f(t) dt \right] \sin c(\Omega t - k) \right|$$

$$\le \left(\sum_{|k|>N} \left| \int_{k/\Omega-\sigma'_k}^{k/\Omega+\sigma''_k} u_k(t) f(t) dt \right|^p \right)^{1/p} \left(\sum_{k\ge|N|} |\sin c(\Omega t - k)|^q \right)^{1/q}$$

$$\le \left(\sum_{|k|>N} \left| \int_{k/\Omega-\sigma'_k}^{k/\Omega+\sigma''_k} u_k(t) f(t) dt \right|^p \right)^{1/p} \left(\sum_{k=-\infty}^{\infty} |\sin c(\Omega t - k)|^q \right)^{1/q}, \tag{13}$$

where $1/p + 1/q = 1$.

Thus, using (11),(12),(13) we obtain

$$\left| f(t) - \sum_{k=-N}^{N} \left[\int_{k/\Omega-\sigma'_k}^{k/\Omega+\sigma''_k} u_k(t) f(t) dt \right] \sin c(\Omega t - k) \right|$$

$$\leq \left(\sum_{k=-N}^{N} \left| f(k/\Omega) - \int_{k/\Omega-\sigma'_k}^{k/\Omega+\sigma''_k} u_k(t) f(t) dt \right|^p \right)^{1/p} \left(\sum_{k=-\infty}^{\infty} |\sin c(\Omega t - k)|^q \right)^{1/q}$$

$$+ 2 \left(\sum_{|k|>N} \left| \int_{k/\Omega-\sigma'_k}^{k/\Omega+\sigma''_k} u_k(t) f(t) dt \right|^p \right)^{1/p} \left(\sum_{k=-\infty}^{\infty} |\sin c(\Omega t - k)|^q \right)^{1/q}. \quad (14)$$

By virtue of Lemma 2.1 we have

$$\left(\sum_{k=-\infty}^{\infty} |\sin c(\Omega t - k)|^q \right)^{1/q} \leq \left(2 + \frac{2}{\pi} + \frac{2}{\pi} \cdot \frac{1}{q-1} \right)^{1/q}. \quad (15)$$

Let N be a positive integer such that $N \geq 4$. Then we have $N \geq 4\sigma\Omega$. It follows that

$$\left(\sum_{k=-N}^{N} \left| f(k/\Omega) - \int_{k/\Omega-\sigma'_k}^{k/\Omega+\sigma''_k} u_k(t) f(t) dt \right|^p \right)^{1/p}$$

$$= \left(\sum_{k=-N}^{N} \left| \int_{k/\Omega-\sigma'_k}^{k/\Omega-\sigma/4} u_k(t) [f(k/\Omega) - f(t)] dt \right.\right.$$

$$+ \int_{k/\Omega-\sigma/4}^{k/\Omega+\sigma/4} u_k(t) [f(k/\Omega) - f(t)] dt$$

$$\left.\left. + \int_{k/\Omega+\sigma/4}^{k/\Omega+\sigma''_k} u_k(t) [f(k/\Omega) - f(t)] dt \right|^p \right)^{1/p}$$

$$\leq \left(\sum_{k=-N}^{N} \left| \int_{-\sigma'_k}^{-\sigma/4} u_k(t + k/\Omega) \omega(f, \sigma/2) dt + \int_{-\sigma/4}^{\sigma/4} u_k(t + k/\Omega) \omega(f, \sigma/4) dt \right.\right.$$

$$\left.\left. + \int_{\sigma/4}^{\sigma''_k} u_k(t + k/\Omega) \omega(f, \sigma/2) dt \right|^p \right)^{1/p}$$

$$\leq (2N+1)^{1/p} (2 - \alpha) \omega(f, \sigma/4)$$

$$\leq (2 + 1/4)^{1/p} N^{1/p} (2 - \alpha) \omega(f, \sigma/4)$$

$$\leq 1.5 N^{1/p} (2 - \alpha) \omega(f, \sigma/4), \quad (16)$$

where we use the inequality $\omega(f, \sigma/2) \le 2\omega(f, \sigma/4)$ in the second step.

