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WRONSKIAN REPRESENTATION OF SOLUTIONS OF THE NLS EQUATION AND HIGHER PEREGRINE BREATHERS

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Abstract

This article contains two basic results. First, we derive a wronskian representation for the well known multi phase N-periodic solutions to the focusing NLS equation. Next, we perform a special passage to the limit when all the periods tend to infinity.

We claim that this provides a new systematic approach for constructing the so called higher Peregrine breathers labelled by the positive integer N. We give explicit representations of all higher Peregrine breathers for $N = 1$ until to $N = 6$ and speculate about further extensions.

1. Introduction

The nonlinear Schrödinger (NLS) equation plays a fundamental role in hydrodynamics, in particular, for waves in deep water in the context of the rogue waves or in nonlinear optics.

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The basic work on the subject is due to Zakharov in 1969 [12]. The first solution, actually called breather, was discovered by Peregrine in 1983 [11]; it was the simplest solution of NLS founded.

The second order Peregrine-like solution was first constructed by Akhmediev et al. in [1].

Same families of higher order were constructed in a series of articles by Akhmediev et al. [2, 3] using Darboux transformations.

Other Peregrine-like solutions were found for reduced self-induced transparency (SIT) integrable systems [10]. In [9], the N-phase quasi-periodic modulations of the plane waves solutions were constructed via appropriate degeneration of the finite gap periodic solutions to the NLS equation. These results in principle provided a way to construct multi-parametric families of multi-Peregrine solutions.

Very recently, it is shown in [6] that rational solutions of NLS equation can be written as a quotient of two wronskians by using modified version of [8], with some different reasoning; moreover, the link between quasi-rational solutions of the focusing NLS equation and the rational solution of the KP-I equation is established.

With this formulation, we recover as particular case, Akhmediev's quasi-rational solutions of NLS equation.

In this paper, we will give a new representation of the solutions of the NLS equation in terms of wronskians. The solutions take the form of a quotient of two wronskians of even order $2N$ of elementary functions depending on a certain number of parameters.

We will call these related solutions, solutions of NLS of order N.

Then, to get quasi-rational solutions of NLS equation, we take the limit when some parameter goes to 0.

For $N = 1$, we recover the well known Peregrine's solution [11] of the focusing NLS equation. For $N = 2, 3$, we recover Akhmediev's breathers. In the cases $N = 4$ to $N = 6$, we find explicit analytic solution of NLS with corresponding graphic of the modulus of the solution in the (x, t) coordinates.

Here, we give a new approach to get higher order Peregrine solutions different from all previous works.

2. Expression of Solutions of NLS Equation in Terms of Fredholm Determinant

2.1. Solutions of NLS equation in terms of θ functions

The solution of the NLS equation

$$iv_t + v_{xx} + 2|v|^2 v = 0, \tag{1}$$

is given in terms of truncated theta function by (see [9])

$$v(x, t) = \frac{\theta_3(x, t)}{\theta_1(x, t)} \exp(2it - i\varphi). \tag{2}$$

The functions $\theta_r(x, t)$, $(r = 1, 3)$ are the functions defined by

$$\theta_r(x, t) = \sum_{k \in \{0;\, 1\}^{2N}} g_{r,k}, \tag{3}$$

with $g_{r,k}$ given by

$$g_{r,k} = \exp\left\{ \sum_{\mu > \nu,\, \mu,\, \nu = 1}^{2N} \ln\left(\frac{\gamma_\nu - \gamma_\mu}{\gamma_\nu + \gamma_\mu}\right)^2 k_\mu k_\nu \right.$$

$$\left. + \left(\sum_{\nu=1}^{2N} i\kappa_\nu x - 2\delta_\nu t + (r-1)\ln\frac{\gamma_\nu - i}{\gamma_\nu + i} + \sum_{\mu=1,\, \mu\neq\nu}^{2N} \ln\left(\frac{\gamma_\nu + \gamma_\mu}{\gamma_\nu - \gamma_\mu}\right) + \pi i \epsilon_\nu \right) k_\nu \right\}. \tag{4}$$

The solutions depend on a certain number of parameters :

φ;

N parameters λ_j, satisfying the relations

$$0 < \lambda_j < 1, \quad \lambda_{N+j} = -\lambda_j, \quad 1 \le j \le N. \tag{5}$$

The terms ϵ_ν, $1 \leq \nu \leq 2N$ are arbitrary numbers equal to 0 or 1.

In the preceding formula, the terms κ_ν, δ_ν, and γ_ν are functions of the parameters λ_ν, $\nu = 1, \ldots, 2N$, and they are given by the following equations:

$$\kappa_\nu = 2\sqrt{1-\lambda_\nu^2}, \quad \delta_\nu = \kappa_\nu \lambda_\nu, \quad \gamma_\nu = \sqrt{\frac{1-\lambda_\nu}{1+\lambda_\nu}}. \tag{6}$$

We also note that

$$\kappa_{N+j} = \kappa_j, \quad \delta_{N+j} = -\delta_j, \quad \gamma_{N+j} = 1/\gamma_j, \quad j = 1, \ldots, N. \tag{7}$$

2.2. Relation between θ and Fredholm determinant

It was already mentioned in [9] that the function θ_r defined in (3) can be rewritten as a Fredholm determinant.

The expression given in [9] is different from which we need in the following. We need different choices of ϵ_ν. It is the crucial point to get quasi-rational solutions of NLS.

Here, we make the following choices:

$$\begin{aligned} \epsilon_\nu &= 0, \quad 1 \leq \nu \leq N, \\ \epsilon_\nu &= 1, \quad N+1 \leq \nu \leq 2N. \end{aligned} \tag{8}$$

The function θ_r defined in (3) can be rewritten with a summation in terms of subsets of $[1, \ldots, 2N]$

$$\theta_r(x, t) = \sum_{J \subset \{1, \ldots, 2N\}} \prod_{\nu \in J} (-1)^{\epsilon_\nu} \prod_{\nu \in J,\, \mu \notin J} \left| \frac{\gamma_\nu + \gamma_\mu}{\gamma_\nu - \gamma_\mu} \right|$$
$$\times \exp\left\{ \sum_{\nu \in J} i\kappa_\nu x - 2\delta_\nu t + x_{r,\nu} \right\},$$

with

$$x_{r,\nu} = (r-1)\ln \frac{\gamma_\nu - i}{\gamma_\nu + i}, \quad 1 \leq j \leq 2N, \tag{9}$$

in particular,

$$x_{r,j} = (r-1)\ln\frac{\gamma_j - i}{\gamma_j + i}, \quad 1 \le j \le N,$$

$$x_{r,N+j} = -(r-1)\ln\frac{\gamma_j - i}{\gamma_j + i} - (r-1)i\pi, \quad 1 \le j \le N. \tag{10}$$

We consider $A_r = (a_{\nu\mu})_{1\le\nu,\mu\le 2N}$ the matrix defined by

$$a_{\nu\mu} = (-1)^{\epsilon_\nu}\prod_{\lambda\neq\mu}\left|\frac{\gamma_\lambda + \gamma_\nu}{\gamma_\lambda - \gamma_\mu}\right|\exp(i\kappa_\nu x - 2\delta_\nu t + x_{r,\nu}). \tag{11}$$

Then $\det(I + A_r)$ has the following form:

$$\det(I + A_r) = \sum_{J\subset\{1,\dots,2N\}}\prod_{\nu\in J}(-1)^{\epsilon_\nu}\prod_{\nu\in J\ \mu\notin J}\left|\frac{\gamma_\nu + \gamma_\mu}{\gamma_\nu - \gamma_\mu}\right|\exp(i\kappa_\nu x - 2\delta_\nu t + x_{r,\nu}). \tag{12}$$

From the beginning of this section, $\tilde{\theta}$ has the same expression as in (12) so, we have clearly the equality

$$\theta_r = \det(I + A_r). \tag{13}$$

Then the solution of NLS equation takes the form

$$v(x, t) = \frac{\det(I + A_3(x, t))}{\det(I + A_1(x, t))}\exp(2it - i\varphi). \tag{14}$$

3. Expression of Solutions of NLS Equation in Terms of Wronskian Determinant

3.1. Link between Fredholm determinants and wronskians

We consider the following functions:

$$\phi_\nu^r(y) = \sin(\kappa_\nu x/2 + i\delta_\nu t - ix_{r,\nu}/2 + \gamma_\nu y), \quad 1 \le \nu \le N,$$

$$\phi_\nu^r(y) = \cos(\kappa_\nu x / 2 + i\delta_\nu t - ix_{r,\nu} / 2 + \gamma_\nu y), \quad N + 1 \le \nu \le 2N. \tag{15}$$

For simplicity, in this section, we denote them $\phi_\nu(y)$.

We use the following notations:

$$\Theta_\nu = \kappa_\nu x / 2 + i\delta_\nu t - ix_{r,\nu} / 2 + \gamma_\nu y, \quad 1 \le j \le 2N.$$

$W_r(y) = W(\phi_1, \ldots, \phi_{2N})$ is the wronskian

$$W_r(y) = \det[(\partial_y^{\mu-1}\phi_\nu)_{\nu,\mu\in[1,\ldots,2N]}]. \tag{16}$$

We consider the matrix $D_r = (d_{\nu\mu})_{\nu,\mu\in[1,\ldots,2N]}$ defined by

$$d_{\nu\mu} = (-1)^{\epsilon_\nu} \prod_{\lambda\neq\mu} \left|\frac{\gamma_\lambda + \gamma_\nu}{\gamma_\lambda - \gamma_\mu}\right| \exp(i\kappa_\nu x - 2\delta_\nu t + x_{r,\nu}),$$

$$1 \le \nu \le 2N, \quad 1 \le \mu \le 2N,$$

with

$$x_{r,\nu} = (r-1)\ln\frac{\gamma_\nu - i}{\gamma_\nu + i}.$$

Then, we have the following statement:

Theorem 3.1.

$$\det(I + D_r) = k_r(0) \times W_r(\phi_1, \ldots, \phi_{2N})(0), \tag{17}$$

where

$$k_r(y) = \frac{2^{2N}\exp(i\sum_{\nu=1}^{2N}\Theta_\nu)}{\prod_{\nu=2}^{2N}\prod_{\mu=1}^{\nu-1}(\gamma_\nu - \gamma_\mu)}.$$

Proof. We start to remove the factor $(2i)^{-1}e^{i\Theta_\nu}$ in each row ν in the wronskian $W_r(y)$ for $1 \le \nu \le 2N$.