Let $p \ge 2$ with $p\gamma \ge 2$. From Lemma 2.2 we know that

$$\left(\sum_{|k|>N} |f(k/\Omega)|^p \right)^{1/p} \le 2^{1/p} A\Omega^{(1+\gamma)} \left(\frac{N^{[1-p(1+\gamma)]}}{p(1+\gamma)-1} \right)^{1/p}$$

$$\le \sqrt{2} A\Omega^{(1+\gamma)} N^{[1-p(1+\gamma)/p]}. \tag{17}$$

Now, if $|k| \ge N+1$, $N \ge 4\sigma\Omega$ and $t \in [-\sigma/4, \sigma/4]$, we have

$$\left| t + \frac{k}{\Omega} \right| \ge -\frac{\sigma}{4} + \left| \frac{k}{\Omega} \right| \ge -\frac{|k|}{16\Omega} + \left| \frac{k}{\Omega} \right| \ge \frac{15|k|}{16\Omega}. \tag{18}$$

If $|k| \ge N+1$, $N \ge 4\sigma\Omega$ and $x \in [-\sigma/2, -\sigma/4] \cup [\sigma/4, \sigma/4]$, we have

$$\left| t + \frac{k}{\Omega} \right| \ge -\frac{\sigma}{2} + \left| \frac{k}{\Omega} \right| \ge -\frac{|k|}{8\Omega} + \left| \frac{k}{\Omega} \right| \ge \frac{7|k|}{8\Omega}. \tag{19}$$

Thus, using (10),(18) and (19) we obtain

$$\left(\sum_{|k|>N} \left| \int_{k/\Omega-\sigma_k'}^{k/\Omega+\sigma_k''} u_k(t) f(t)\, dt \right|^p \right)^{1/p}$$

$$\le A \left(\sum_{|k|>N} \left| \int_{k/\Omega-\sigma_k'}^{k/\Omega+\sigma_k''} \frac{u_k(t)\, dt}{t^{(1+\gamma)}} \right|^p \right)^{1/p}$$

$$\le A \left(\alpha \left(\frac{16}{15} \right)^{(1+\gamma)} + (1-\alpha) \left(\frac{8}{7} \right)^{(1+\gamma)} \right) \left(\sum_{|k|>N} \left| \frac{k}{\Omega} \right|^{-(1+\gamma)p} \right)^{1/p}$$

$$\le 2A \left(\alpha \left(\frac{16}{15} \right)^{(1+\gamma)} + (1-\alpha) \left(\frac{8}{7} \right)^{(1+\gamma)} \right) \left(\int_N^\infty \left(\frac{t}{\Omega} \right)^{-(1+\gamma)p} dt \right)^{1/p}$$

$$\le 2^{1/p} A \left(\alpha \left(\frac{16}{15} \right)^{(1+\gamma)} + (1-\alpha) \left(\frac{8}{7} \right)^{(1+\gamma)} \right) \Omega^{(1+\gamma)} \left(\frac{N^{1-(1+\gamma p)}}{p(1+\gamma)-1} \right)^{1/p}$$

$$\leq \sqrt{2}A\left(\alpha\left(\frac{16}{15}\right)^{(1+\gamma)} + (1-\alpha)\left(\frac{8}{7}\right)^{(1+\gamma)}\right)\Omega^{(1+\gamma)}N^{[1-(1+\gamma)p]/p}. \tag{20}$$

Suppose $\omega(f, \sigma/4) \leq \min\{64^{-1}, \Omega^{-1}\}$ and $\Omega \geq 2$, let

$$N = \left\lfloor [\omega(f, \sigma/4)^{-1/(1+\gamma)}\Omega] \right\rfloor > 8, \tag{21}$$

$$p = \ln N,\ q = \frac{\ln N}{\ln N - 1}, \tag{22}$$

where $\lfloor t \rfloor$ stands for the smallest integer that is greater than or equal to a given real number t. Then we see from (21) and (22) that

$$\omega(f, \sigma/4) \leq N^{-(1+\gamma)}\Omega^{(1+\gamma)}, \tag{23}$$

$$N^{1/p} = N^{(1/\ln N)} = e \tag{24}$$

$$\left(2 + \frac{2}{\pi} + \frac{2}{\pi}\cdot\frac{1}{q-1}\right)^{1/q} = \left(2 + \frac{2}{\pi}\ln N\right)^{1-(1/\ln N)} \leq \frac{2}{\pi}(\pi + \ln N). \tag{25}$$

So we have

$$\left(\sum_{k=-N}^{N}\left| f(k/\Omega) - \int_{k/\Omega-\sigma'_k}^{k/\Omega+\sigma''_k} u_k(t)f(t)dt \right|^p\right)^{1/p} \leq \frac{3}{2}e(2-\sigma)N^{-(1+\gamma)}\Omega^{(1+\gamma)}, \tag{26}$$

$$\left(\sum_{|k|>N}| f(k/\Omega) |^p\right)^{1/p} \leq 2^{1/2}A\Omega^{(1+\gamma)}\left(\frac{N^{[1-p(1+\gamma)]}}{p(1+\gamma)-1}\right)^{1/p}$$

$$\leq \sqrt{2}eA\Omega^{(1+\gamma)}N^{-(1+\gamma)} \tag{27}$$

and

$$\left(\sum_{|k|>N}\left|\int_{k/\Omega-\sigma'_k}^{k/\Omega+\sigma''_k} u_k(t)f(t)dt \right|^p\right)^{1/p}$$

$$\leq \sqrt{2}eA\Omega^{(1+\gamma)}\left(\alpha\left(\frac{16}{15}\right)^{(1+\gamma)} + (1-\alpha)\left(\frac{8}{7}\right)^{(1+\gamma)}\right)N^{-(1+\gamma)}. \tag{28}$$