Then

$$W_r = \prod_{\nu=1}^{2N} e^{i\Theta_\nu}(2i)^{-N}(2)^{-N} \times W_{r1}, \tag{18}$$

with

$$W_{r1} = \begin{vmatrix} (1-e^{-2i\Theta_1}) & i\gamma_1(1+e^{-2i\Theta_1}) & \cdots & (i\gamma_1)^{2N-1}(1+(-1)^{2N}e^{-2i\Theta_1} \\ (1-e^{-2i\Theta_2}) & i\gamma_2(1+e^{-2i\Theta_2}) & \cdots & (i\gamma_2)^{2N-1}(1+(-1)^{2N}e^{-2i\Theta_2} \\ \vdots & \vdots & \vdots & \vdots \\ (1-e^{-2i\Theta_{2N}}) & i\gamma_{2N}(1+e^{-2i\Theta_{2N}}) & \cdots & (i\gamma_{2N})^{2N-1}(1+(-1)^{2N}e^{-2i\Theta_{2N}} \end{vmatrix}.$$

The determinant W_{r1} can be written as

$$W_{r1} = \det(\alpha_{jk}e_j + \beta_{jk}),$$

where $\alpha_{jk} = (-1)^k(i\gamma_j)^{k-1}$, $e_j = e^{-2i\Theta_j}$, and $\beta_{jk} = (i\gamma_j)^{k-1}$, $1 \le j \le N$, $1 \le k \le 2N$,

$\alpha_{jk} = (-1)^{k-1}(i\gamma_j)^{k-1}$, $e_j = e^{-2i\Theta_j}$, and $\beta_{jk} = (i\gamma_j)^{k-1}$, $N+1 \le j \le 2N$, $1 \le k \le 2N$.

Denoting $U = (\alpha_{ij})_{i,\, j\in[1,\dots,2N]}$, $V = (\beta_{ij})_{i,\, j\in[1,\dots,2N]}$, the determinant of U is clearly equal to

$$\det(U) = (i)^{\frac{2N(2N-1)}{2}} \prod_{2N\ge l>m\ge 1} (\gamma_l - \gamma_m). \tag{19}$$

Then we use the following lemma:

Lemma 3.1. *Let* $A = (a_{ij})_{i,\, j\in[1,\dots,N]}$, $B = (b_{ij})_{i,\, j\in[1,\dots,N]}$, *and* $(H_{ij})_{i,\, j\in[1,\dots,N]}$, *the matrix formed by replacing the j-th row of A by the i-th row of B.*

Then

$$\det(a_{ij}x_i + b_{ij}) = \det(a_{ij}) \times \det(\delta_{ij}x_i + \frac{\det(H_{ij})}{\det(a_{ij})}). \tag{20}$$

Proof. For $\widetilde{A} = (\widetilde{a}_{ji})_{i,\, j\in[1,\dots,N]}$, the transposed matrix in cofactors of A, we have the well known formula $A \times^t \widetilde{A} = \det A \times I$.

So, it is clear that $\det(\widetilde{A}) = (\det(A))^{N-1}$.

The general term of the product $(c_{ij})_{i,\, j\in[1,\dots,N]} = (a_{ij}x_i + b_{ij})_{i,\, j\in[1,\dots,N]} \times (\widetilde{a}_{ji})_{i,\, j\in[1,\dots,N]}$ can be written as

$$\begin{aligned} c_{ij} &= \sum_{s=1}^{N} (a_{is}x_i + b_{is}) \times \widetilde{a}_{js} \\ &= x_i \sum_{s=1}^{N} a_{is}\widetilde{a}_{js} + \sum_{s=1}^{N} b_{is}\widetilde{a}_{js} \\ &= \delta_{ij} \det(A) x_i + \det(H_{ij}). \end{aligned}$$

We get

$$\det(c_{ij}) = \det(a_{ij}x_i + b_{ij}) \times (\det(A))^{N-1} = (\det(A))^{N} \times \det(\delta_{ij}x_i + \frac{\det(H_{ij})}{\det(A)}).$$

Thus $\det(a_{ij}x_i + b_{ij}) = \det(A) \times \det(\delta_{ij}x_i + \frac{\det(H_{ij})}{\det(A)})$.

□

Using the previous lemma's Equation (20), we get

$$\det(\alpha_{ij}e_i + \beta_{ij}) = \det(\alpha_{ij}) \times \det(\delta_{ij}e_i + \frac{\det(H_{ij})}{\det(\alpha_{ij})}),$$

where $(H_{ij})_{i,\, j\in[1,\dots,N]}$ is the matrix formed by replacing the j-th row of U by the i-th row of V defined previously.

We compute $\det(H_{ij})$ and we get

$$\det(H_{jk}) = (-1)^{E_k} (i)^{\frac{2N(2N-1)}{2}} \prod_{2N \geq l > m \geq 1,\, l \neq k,\, m \neq k} (\gamma_l - \gamma_m) \prod_{l \neq k} (\gamma_l + \gamma_j), \quad (21)$$

with $E_k = k - 1$ if $1 \leq k \leq N$ and $E_k = k$ if $N + 1 \leq k \leq 2N$.

We can simplify the quotient $q = \dfrac{\det(H_{jk})}{\det(\alpha_{jk})}$.

$$q = \frac{(-1)^{\epsilon(k)} \prod_{l \neq k} (\gamma_l + \gamma_j)}{\prod_{l \neq k} (\gamma_l - \gamma_k)}. \tag{22}$$

So $\det(\delta_{jk} e_j + \dfrac{\det(H_{jk})}{\det(\alpha_{jk})})$ can be expressed as

$$\det(\delta_{jk} e_j + \frac{\det(H_{jk})}{\det(\alpha_{jk})}) = \prod_{j=1}^{2N} e^{-2i\Theta_j} \det(\delta_{jk} + (-1)^{\epsilon(k)} \prod_{l \neq k} \left| \frac{\gamma_l + \gamma_j}{\gamma_l - \gamma_k} \right| e^{2i\Theta_j}).$$

Then the wronskian

$$W_r(\phi_1, \dots, \phi_N)(0),$$

can be written as

$$\prod_{j=1}^{2N} e^{i\Theta_j|_{y=0}} (2)^{-2N} (i)^{\frac{2N(2N-2)}{2}} \prod_{j=2}^{2N} \prod_{i=1}^{j-1} (\gamma_j - \gamma_i) \prod_{j=1}^{2N} e^{-2i\Theta_j|_{y=0}} \det(I + D_r).$$

It follows that

$$\det(I + D_r) = \frac{e^{i\sum_{j=1}^{2N} \Theta_j|_{y=0}} 2^{2N}}{\prod_{j=2}^{2N} \prod_{i=1}^{j-1} (\gamma_j - \gamma_i)} W_r(\phi_1, \dots, \phi_{2N})(0)$$

$$= k_r(0) W_r(\phi_1, \dots, \phi_{2N})(0). \tag{23}$$

□

So, the solution of NLS equation takes the form

$$v(x, t) = \frac{W_3(0)}{W_1(0)} \exp(2it - i\varphi). \tag{24}$$

3.2. Wronskian representation of solutions of NLS equation

From the previous section, we get the following result:

Theorem 3.2. *The function v defined by*

$$v(x,\ t) = \frac{W_3(0)}{W_1(0)} \exp(2it - i\varphi) \tag{25}$$

is solution of the NLS equation (1)

$$iv_t + v_{xx} + 2|v|^2 v = 0.$$

Remark 3.1. In formula (25), $W_r(y)$ is the wronskian defined in (16); the functions ϕ_ν^r are given by (15); κ_ν, δ_ν, γ_ν are defined by (6); and λ_ν are arbitrary parameters given by (5).

4. Construction of Quasi-rational Solutions of NLS Equation

4.1. Method to get quasi-rational solutions of NLS equation

In the following, we show how we can obtain quasi-rational solutions of NLS equation by a simple limiting procedure.

For simplicity, we denote d_j the term $\frac{c_j}{\sqrt{2}}$.

We consider the parameter λ_j written in the form

$$\lambda_j = 1 - \epsilon^2 c_j^2, \quad 1 \leq j \leq N. \tag{26}$$

When ϵ goes to 0, we realize limited expansions at order p, for $1 \leq j \leq N$, of the terms

$$\kappa_j = 4d_j\epsilon(1 - \epsilon^2 d_j^2)^{1/2}, \quad \delta_j = 4d_j\epsilon(1 - 2\epsilon^2 d_j^2)(1 - \epsilon^2 d_j^2)^{1/2},$$

$$\gamma_j = d_j\epsilon(1 - \epsilon^2 d_j^2)^{-1/2}, \quad x_{r,j} = (r-1)\ln\frac{1 + i\epsilon d_j(1 - \epsilon^2 d_j^2)^{-1/2}}{1 - i\epsilon d_j(1 - \epsilon^2 d_j^2)^{-1/2}},$$

$$\kappa_{N+j} = 4d_j\epsilon(1-\epsilon^2 d_j^2)^{1/2}, \quad \delta_{N+j} = -4d_j\epsilon(1-2\epsilon^2 d_j^2)(1-\epsilon^2 d_j^2)^{1/2},$$

$$\gamma_{N+j} = 1/(d_j\epsilon)(1-\epsilon^2 d_j^2)^{1/2}, \quad x_{r,N+j} = (r-1)\ln\frac{1-i\epsilon d_j(1-\epsilon^2 d_j^2)^{-1/2}}{1+i\epsilon d_j(1-\epsilon^2 d_j^2)^{-1/2}}.$$

For example, the expansions at order 1 gives

$$\kappa_j = 4d_j\epsilon + O(\epsilon^2),\ \gamma_j = d_j\epsilon + O(\epsilon^2), \quad \delta_j = 4d_j\epsilon + O(\epsilon^2),$$

$$x_{r,j} = (r-1)(2id_j\epsilon + O(\epsilon^2)),$$

$$\kappa_{N+j} = 4d_j\epsilon + O(\epsilon^2), \quad \gamma_{N+j} = 1/(d_j\epsilon) - (d_j\epsilon)/2 + O(\epsilon^2),$$

$$\delta_{N+j} = -4d_j\epsilon + O(\epsilon^2),$$

$$x_{r,N+j} = -(r-1)(2id_j\epsilon + O(\epsilon^2)),$$

$$1 \le j \le N.$$

We choose the following notations:

$$B_j = \kappa_j(x-x_{0j})/2 + i\delta_j(t-t_{0j}) - ix_j/4, \text{ and}$$

$$B'_j = \kappa_j(x-x_{0j})/2 - i\delta_j(t-t_{0j}) + ix_j/4.$$

Then, we realize limited expansions at order p in ϵ of the functions $\phi_j^r(0)$ and $\phi_{N+j}^r(0)$, for $1 \le j \le N$.

$$\phi_j^1(0) = \sin(B_j + ix_j/4) = P_j + O(\epsilon^{p+1}),$$

$$\phi_j^3(0) = \sin(B_j - ix_j/4) = Q_j + O(\epsilon^{p+1}),$$

$$\phi_{N+j}^1(0) = \cos(B'_j - ix_j/4) = P'_j + O(\epsilon^{p+1}),$$

$$\phi_{N+j}^3(0) = \cos(B'_j + ix_j/4) = Q'_j + O(\epsilon^{p+1}).$$

Then the function v defined by

$$v(x,\, t) = \exp(2it - i\varphi) \lim_{\epsilon \to 0} \frac{W_3(0)}{W_1(0)} \tag{27}$$

is a quasi-rational solution of the NLS equation (1)

$$iv_t + v_{xx} + 2|v|^2 v = 0.$$

Remark 4.1. In formula (27), $W_r(y)$ is the wronskian defined in (16); the functions ϕ_ν^r are given by (15); κ_ν, δ_ν, γ_ν are defined by (6); and λ_ν are arbitrary parameters given by (5).

In the following, we apply this method to get Peregrine breathers of order 1, until to 6.

4.2. Quasi-rational solutions of order *N*

To compare our solutions of NLS equation written in the context of hydrodynamic (1) with recent studies in fiber optics, we can make the following changes of variables:

$$t \to X/2,$$

$$x \to T. \tag{28}$$

The Equation (1) becomes

$$iu_x + \frac{1}{2} u_{tt} + u|u|^2 = 0. \tag{29}$$

We give all the solutions for (1), but the reader can easily get the analogues for (29) by using the formulas (28).

In the following, the solution of NLS equation will be written as

$$v(x,\, t) = \frac{N(x,\, t)}{D(x,\, t)} \exp(2it).$$

We will give in each case the analytic expression of the polynomials *N* and *D*.

4.2.1. Case $N = 1$

If we consider the case $N = 1$, from (27), we realize an expansion at order 1 of W_3 and W_1 in ϵ. We have $N(x, t) = -4x^2 - 16t^2 + 16it + 3$ and $D(x, t) = 4x^2 + 16t^2 + 1$, i.e.,

$$v(x, t) = \frac{-4x^2 - 16t^2 + 16it + 3}{4x^2 + 16t^2 + 1}\exp(2it).$$

If we make the preceding change of variable (28), we get exactly first order Akhmediev's solution (see [3]).

In particular,

$$v(x, 0) = \frac{3 - 4x^2}{1 + 4x^2}.$$

We represent here the modulus of v in function of $x \in [-5; 5]$ and $t \in [-5; 5]$ by the Figure 1. The maximum of modulus of v is equal to 3.

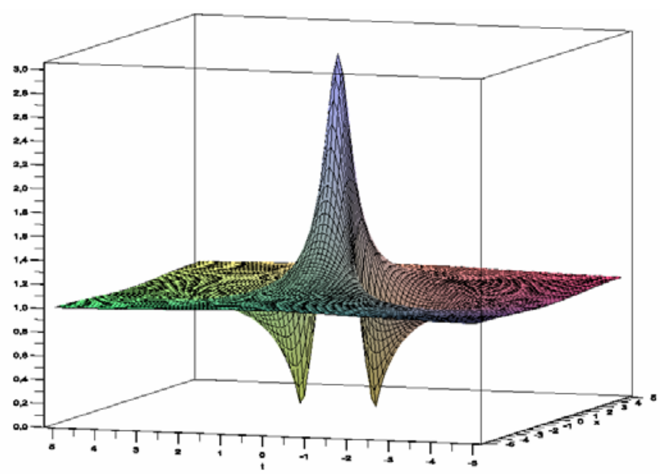

Figure 1. Solution to NLS equation for $N = 1$.

4.2.2. Case $N = 2$

In the case $N = 2$, we realize an expansion at order 3 in ϵ. Polynomials N and D are given by

$$N(x,\, t) = 64x^6 - (144 + 768it - 768t^2)x^4 - (180 + 6144it^3 + 5760t^2 - 1152it$$

$$- 3072t^4)x^2 + 45 + 720it - 1536it^3 - 12288it^5 + 4096t^6$$

$$- 8448t^4 - 1872t^2;$$

and

$$D(x,\, t) = 64x^6 + (48 + 768t^2)x^4 + (108 - 1152t^2 + 3072t^4)x^2 + 9$$

$$+ 4096t^6 + 6912t^4 + 1584t^2.$$

If we make the preceding changes of variables defined by (28), it can be reduced exactly at the second order Akhmediev's solution (see [3]).

In particular,

$$v(x,\, 0) = \frac{45 - 180x^2 - 144x^4 + 64x^6}{9 + 108x^2 + 48x^4 + 64x^6}.$$

We represent here the modulus of v in function of $x \in [-5;\, 5]$ and $t \in [-5;\, 5]$ in Figure 2. The maximum of modulus of v is equal to 5.

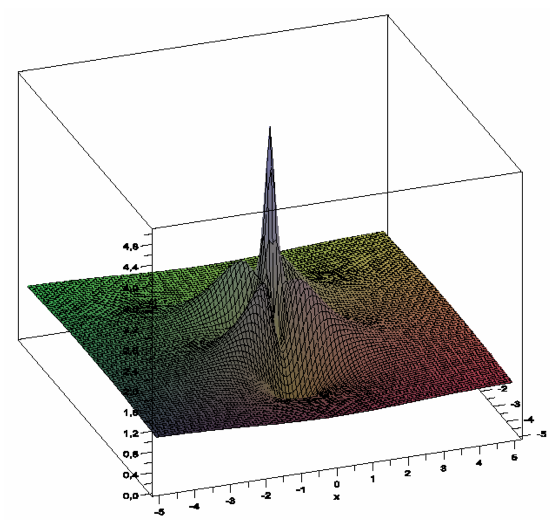

Figure 2. Solution to NLS equation for $N = 2$.

4.2.3. Case $N = 3$

In the case $N = 3$, we make an expansion at order 5 in ϵ. We give expressions of polynomials N and D.

$$N(x, t) = -4096x^{12} + (18432 + 98304it - 98304t^2)x^{10}$$
$$+ (1843200t^2 + 1966080it^3 - 368640it - 983040t^4 + 57600)x^8$$
$$+ (22609920t^4 - 13762560it^3 - 5242880t^6 - 921600it$$
$$+ 172800 - 2027520t^2 + 15728640it^5)x^6 + (-226800$$
$$+ 62914560it^7 - 691200t^2 - 2073600it - 15728640t^8$$
$$- 9216000t^4 - 82575360it^5 + 106168320t^6 - 11059200it^3)x^4$$
$$+ (-18403200t^2 + 212336640t^8 - 25165824t^{10} + 85708800t^4$$
$$- 38707200it^3 - 56033280t^6 + 125829120it^9 + 1814400it$$
$$+ 168099840it^5 - 94371840it^7 - 113400)x^2 + 14175$$
$$- 83808000t^4 - 3801600it^3 + 100663296it^{11} + 157286400it^9$$
$$+ 144703488t^{10} - 2268000t^2 - 342097920it^7 - 16777216t^{12}$$
$$+ 533790720t^8 - 236666880it^5 + 152616960t^6 + 453600it;$$

and

$$D(x, t) = 4096x^{12} + (98304t^2 + 6144)x^{10} + (-368640t^2 + 34560$$
$$+ 983040t^4)x^8 + (-2949120t^4 + 149760 + 5242880t^6$$
$$+ 552960t^2)x^6 + (3456000t^2 + 15728640t^8 + 54000$$
$$- 5529600t^4 + 3932160t^6)x^4 + (25165824t^{10} + 80179200t^4$$
$$+ 221184000t^6 + 70778880t^8 - 2332800t^2 + 48600)x^2$$

$$+\ 2025 + 36806400t^4 + 1490400t^2 + 62668800t^6$$

$$+ 244776960t^8 + 132120576t^{10} + 16777216t^{12}.$$

The solution with initial condition $t = 0$ takes the form

$v(x,\ 0) =$

$$\frac{14175 - 113400x^2 - 226800x^4 + 172800x^6 + 57600x^8 + 18432x^{10} - 4096x^{12}}{2025 + 48600x^2 + 54000x^4 + 149760x^6 + 34560x^8 + 6144x^{10} + 4096x^{12}}.$$

We get the following graphic for the modulus of v in function of $x \in [-5;\ 5]$ and $t \in [-5;\ 5]$ given by Figure 3. The maximum of modulus of v is equal to 7.

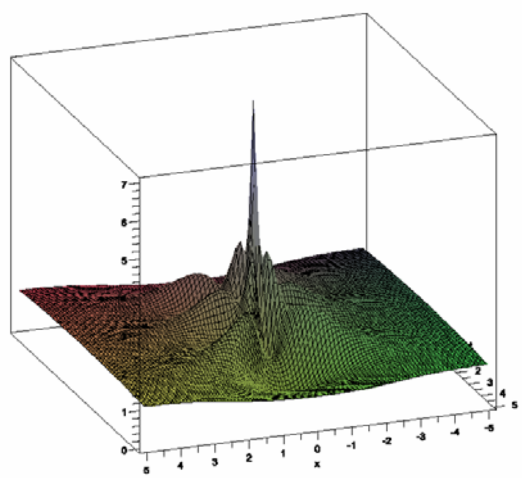

Figure 3. Solution to NLS equation for $N = 3$.

If we make the preceding changes of variables defined by (28), we still recover the solution given recently by Akhmediev et al. [3].

The results presented here are in accordance with the preceding work [6]; here, we give moreover an explicit expression of the solutions of NLS equation for the order 3.