Combining (14), (26), (27) and (28), we get

$$\left| f(t) - \sum_{k=-N}^{N} \left(\int_{-\sigma'_k}^{\sigma''_k} u_k(t) f(t + k/\Omega) dt \right) \sin c(\Omega t - k) \right|$$

$$\leq \left[2A \left(\alpha \left(\frac{16}{15} \right)^{(1+\gamma)} + (1 - \alpha) \left(\frac{8}{7} \right)^{(1+\gamma)} \right) + \frac{3\sqrt{2}}{4} (2 - \sigma) \right] \cdot \Omega^{(1+\gamma)} \cdot \frac{2\sqrt{2}e}{\pi} \cdot \frac{(\pi + \ln N)}{N^{1+\gamma}}$$

which completes the proof of Theorem 1.2.

Remark 1. If we assume that

$$A > \frac{3\sqrt{2}(2 - \sigma)}{8 \left[1 - \alpha \left(\frac{16}{15} \right)^{(1+\gamma)} - (1 - \alpha) \left(\frac{8}{7} \right)^{(1+\gamma)} \right]}$$

in Theorem 1.2 ,then our result will be better than Li's in [10].

Remark 2. If we suppose

$$u_k(t) = \frac{1}{\sigma'_k + \sigma''_k} \chi[t_k - \sigma'_k, \, t_k + \sigma''_k],$$

where χ denotes the characteristic function, then the error bound can be estimated as

$$K = \left[A \left(3 \left(\frac{16}{15} \right)^{(1+\gamma)} - \left(\frac{8}{7} \right)^{(1+\gamma)} \right) + \frac{3\sqrt{2}}{8} \right].$$

Acknowledgment

The author expresses his sincere gratitude to the referees, the editors and professor W. Sun for very valuable suggestions and carefully reading, which helped him to improve the paper.

References

[1] F. J. Beutler, On the truncation error of the cardinal sampling expansion signals, IEEE Trans. Inform. Theory IT-22 (1976), 568-573.

[2] J. L. Brown, Bounds for truncation error in sampling expansion of band-limited signal, IEEE Trans. Inform. Theory IT-15 (1969), 440-444.

[3] P. L. Butzer, W. Engels and U. Scheben, Magnitude of the truncation error in sampling expasions of bandlimited siagnals, IEEE Trans. Acout. Speech Signal Process, ASSP-30 (1982), 906-912.

[4] P. L. Butzer and J. Lei, Errors in truncated sampling series with measured sampled values for non-necessarily bandlimited functions, Funct. Approx. Comment. Math. 26 (1998), 18-32.

[5] P. L. Butzer and J. Lei, Errors in truncated sampling series with measured sampled values for non-necessarily bandlimited functions, Funct. Approx. 26 (1998), 25-39.

[6] P. L. Butzer and J. Lei, Approximation of signals using measured sampled values and error analysis, Commun. Appl. Anal. 4 (2000), 245-255.

[7] S. Cambanis, Truncation error bounds for the cardinal sampling series for bandlimited signals, IEEE Trans. Inform. Theory IT-28 (1982), 605-612.

[8] K. Gröchenig, Reconstruction algorithms in irregular sampling, Math. Comput. 59 (1992), 181-194.

[9] D. Jagerman, Bounds for truncation error of the sampling expansions, SIAM. J. Appl. Math. 14 (1996), 714-723.

[10] X. M. Li, Uniform bounds for sampling expansions, J. Approx. Theory 93 (1998), 110-113.

[11] Z. Song and X. Zhou, Error estimate on no-bandlimited random signals by local average, Lecture Notes in Computer Science 3991 (2006), 822-825.

[12] Z. Song and X. Zhou, Approximation of weak sense stationary stochastic processes from local averages, Science in China: Series A Math 50 (2007), 457-463.

[13] W. Splettstösser, R. L. Stens and G.Wilmes, On the approximation of the interpolating series of G. Valiron , Funct. Approx. Comment. Math. 11 (1980), 39-56.

[14] W. Sun and X. Zhou, Reconstruction of bandlimited function from local averages, Constr. Approx. 18 (2002), 205-222.

[15] W. Sun and X. Zhou, Reconstruction of bandlimited signals from local averages, IEEE Trans. Inform. Theory 48 (2002), 2955-2963.

[16] K. Yao and J. B. Thomas, On truncation error bounds for sampling represntations of band-limited signals, IEEE Trans. on Aerospace and Electronic Systems AES-2(6) (1996), 640-647.

■