4.3. Case $N = 4$

For this case, we realize an expansion at order 7 in ϵ. The analytical expression of polynomials N and D being too long, we give only in the Appendix. We recover a result of Akhmediev formulated in [3] in the case, where $x = 0$. Here, we give the complete solution in x and t.

We get the following graphic for the modulus of v in function of $x \in [-5; 5]$ and $t \in [-5; 5]$. The maximum of modulus of v is equal to 9. It is given in Figure 4.

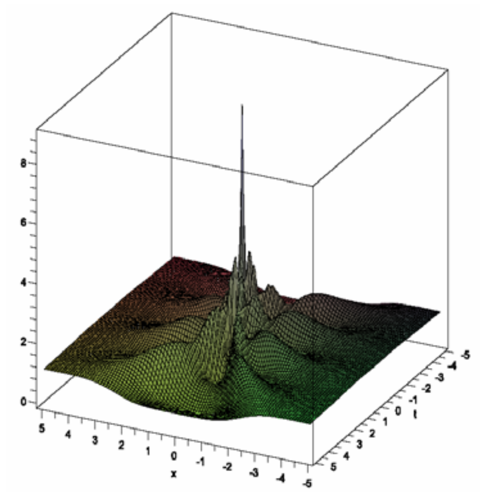

Figure 4. Solution to NLS equation for $N = 4$.

4.4. Case $N = 5$

In the case $N = 5$, we realize an expansion at order 9 in ϵ. Analytical expression of v is given in the Appendix. Here, we obtain the Akhmediev's breather of order 5. Again, we can note the presence of $N(N-1)-1$ local maximums; the global maximum of $|v|$ is equal to 11. We represent the modulus of v in the (x, t) coordinates and we obtain

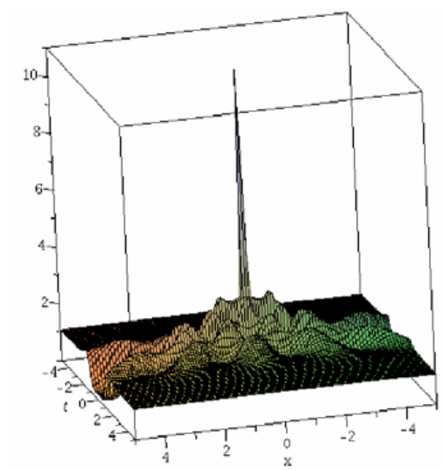

Figure 5. Solution to NLS equation for $N = 5$.

4.5. Case $N = 6$

In the case $N = 6$, we realize an expansion at order 11 in ϵ. We give analytical expression of v in the Appendix.

We get the following plot for the modulus of v in function of (x, t). We recognize again $N(N + 1) - 1$ local maximums and the global maximum is equal to 13.

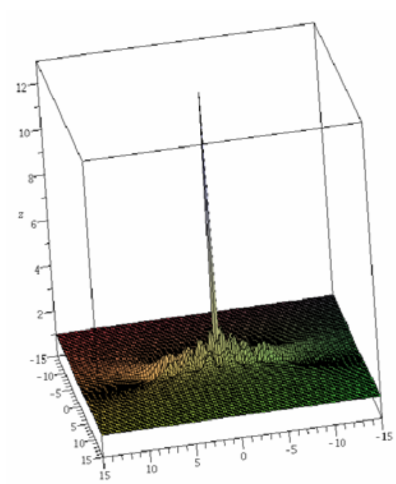

Figure 6. Solution to NLS equation for $N = 6$.

5. Conclusion

Here is given a new formulation of solutions of NLS equation that provides as particular case, Akhmediev's solutions. These solutions expressed as a quotient of wronskians are similar to a previous result of Eleonski et al. [8]. It has been rewritten recently and their formulation can also be seen as a quotient of two wronskians (see [6]); the method describe in [6] can be also used to get solutions of NLS equation, but it is more complicated to find explicit solution in x and t; the computation gives complicated results. We can make the following conjecture: at the order N, we get with this method a quasi-rational solution v quotient of two polynomials of degree $N(N+1)$.

There is $N(N+1)-1$ local maximum for the modulus and the global maximum modulus is equal to $2N+1$.

This method described in the present paper provides a powerful tool to get explicitly solutions in analytical form. This new formulation gives an infinite set of non singular solution of NLS equation.

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Appendix

Analytical expressions of solutions to NLS equation for $N = 4, 5, 6$

The solutions of NLS equation can be written as

$$v(x, t) = \frac{N(x, t)}{D(x, t)} \exp(2it - i\varphi).$$

We define N and D in the following cases:

(1) Case $N = 4$

$$\begin{aligned} N(x, t) = & -1048576x^{20} + (7864320 + 41943040it - 41943040t^2)x^{18} \\ & + (44236800 - 283115520it + 1509949440it^3 - 754974720t^4 \\ & + 1415577600t^2)x^{16} + (324403200 - 4246732800t^2 \\ & + 24159191040it^5 + 37748736000t^4 - 1415577600it \\ & - 25165824000it^3 - 8053063680t^6)x^{14} \\ & + (13212057600it^3 + 225485783040it^7 - 493250150400it^5 \\ & - 9083289600it + 465064427520t^6 - 10734796800t^2 \\ & - 56371445760t^8 - 235615027200t^4 + 1354752000)x^{12} \end{aligned}$$

$$+ (-3223742054400t^6 + 1352914698240it^9$$

$$- 4396972769280it^7 - 138726604800it^3$$

$$- 74008166400t^2 - 6096384000 + 9909043200t^4$$

$$+ 3297729576960t^8 + 10886735462 40it^5$$

$$- 270582939648t^{10} - 32514048000it)x^{10}$$

$$+ (14543833006080t^{10} - 402554880000it^3$$

$$- 9398592000 - 804722688000t^2$$

$$+ 5411658792960it^{11} + 10477161676800t^6$$

$$+ 12049396531200it^7 - 1086898176000t^4$$

$$- 901943132160t^{12} - 18232639488000t^8$$

$$- 20857434931200it^9 - 5588700364800it^5$$

$$+ 121927680000it)x^8 + (48197586124800it^9$$

$$+ 16161649459200t^6 - 2061584302080t^{14}$$

$$- 19920273408000t^4 + 47933344972800t^8$$

$$+ 14431090114560it^{13} - 19074908160000it^5$$

$$+ 7153090560000it^3 - 36923296972800t^{10}$$

$$+ 150377472000it - 11430720000$$

$$+ 40587440947200t^{12} - 52312701665280it^{11}$$

$$+ 678730752000t^2 + 12472182374400it^7)x^6$$

$$+ (-98297708544000it^{9} + 273331047628800t^{10}$$
$$+ 24739011624960it^{15} - 207309570048000t^{6}$$
$$+ 350542080000t^{2} + 70490456064000it^{5}$$
$$+ 52425444556800t^{12} + 291375415296000t^{8}$$
$$- 3092376453120t^{16} + 137168640000it$$
$$+ 9644670000 - 54116587929600it^{13}$$
$$- 344240160768000it^{7}$$
$$+ 37205154201600it^{11} + 69578470195200t^{14}$$
$$+ 3170119680000it^{3} + 3292047360000t^{4})x^{4}$$
$$+ (24739011624960it^{17} + 1181263645900800t^{12}$$
$$+ 66486093742080t^{16} - 1542243483648000t^{10}$$
$$- 1290950148096000it^{9} + 206158430208000it^{15}$$
$$+ 330497733427200t^{14} + 1311675120000t^{2}$$
$$- 19203609600000t^{4} - 73156608000000it^{5}$$
$$- 77157360000it - 202937204736000it^{13}$$
$$+ 546681913344000it^{7} - 2748779069440t^{18}$$
$$+ 6958006272000t^{6} - 2693991392870400it^{11}$$
$$- 1756650405888000t^{8} + 4511324160000it^{3}$$
$$+ 2679075000)x^{2} - 200930625 + 1099511627776t^{20}$$

$$+ 1883933717299200it^{13} - 26800595927040t^{18}$$

$$+ 569274531840000it^{9} - 379138238054400t^{16}$$

$$- 395779571712000it^{7} - 796770120499200t^{14}$$

$$- 10995116277760it^{19} + 832835262873600t^{12}$$

$$- 53014155264000it^{5} + 523098390528000t^{10}$$

$$- 63909113364480it^{17} + 963558457344000t^{8}$$

$$+ 369538986147840it^{15} + 48039505920000t^{6}$$

$$+ 10716300000it - 11341179360000t^{4}$$

$$- 240045120000it^{3} - 87873660000t^{2}$$

$$+ 806992478208000it^{11};$$

$$D(x, t) = 1048576x^{20} + (41943040t^{2} + 2621440)x^{18} + (283115520t^{2}$$

$$+ 754974720t^{4} + 26542080)x^{16} + (471859200t^{2}$$

$$+ 8053063680t^{6} + 265420800 - 7549747200t^{4})x^{14}$$

$$+ (6606028800t^{4} + 7431782400t^{2} + 56371445760t^{8}$$

$$- 70464307200t^{6} + 1741824000)x^{12} + (270582939648t^{10}$$

$$+ 49235558400t^{2} + 264241152000t^{6} + 1683763200$$

$$+ 188271820800t^{4} - 253671505920t^{8})x^{10}$$

$$+ (4280706662400t^{8} + 338228674560t^{10} - 128894976000t^{2}$$

$$- 594542592000t^{6} + 4463424000 + 1532805120000t^{4}$$

$$
\begin{aligned}
&+901943132160t^{12})x^8+(8382528000+134120448000t^2\\
&+68966940672000t^8+2061584302080t^{14}\\
&-2536095744000t^4+42658430976000t^6\\
&+6313601925120t^{12}+31962609745920t^{10})x^6\\
&+(161269678080000t^8+46039891968000t^6\\
&-365783040000t^4+332943851520000t^{10}\\
&+123453466214400t^{12}+1786050000+23192823398400t^{14}\\
&+320060160000t^2+3092376453120t^{16})x^4\\
&+(15545779200000t^4+158993694720000t^6\\
&+241591910400000t^{14}-417368899584000t^{10}\\
&-58785398784000t^8+893025000+2748779069440t^{18}\\
&+38654705664000t^{16}-122880240000t^2\\
&+296795661926400t^{12})x^2+22325625+46437300000t^2\\
&+1053529473024000t^{12}+1099511627776t^{20}\\
&+190374425395200t^{16}+25426206392320t^{18}\\
&+3594961440000t^4+435233046528000t^8\\
&+17801441280000t^6+592866548121600t^{14}\\
&+727066135756800t^{10}.
\end{aligned}
$$

(**2**) **Case** $N = 5$

$$
\begin{aligned}
N(x, t) = {} & 3593187392256000000 + 6898919793131520000000x^6 \\
& - 35069508948418560000000t^2 \\
& + 100162094774353920000000x^8 \\
& - 113234270204598681600000 0t^4 \\
& - 1888946593147858085 4784t^{30} \\
& + 938022663299058892800 00000t^6 \\
& - 4024369879326720000000x^4 \\
& - 71863747845120000000x^2 \\
& + 605412580955104110182400000t^8 \\
& - 17592186044416x^{30} \\
& + 123919598989086405584486400t^{22} \\
& + 2970660206865672371699712 00t^{24} \\
& + 382345664415777876344832 00t^{26} \\
& + 10448235843349090035302 40t^{28} \\
& - 4564646798971790373263769600t^{20} \\
& - 20503770921695624558542848000t^{18} \\
& - 217051957827992241045504 00000t^{16} \\
& - 16464595962633925070684160000t^{14} \\
& + 9740812251630034401361920000t^{12}
\end{aligned}
$$

$+ 9074203500156029396582400000t^{10}$

$+ 238654152769536000x^{22}$

$+ 1062519926119464960000x^{10}$

$- 2506043399602176000000x^{12}$

$- 1710339224135270400000x^{14}$

$- 498376992463257600000x^{16}$

$+ 81262980499310844995174400t^{24}x^{2}$

$- 15139622816317440000x^{18}$

$+ 1024508184389222400x^{20} + 22079567875276800x^{24}$

$+ 1731730813747200x^{26} + 197912092999680x^{28}$

$- 6386388776384723297501184000t^{18}x^{2}$

$+ 416086906642894976502988800t^{20}x^{4}$

$- 1190425059366988941361152000t^{20}x^{2}$

$+ 26260842509408148022886400t^{20}x^{6}$

$+ 3904806785522837886074880t^{26}x^{2}$

$+ 6779178447088260218880000t^{22}x^{6}$

$+ 69481736812335318486220800t^{22}x^{4}$

$+ 530962203922070260993228800t^{22}x^{2}$

$+ 4681991171778313432596480t^{20}x^{8}$

$$+\ 7373349604926318 18240t^8x^{20}$$

$$-\ 151886923289069617 15200t^8x^{18}$$

$$+\ 917409474044762259456 00t^8x^{16}$$

$$-\ 904462223250397593 6000t^8x^{14}$$

$$-\ 120069604459624896921 6000t^8x^{12}$$

$$-\ 326972085024576786923 52000t^8x^{10}$$

$$+\ 118510986917599437127 680000t^8x^8$$

$$-\ 262466757462293993226 240000t^8x^6$$

$$-\ 332595057410306998272 000000t^8x^4$$

$$+\ 209932781297876210810 8800000t^8x^2$$

$$+\ 54262746010280263680t^6x^{22}$$

$$-\ 1016318179971956736000t^6x^{20}$$

$$+\ 4368616543070296473600t^6x^{18}$$

$$-\ 11325789671913632563200t^6x^{16}$$

$$-\ 138603785986070544384000t^6x^{14}$$

$$-\ 1861193095820391481344000t^6x^{12}$$

$$+\ 9309545291393500446720000t^6x^{10}$$

$$-\ 74683091121266176819200000t^6x^6$$

$$-\ 1007192226424541518233360t^{22}x^8$$

$$- 1342922968566055357644480t^{24}x^{6}$$

$$- 1239621201753281868595 20t^{26}x^{4}$$

$$- 70835497243044678205440t^{28}x^{2}$$

$$- 55395572453349783502848t^{20}x^{10}$$

$$- 7419049882145060290560t^{16}x^{14}$$

$$- 1854762470536265072640t^{14}x^{16}$$

$$- 360648258159829319680t^{12}x^{18}$$

$$- 54097238723974397952t^{10}x^{20}$$

$$- 6147413491360727040t^{8}x^{22} - 512284457613393920t^{6}x^{24}$$

$$- 29554872554618880t^{4}x^{26} - 1055531162664960t^{2}x^{28}$$

$$- 1575925981913153343062016000 0t^{12}x^{2}$$

$$- 14071308503010055815168000 00t^{12}x^{4}$$

$$- 316865583183079637463859200 0t^{12}x^{6}$$

$$- 12392443143241062993100800 0t^{12}x^{8}$$

$$+ 48671270548965163008000 00t^{6}x^{2}$$

$$+ 1366020980652248137728000 00t^{6}x^{4}$$

$$- 23081488522229076459520t^{18}x^{12}$$

$$+ 6624225796869099985305600t^{24}x^{4}$$

$$- 2389036865684949172224000 0t^{6}x^{8}$$

$$- 5392376344402780697395200000t^{18}x^4$$

$$+ 1999402165839123022086144000t^{18}x^6$$

$$- 75403228129108930068480 0t^{18}x^8$$

$$+ 230903660178145491812352 0t^{18}x^{10}$$

$$- 10050849128814676725989376000t^{16}x^4$$

$$- 1234834967472296668692480 0t^{16}x^6$$

$$+ 7246872719755062584279040 0t^{16}x^8$$

$$- 549760351264624575774720 0t^{16}x^{10}$$

$$+ 83792461457380652089344 0t^{16}x^{12}$$

$$+ 11166360257961750171746304000t^{16}x^2$$

$$+ 659914789640905477324800 0t^{14}x^8$$

$$+ 2212234700166880171130880 0t^{14}x^{10}$$

$$- 2792221195313134708981760 0t^{14}x^{12}$$

$$+ 226851717550204728115200t^{14}x^{14}$$

$$- 13607713531342733967360 0t^{10}x^{16}$$

$$+ 685578467674983235584 0t^{10}x^{18}$$

$$+ 7607150377746091632230400 00t^{10}x^6$$

$$- 5741101174499905948876800 00t^{10}x^8$$

$$+ 13694342076892712219443200t^{10}x^{10}$$

$$- 77272808594707429982 2080000t^{14}x^2$$

$$- 603468237422487586996 2240000t^{14}x^4$$

$$- 33859862540517161419407 36000t^{14}x^6$$

$$+ 694469841659962526269 4400000t^{10}x^2$$

$$- 100487493230237224796160000t^{10}x^4$$

$$- 30603590263628837683 2000t^{10}x^{12}$$

$$+ 90845412599916857917 4400t^{10}x^{14}$$

$$- 29749789894603846975 48800t^{12}x^{10}$$

$$+ 53828431247115628314 62400t^{12}x^{12}$$

$$- 77636157860355495690 2400t^{12}x^{14}$$

$$+ 45878619763985979801 600t^{12}x^{16}$$

$$- 217311652821860352000t^2x^{18}$$

$$- 152465910169337856 00t^2x^{20}$$

$$- 1516996192842547200t^2x^{22}$$

$$- 297857699964518400t^2x^{24} + 55415386039910400t^2x^{26}$$

$$- 82327109531369472000000t^2x^2$$

$$- 45686179963404288000000t^2x^4$$

$$- 11896060591975956480 0000t^2x^6$$

$$+ 18744779875905699840 0000t^2x^8$$

$+ 5495965551886860288000t^{2}x^{10}$

$- 73112754300597043200000t^{4}x^{4}$

$+ 76578003509443559424000000t^{4}x^{6}$

$+ 14699629392703782912000000t^{4}x^{8}$

$+ 10246429600603176960000t^{4}x^{10}$

$+ 54136427341317931008000t^{4}x^{12}$

$- 683212289858548531200000t^{4}x^{14}$

$+ 28268010265093079040000000t^{4}x^{2}$

$+ 1536360346766278656000t^{2}x^{12}$

$+ 787737694754045952000t^{2}x^{14}$

$- 231672666464767180800t^{2}x^{16}$

$- 639415843480299110400t^{4}x^{16}$

$- 30784112810577100800t^{4}x^{18}$

$+ 2763842378740531200t^{4}x^{20}$

$- 341912931866247168000t^{4}x^{22}$

$+ 24752205764493312000t^{4}x^{24}$

$- (447473670587702483248742400 0i)t^{21}$

$- (973325387811016309682995200i)t^{23}$

$- (483684697659108669102489 60i)t^{25}$

$$+ (2052227774063702546841600000i)t^{11}$$

$$+ (51116504404320269107200000 0i)t^{9}$$

$$- 1702429528194885353472000000i)t^{7}$$

$$- (7916247049317226905600000i)t^{5}$$

$$- (14104458243735552000000i)t^{3}$$

$$+ (287454991380480000000i)t$$

$$+ (14624828633331484330033152000i)t^{15}$$

$$+ (4387979219806808916885504000i)t^{17}$$

$$- (8783473670339729855348736000i)t^{19}$$

$$+ (59109745109237760i)t^{3}x^{26}$$

$$- (4194543234223964160000000i)t^{3}x^{2}$$

$$- (16025935163896627200000i)tx^{6}$$

$$+ (421471208596115835322368 0i)t^{27}$$

$$+ (2163889548958975918080i)t^{11}x^{18}$$

$$- (19306349665407866045988864 00i)t^{21}x^{2}$$

$$- (1003657369091416925405184 00i)t^{21}x^{4}$$

$$- (6732692652022512148807680i)t^{21}x^{6}$$

$$- (1285913846860423737901056 0000i)t^{15}x^{2}$$

$$+ (634352727169645009698816000i)t^{13}x^{6}$$

$$- (113309125472760920801280000i)t^{23}x^2$$

$$- (135629797491220237044940800i)t^{15}x^8$$

$$+ (593523990571604823244480i)t^{15}x^{14}$$

$$+ (927956795300510485708800i)t^{11}x^{14}$$

$$- (215675391510498091047321600 0i)t^{15}x^6$$

$$+ (328301860508856578211840000i)t^9x^8$$

$$- (10598192580132864 00i)tx^{22}$$

$$- (265249548992128909049856000i)t^{13}x^8$$

$$- (6400477087609651200i)t^3x^{20}$$

$$- (900500023148544 00i)tx^{24}$$

$$+ (991696961402625494876160i)t^{27}x^2$$

$$- (5711637913599360016318464 00i)t^{17}x^6$$

$$- (24513482783505340012953600i)t^{13}x^{10}$$

$$+ (38919369389396937670656 00i)t^{13}x^{12}$$

$$- (509346309216497408409600i)t^{13}x^{14}$$

$$+ (129833372937538555084 80i)t^{13}x^{16}$$

$$- (438134268982282577510400000i)t^7x^2$$

$$- (128480941969439195136 00i)t^9x^{18}$$

$$- (409803273755688960 00i)tx^{18}$$

$$+ (72020006110872207360000i)t^3x^{10}$$

$$+ (54502642138742784000 0i)tx^{16}$$

$$+ (321949590346137600000 0i)tx^2$$

$$- (3579604828806168434442240000i)t^{19}x^2$$

$$- (1460298592300752703586304000i)t^{19}x^4$$

$$- (3604828139239182371389440 0i)t^{19}x^6$$

$$+ (60145041590452224000 00i)tx^{10}$$

$$+ (852750218304222658560000 0i)t^5x^6$$

$$+ (3200493777375232870318080000i)t^{11}x^4$$

$$+ (3120142857158167741071360000i)t^{11}x^6$$

$$- (422325775175211804524544000i)t^{11}x^8$$

$$- (82787037517578240000 00i)tx^4$$

$$+ (1107911449066995670056960i)t^{21}x^8$$

$$- (93740181582407270400000i)t^3x^8$$

$$- (62065232364699648000i)t^5x^{22}$$

$$+ (4627795739945763568877568000i)t^{15}x^4$$

$$- (47698783780744933775769 60i)t^{17}x^{10}$$

$$+ (458346892167028408320 00i)t^5x^{14}$$

$$+ (47889498275787571200 00i)tx^{12}$$

$+\ (52690745717466071040000i)t^3x^{14}$

$+\ (880706072845252791631872 0i)t^{15}x^{10}$

$-\ (18775903163274806427648 00i)t^{15}x^{12}$

$-\ (20898944158028267520000i)t^3x^{12}$

$-\ (8821021149832937472 00i)t^5x^{18}$

$+\ (1611507562279266429173760i)t^{23}x^6$

$+\ (15368533728401817 60i)t^5x^{24}$

$-\ (42014686657889238515712 00i)t^{11}x^{12}$

$+\ (70552107660537275351040 0000i)t^9x^4$

$+\ (73794107385376564838400 00i)t^7x^6$

$-\ (47214001120669492838400000i)t^7x^8$

$+\ (253073074819808844840960 00i)t^7x^{10}$

$-\ (17862567828411698380 8000i)t^7x^{12}$

$-\ (209440234922399760384000i)t^7x^{14}$

$-\ (4140327318740281589 7600i)t^7x^{16}$

$+\ (27048619361987198976 0i)t^9x^{20}$

$+\ (165388898404536993762508 80000i)t^{11}x^2$

$+\ (24589653965442908160i)t^7x^{22}$

$-\ (27286548100357187174400000i)t^5x^4$

$$+ (1086141566382243840000 0i)t^7x^{18}$$

$$+ (13209786727267345367040 0i)t^9x^{16}$$

$$- (519133261524369408000000i)t^3x^4$$

$$+ (1814044308651638784000i)t^3x^{16}$$

$$+ (2077333967000616881356 80i)t^{17}x^{12}$$

$$+ (43476695271472103424 0i)t^5x^{20}$$

$$- (1699405171890585600i)t^3x^{24}$$

$$+ (20610359380051671121920 00000i)t^9x^2$$

$$+ (1055531162664960i)tx^{28}$$

$$+ (161150756227926642917376 0i)t^{25}x^4$$

$$- (2392955678529474999091200 0i)t^9x^{10}$$

$$+ (343809808336711109836800000i)t^7x^4$$

$$+ (604286427864443331280896 0000i)t^{13}x^4$$

$$- (57215277778486689792 0000i)t^9x^{14}$$

$$- (48429500814175135334400 0i)t^{11}x^{10}$$

$$+ (46485795065748070072320 0i)t^{23}x^4$$

$$+ (786261290669706313728 0000i)t^5x^8$$

$$+ (6010499882222309854937088000i)t^{17}x^2$$

$$+ (1550087360736468664320000i)t^5x^{12}$$

$+ (159480637588242432000 0i)tx^{14}$

$- (182675243772591420211200 0i)t^5x^{10}$

$+ (650801130920472981012480 0i)t^{25}x^2$

$- (501074070624525498502348800 0i)t^{17}x^4$

$+ (17190268412538912768000000i)t^5x^2$

$+ (268439891780128389621350400 00i)t^{13}$

$- (653521367624631032217600 00i)t^9x^6$

$- (111644685562281984000i)t^3x^{18}$

$+ (361581404002620486451200 0i)t^{17}x^8$

$- (922199602734360390795264000 0i)t^{13}x^2$

$- (782995110638694055280640 0i)t^{19}x^8$

$- (11083077207982080i)tx^{26}$

$- (113916300774523011072 0i)t^7x^{20}$

$+ (553955724533497835028480i)t^{19}x^{10}$

$- (183423121376170475520000 0i)t^3x^6$

$+ (3657415478634086400i)t^3x^{22}$

$- (483745148973770342400 0i)t^5x^{16}$

$+ (2833419889721787128217 60i)t^{29}$

$- (212503985223892992000 00i)tx^8$

$$
\begin{aligned}
& - (1050078272185958400 0i)tx^{20} \\
& - (970005103581417858 66240i)t^{11}x^{16} \\
& - (837112579672932089856000i)t^{9}x^{12};
\end{aligned}
$$

$$
\begin{aligned}
D(x,t) = \; & 43251813652561920000 0x^{6} + 1541804044677120000000t^{2} \\
& + 392631163984281600000x^{8} \\
& + 28440274201745817600 0000t^{4} \\
& + 1888946593147858085478 4t^{30} \\
& + 336202712181374976000 00000t^{6} \\
& + 609753012019200000 00x^{4} + 195992039577600000 00x^{2} \\
& + 24161472903766364651520 0000t^{8} + 17592186044416x^{30} \\
& + 11704410475892734898995 20000t^{22} \\
& + 2049515124015958270672 89600t^{24} \\
& + 2033753534126478065664 0000t^{26} \\
& + 1009405835713386664427 520t^{28} \\
& + 3738595696032685094653 132800t^{20} \\
& + 6669056746644308068663 296000t^{18} \\
& + 5450887469422271161958 400000t^{16} \\
& + 7441435294537031645921 280000t^{14} \\
& + 5807382063900108049612 800000t^{12}
\end{aligned}
$$

$$+ 10407566500994816881459200000t^{10}$$

$$+ 2548891291484160000x^{22} + 245908458260398080000x^{10}$$

$$+ 374569324196659200000x^{12}$$

$$+ 124432437403975680000x^{14}$$

$$+ 25999984897818624000x^{16}$$

$$+ 4541081105485264595189760 0t^{24}x^{2}$$

$$+ 4792260621238272000x^{18} + 229619388414689280 0x^{20}$$

$$+ 17750240840908800x^{24} + 1039038488248320x^{26}$$

$$+ 65970697666560x^{28}$$

$$+ 6309831064001936822894592000t^{18}x^{2}$$

$$+ 273889401728689288249344000t^{20}x^{4}$$

$$+ 1575376470498366367727616000t^{20}x^{2}$$

$$+ 26982825014023047736197120t^{20}x^{6}$$

$$+ 27891477039448842043392 00t^{26}x^{2}$$

$$+ 24869900360175217488691 20t^{22}x^{6}$$

$$+ 45454391487726784767590400t^{22}x^{4}$$

$$+ 342128188278426772596326400t^{22}x^{2}$$

$$+ 1134543935823413835202560t^{20}x^{8}$$

$$- 1287410248479198412 80t^{8}x^{20}$$

$$+\ 102717959563577917440 0t^8x^{18}$$

$$+\ 29674939224372019200000t^8x^{16}$$

$$+\ 510583513125222448000000t^8x^{14}$$

$$+\ 2814087578532150509568000t^8x^{12}$$

$$+\ 3227192189276550856704000t^8x^{10}$$

$$+\ 5820250701529501138944 0000t^8x^8$$

$$+\ 215456891557737450700800000t^8x^6$$

$$+\ 2311001298251714396160 00000t^8x^4$$

$$-\ 8565496632667652751360 0000t^8x^2$$

$$-\ 112308515707551744 00t^6x^{22}$$

$$+\ 605136015555821568 00t^6x^{20}$$

$$+\ 3879077022793728000t^6x^{18}$$

$$+\ 8930847518040784896000t^6x^{16}$$

$$+\ 87545434673548099584000t^6x^{14}$$

$$-\ 327338035938695577600000t^6x^{12}$$

$$+\ 716792974760245985280000t^6x^{10}$$

$$+\ 4539966723077355601920000 0t^6x^6$$

$$+\ 1007192226424541518233 60t^{22}x^8$$

$$+\ 134292229685660553576 4480t^{24}x^6$$

$+\ 1239621201753281868595 20t^{26}x^{4}$

$+\ 708354972430446782054 40t^{28}x^{2}$

$+\ 553955724533497835028 48t^{20}x^{10}$

$+\ 741904988214506029056 0t^{16}x^{14}$

$+\ 185476247053626507264 0t^{14}x^{16}$

$+\ 360648258159829319680t^{12}x^{18}$

$+\ 5409723872397439795 2t^{10}x^{20}$

$+\ 614741349136072704 0t^{8}x^{22}$

$+\ 51228445761339392 0t^{6}x^{24} + 29554872554618880t^{4}x^{26}$

$+\ 1055531162664960t^{2}x^{28}$

$+\ 796142652981714691817472 0000t^{12}x^{2}$

$+\ 843514205937109743697920000t^{12}x^{4}$

$+\ 541889875773072825384960 00t^{12}x^{6}$

$-\ 584277209844233766174720 00t^{12}x^{8}$

$+\ 518562393333002403840000 00t^{6}x^{2}$

$+\ 316264149507272343552000 00t^{6}x^{4}$

$+\ 230814885222290764595 20t^{18}x^{12}$

$+\ 326653399296000000$

$+\ 344769646737631519703040 0t^{24}x^{4}$

$$+ 2795623085722986086400000t^{6}x^{8}$$

$$+ 50610969601329598562304 0000t^{18}x^{4}$$

$$+ 1399635537665636350033920 00t^{18}x^{6}$$

$$+ 1062136862454401948712960 0t^{18}x^{8}$$

$$+ 3222530897526597982617 60t^{18}x^{10}$$

$$+ 1240230918774083656089600 0t^{16}x^{4}$$

$$+ 16655056148509475261644800 0t^{16}x^{6}$$

$$+ 4899096063906974112153600 0t^{16}x^{8}$$

$$+ 29508719665637587156992 00t^{16}x^{10}$$

$$+ 4494232140145565368320 0t^{16}x^{12}$$

$$+ 149309638248232481351270400 00t^{16}x^{2}$$

$$+ 1106466866006271008440320 00t^{14}x^{8}$$

$$+ 11484295242665150958796 800t^{14}x^{10}$$

$$+ 605018978260505277235200t^{14}x^{12}$$

$$- 4280221085852919398400t^{14}x^{14}$$

$$+ 1145582326064093331456 0t^{10}x^{16}$$

$$- 90508841711264858112 0t^{10}x^{18}$$

$$+ 32915751992554474281369600 0t^{10}x^{6}$$

$$- 3781683920747801817907200 0t^{10}x^{8}$$

$$+ 2614998236717372607037440 0t^{10}x^{10}$$

$$+ 141082896741091672640716800 00t^{14}x^{2}$$

$$- 534419122544104156692480000t^{14}x^{4}$$

$$- 38604512465610816356352000 0t^{14}x^{6}$$

$$- 125245364064367851601920000 0t^{10}x^{2}$$

$$+ 166058778941564009840640000 0t^{10}x^{4}$$

$$+ 2963793282957456703488000t^{10}x^{12}$$

$$+ 127134421975850758963200t^{10}x^{14}$$

$$+ 3168993043875464856207360 0t^{12}x^{10}$$

$$+ 1667568663174750142464000t^{12}x^{12}$$

$$+ 94736370681477542707200t^{12}x^{14}$$

$$- 36827735592859493990 40t^{12}x^{16}$$

$$+ 659367330651832320 00t^{12}x^{18}$$

$$+ 957344087109795840 0t^{2}x^{20} + 602642323184025600t^{2}x^{22}$$

$$+ 20780769764966400t^{2}x^{24} - 1108307720798208 0t^{2}x^{26}$$

$$- 6146310361153536000000t^{2}x^{2}$$

$$+ 2472461356164710400000 0t^{2}x^{4}$$

$$+ 1870187981092945920000 0t^{2}x^{6}$$

$$- 2485230496791920640000 0t^{2}x^{8}$$

$$+ 1331195151535570944 0000t^2x^{10}$$

$$+ 900901364501053440000 00t^4x^4$$

$$- 846074231955180748800000t^4x^6$$

$$+ 785872556615584972800000t^4x^8$$

$$+ 253290289902932459520000t^4x^{10}$$

$$+ 30704050024564654080000t^4x^{12}$$

$$- 21963682613884354560000t^4x^{14}$$

$$+ 1980979522659680256000000t^4x^2$$

$$+ 52077878656892928000000t^2x^{12}$$

$$+ 8215669209091276800000t^2x^{14}$$

$$- 4700037837918044160000t^2x^{16}$$

$$+ 2114748541141254144000t^4x^{16}$$

$$+ 3049318203386757120000t^4x^{18}$$

$$+ 17746777379281305600t^4x^{20}$$

$$+18287077393170432000t^4x^{22} - 5541538603991040000t^4x^{24}.$$

(3) Case $N = 6$

$$N(x,t) = (1306769913293710881315518152704000000t^{20}$$

$$+ 304159304532499302649143125606400000t^{18}$$

$$+ 82274724986110217960494517452800000t^{16}$$

$$
\begin{aligned}
&+ 24522724634493022093505986560000000t^{14} \\
&- 8374137387902637169390387200000000t^{12} \\
&+ 13029752662919612928000000x^{12} \\
&+ 17927619210928982016000000x^{14} \\
&+ 8652194022058481664000000x^{16} \\
&+ 3987077385111625728000000x^{18} \\
&- 492429563900854272000000x^{20} \\
&- 1258173898837194035923330793472000 0t^{22} \\
&- 142471752915773030400000x^{22} \\
&- 50768812585652541528257593344000000t^{24} \\
&- 6003371153223348445863129120768000t^{28}x^{2} \\
&- 30346203387962327040000x^{24} \\
&+ 319446516073671267297553022976 0t^{36} \\
&+ 16667909082316800x^{36} \\
&- 50820486401353805136000000x^{10} \\
&+ 5332446739393215316875000 0t^{2} \\
&- 32661858646516799700000000x^{8} \\
&+ 3577668467841632864700000000 0t^{4} \\
&- 14737667925867336450000000x^{6}
\end{aligned}
$$

$$- 524237317699453990853760000000t^{6}$$

$$+ 62734667522273121375000000x^{4}$$

$$- 82190313059811212101063680000000t^{8}$$

$$+ 7841833440284140171875000x^{2}$$

$$- 290232638614071372247547904000000t^{10}$$

$$+ 865865406873600x^{38}$$

$$+ 162844574638002327262624481280t^{38}$$

$$- 35154585683572853767471015919616 00t^{30}$$

$$+ 9540199756962201600x^{30}$$

$$- 177938969354401976005008021258240 00t^{28}$$

$$- 386905445327925385222176406241280 00t^{26}$$

$$+ 4179430850376499200x^{32}$$

$$+ 311955071120179200x^{34}$$

$$- 2381524983745152523723408932864 00t^{32}$$

$$+ 162034684114671147780000000 0t^{2}x^{4}$$

$$+ 179958874835320611030000000 0t^{2}x^{2}$$

$$+ 1385039847833432501738117529600 0t^{34}$$

$$- 6406275691988765429760000000t^{2}x^{10}$$

$$- 11743071850186317748800000000t^{2}x^{8}$$

$$+ 570769963001374552416000000 0t^2x^6$$

$$- 18100767058091434141286400000 0t^4x^{12}$$

$$- 3922239420529166499840 0000000t^4x^{10}$$

$$- 25706817195917802835968000000 0t^4x^8$$

$$- 7094299173254926777267200000 00t^4x^6$$

$$+ 4554276425157839688000000000 0t^4x^4$$

$$- 12337578956837438544240000000 00t^4x^2$$

$$+ 172356479738448445440000000t^2x^{18}$$

$$+ 5765220303526217318400000000t^2x^{16}$$

$$- 2052834294940463333376000000 0t^2x^{14}$$

$$- 28891023802344350760960000000 t^2x^{12}$$

$$+ 2352929303194724420812800000000t^6x^6$$

$$- 190831057259601794395852800000000t^6x^4$$

$$- 8376527185464388875724800000000t^6x^2$$

$$+ 60221837251618063515648000000t^4x^{16}$$

$$+ 148226106514706304860160000000t^4x^{14}$$

$$- 44495881740233359362533376000000000t^8x^2$$

$$+ 2398401005860175946448896000000t^6x^{14}$$

$$+ 9766830495599016957444096000000t^6x^{12}$$

$$- 38793474442492103777845248000000t^{6}x^{10}$$

$$+ 76863585410991583525601280000000t^{6}x^{8}$$

$$+ 3978001275012742347349995552768000000t^{18}x^{2}$$

$$+ 14161170171803730690313121955840000 00t^{16}x^{4}$$

$$- 7712215878432479348291650191360000 0t^{16}x^{2}$$

$$+ 3005659069034411543637597880320000 0t^{14}x^{6}$$

$$+ 2182759648944286926574190592000000 0t^{14}x^{4}$$

$$+ 41850340863817061414574489600000 0t^{14}x^{2}$$

$$+ 1492444278814740435060129792000000 t^{12}x^{8}$$

$$+ 8868606698708992701052747776000000 t^{12}x^{6}$$

$$+ 2783454017978870795480334336000000 t^{12}x^{4}$$

$$+ 17074228651321446767511207936000000 t^{12}x^{2}$$

$$+ 395251485904236030745116672000000 t^{10}x^{10}$$

$$+ 1024797651488330815547375616000000 t^{10}x^{8}$$

$$- 1130716815140272736758136832000000 t^{10}x^{6}$$

$$- 5056380659338243211746344960000000 t^{10}x^{4}$$

$$- 3665884543785271110625394688000000 t^{10}x^{2}$$

$$+ 446812059452229958521323520000 0t^{8}x^{12}$$

$$+ 29910409415857677027350937600000 t^{8}x^{10}$$

$$- 8007110622834682406790758400000 0t^8x^8$$

$$+ 14043162796463079346352947200000 0t^8x^6$$

$$+ 602351495268321665109196800000 00t^8x^4$$

$$+ 21071577035882986515128647 68t^{40}$$

$$+ 69269232549888x^{40} - 4398046511104x^{42}$$

$$- 1934281311383406679529881 6t^{42}$$

$$+ 1356439377244452689018880 0t^{10}x^{26}$$

$$+ 52510914260099562217937043456 00t^{28}x^8$$

$$+ 93745820345475187401308504064 00t^{30}x^6$$

$$+ 118603234621095744958378278912 00t^{32}x^4$$

$$+ 24065358386236623216710988595 20t^{26}x^{10}$$

$$+ 41912274138712392347355709440 0t^{22}x^{14}$$

$$+ 102889980951861123953668915200 0t^{24}x^{12}$$

$$+ 14895975130387571318508748800 0t^{20}x^{16}$$

$$+ 1707102478364557606649856 00t^{12}x^{24}$$

$$+ 8793382726045532160 0t^4x^{32}$$

$$+ 1741581452769654472704 0t^6x^{30}$$

$$+ 678222481557682755993600t^8x^{28}$$

$$+ 14758389320498992272900009600t^{14}x^{22}$$

$$- 29684566402368649311509544960000t^{14}x^{14}$$

$$- 6181670155156025015260938240000t^{14}x^{16}$$

$$+ 267681531056024685819985920000t^{14}x^{18}$$

$$+ 12440386852572231882976002048000000t^{16}x^{6}$$

$$- 5319016096575029456694474178560000t^{16}x^{8}$$

$$+ 1279503167224870826982607159296000t^{16}x^{10}$$

$$- 103519019531854247320734400512000t^{16}x^{12}$$

$$- 1559043058209209727088538419200 0t^{16}x^{14}$$

$$- 33947568448408047679402357555200000t^{18}x^{4}$$

$$- 10511849301398468930982421463040000t^{18}x^{6}$$

$$- 11014089385299696758960632627200000t^{18}x^{8}$$

$$+ 21794914383199203466590289920000t^{12}x^{14}$$

$$- 1882705701688384657758879744000t^{12}x^{16}$$

$$- 56580697170227032951357440000t^{12}x^{18}$$

$$+ 3878882819130028045683916800t^{12}x^{20}$$

$$+ 382664960470760535165291724800000000t^{14}x^{8}$$

$$- 26009395064146611068416819200000 0t^{14}x^{10}$$

$$+ 23947618677915981385159409664 0000t^{14}x^{12}$$

$$+ 22690428060348070414667218944 0000t^{12}x^{10}$$

$$- 544002783999933644052155596800000t^{12}x^{12}$$

$$+ 157881302174893351712587776000 0t^{10}x^{16}$$

$$- 5642124236456428629393408 0000t^{10}x^{18}$$

$$- 547356797397671121864622 0800t^{10}x^{20}$$

$$+ 101282529609939830597222400t^{10}x^{22}$$

$$+ 866978725483223130293340733440000t^{22}x^{2}$$

$$- 2155498822840068717491389464576 0000t^{22}x^{4}$$

$$+ 58332801268119996911542043934720 00t^{22}x^{6}$$

$$- 16927549455795092342063706931200 00t^{22}x^{8}$$

$$- 1479852315629783449728731578368 00t^{22}x^{10}$$

$$- 11722006937659845115300691312640 0000t^{24}x^{2}$$

$$- 36066919971440602268502589440000 00t^{24}x^{4}$$

$$- 19383193295153523979916389908480 00t^{24}x^{6}$$

$$- 4673336618709173020368473948160 00t^{24}x^{8}$$

$$- 40196491256559835664946806390784 000t^{26}x^{2}$$

$$- 31769513300810703574386176163840 00t^{26}x^{4}$$

$$- 9045373578284236291392657162240 00t^{26}x^{6}$$

$$+ 2805070566462195870203904000 00t^{10}x^{12}$$

$$- 783477666559197712264200192000 0t^{10}x^{14}$$

$$+\ 35141764705881832871347145932800 0t^{18}x^{10}$$

$$-\ 1671643944383811238550663331840 00t^{18}x^{12}$$

$$-\ 327513896254778576522025369600 0t^{18}x^{14}$$

$$+\ 2093489118602077223058596167680 00000t^{20}x^{2}$$

$$-\ 691828934657964043482845046374 40000t^{20}x^{4}$$

$$-\ 517978265084974532786839879680 0000t^{20}x^{6}$$

$$+\ 310703503348776213092637868032 0000t^{20}x^{8}$$

$$-\ 780645876774648274975414498099 200t^{20}x^{10}$$

$$-\ 290644182374001900047799681024 00t^{20}x^{12}$$

$$-\ 102116686591361021432951734272 0000t^{28}x^{4}$$

$$-\ 643855095469597985910711071539 200t^{30}x^{2}$$

$$+\ 282169621610431827148800 00t^{2}x^{20}$$

$$+\ 455392656468291354624 0000t^{2}x^{22}$$

$$+\ 108935318524284370944 0000t^{2}x^{24}$$

$$-\ 109543867036748218368 000t^{2}x^{26}$$

$$-\ 106051939136962560000 00t^{2}x^{28}$$

$$-\ 551741126442850713600t^{2}x^{30}$$

$$-\ 771081978815447040000x^{28}$$

$$-\ 456826698508271616000 0x^{26}$$

$$+ 354295859065251043265740800000t^8x^{14}$$

$$- 504281339250167885290536960000t^8x^{16}$$

$$+ 89125941971739511955128320000t^8x^{18}$$

$$- 1477195572245748938440704000t^8x^{20}$$

$$- 257979458356789691547648000t^8x^{22}$$

$$- 12577694624499351158784000t^8x^{24}$$

$$+ 413339611507616592691200000000t^6x^{16}$$

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$$- 99171740382105163530240000t^6x^{24}$$

$$- 5747402953321516892160000t^6x^{26}$$

$$+ 76949563379137118208000000000t^4x^{18}$$

$$- 3514629307399762083840000000t^4x^{20}$$

$$+ 116740110638455833231360000t^4x^{22}$$

$$- 1156846770416611491840000t^4x^{24}$$

$$- 341862050367943999488000t^4x^{26}$$

$$- 17602857560677810176000t^4x^{28}$$

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$$- 28006548001014786328125$$

$$+ 918279319269397242857914368 0t^{16}x^{20}$$

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$$- 80371115134570900154 9964902400t^{20}x^{14}$$

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$$- 140574362151112277856482230272 00t^{30}x^{4}$$

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$$+ 62687571282940133376000000itx^{20}$$

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$$+\ 52258973834426879520000000 0itx^6$$

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$$- 57241198541773209600 0 itx^{28}$$

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$$- 1132736600390108184576 0000it^3x^{22}$$

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$$- 6215858871920958443272525578 2400it^{31}x^4$$

$$- 4414134163399294768582943047 6800it^{33}x^2$$

$$- 405997282491043118068594114 5600it^{23}x^{12}$$

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$$+ 419844825579415529510771097 60it^{35}x^{4}$$

$$+ 384619749510394271932769894 40it^{37}x^{2}$$

$$+ 3694359069327 36itx^{40} + 2955487255461888 0it^{3}x^{38}$$

$$+ 586519992431118227865 6it^{11}x^{30}$$

$$+ 5865199924311182278656 0it^{13}x^{28}$$

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$$+ 5450002512461762172939141120it^{17}x^{20}$$

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$$+ 97395245235981275627 5200it^{9}x^{28}$$

$$- 409985192077673103360 00it^{9}x^{30}$$

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$- 763256892697450905600 0it^5x^{28}$

$- 121754530052938137600it^3x^{30}$

$$+\ 13237501624287485755392000 i t^{9} x^{24}$$

$$-\ 833305594148400184543936512 00 i t^{17} x^{16}$$

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$$+ 27707693019955200t^{2}x^{38}$$

$$+ 1836096457455697920t^{4}x^{36}$$

$$+ 61444580041052651520t^{6}x^{34}$$

$$+ 2000727737339044744396 8t^{10}x^{30}$$

$$+ 2286270365233141776384 00t^{12}x^{28});$$

$$D(x,\ t) = (1214628889736129551139136444825600 00t^{20}$$

$$+ 9453765404227993489416671723520000 0t^{18}$$

$$+ 1625293770243748458252573081600000 0t^{16}$$

$$+ 9805423648517498549009645568000000 t^{14}$$

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$$+\ 28825577773415107644072960000000t^{8}$$

$$+\ 1809653870834801578125 00x^{2}$$

$$+\ 2670894577683727415585832960000000t^{10}$$

$$+\ 519519244124160x^{38}$$

$$+\ 8963353815941490272888 6845440t^{38}$$

$$+\ 1867092791250438641571509934489600t^{30}$$

$$+\ 69931633115293286400x^{30}$$

$$+\ 8259327114186084142196937719808000t^{28}$$

$$+\ 26590369766887902946198699376640000t^{26}$$

$$+\ 59412098995716 09600x^{32}$$

$$+\ 315364416159744000x^{34}$$

$$+\ 2940674255093901075656900345856 00t^{32}$$

$$+\ 64159410887247822300000000t^2x^4$$

$$-\ 111957252808979724300000000t^2x^2$$

$$+\ 3114281580398828498167435100160 0t^{34}$$

$$+\ 9602939332939781283840000000t^2x^{10}$$

$$-\ 13250612282724341078400000000t^2x^8$$

$$+\ 74848969278051556896 0000000t^2x^6$$

$$+\ 11695926375914239033344000000t^4x^{12}$$

$$+\ 51630769268674678849536000000t^4x^{10}$$

$$+\ 894241105346914014412800000000t^4x^8$$

$$-\ 679001925546115910323200000000t^4x^6$$

$$+\ 9983130798914343100800000000 0t^4x^4$$

$$+\ 7564265857109036663280000000 0t^4x^2$$

$$+\ 1577687904905362145280 0000t^2x^{18}$$

$$-\ 1190769193846031450112 00000t^2x^{16}$$

$$+\ 18788518164614381568 0000000t^2x^{14}$$

$$+\ 68034145969753038028 8000000t^2x^{12}$$

$$+\ 848834975457657580879872000000000t^6x^6$$

$$+\ 411025594596624438875136000000000t^6x^4$$

$$+\ 416404209761764131156480000000000t^6x^2$$

$$+\ 708618775142017164902400000t^4x^{16}$$

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$$-\ 158698608490678649483059200000000t^8x^2$$

$$+\ 804341607381595406204928000000t^6x^{14}$$

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$$+\ 2056382819164484226175205376t^{40}$$

$$+\ 230897441832 96x^{40}+4398046511104x^{42}$$

$$+\ 193428131138340667952988 16t^{42}$$

$$-\ 15553298969664272990 2080t^{10}x^{26}$$

$$+\ 462417330202041262674228019200 0t^{28}x^{8}$$

$$+\ 7692422509798564361512880701440t^{30}x^{6}$$

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$$+\ 919392283596423168000 00t^{2}x^{24}$$

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$$+ 1658936156076638208000t^2x^{28}$$

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$$+ 38039813824827642347520 0000t^4x^{18}$$

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$+\ 845412296695190913024 0000t^4x^{22}$

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$+\ 171416692930800481468416000t^{12}x^{22}$

$+\ 2689023734168603609530368000t^{14}x^{20}$

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$+\ 2600989626560585833591603 2000t^{16}x^{18}$

$+\ 21543498462319066406 25$

$+\ 674120776546396850356224000t^{16}x^{20}$

$+\ 17048781845670715006648320 0000t^{18}x^{16}$

$+\ 82410453864340473061048320 0000t^{20}x^{14}$

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$$+\ 17437999369303766701902397440t^{36}x^{4}$$

$$+\ 8758667563107988370746245120t^{38}x^{2}$$

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$$+ 37499813394751527321 60t^{8}x^{30}$$

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$$- 3877977581534696177664t^{10}x^{30}$$

$$- 38008038983200753582080t^{12}x^{28}).$$

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