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EXACT EXPLICIT TRAVELLING WAVE SOLUTIONS FOR A GENERALIZED NEW COUPLED KP EQUATION

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Abstract

The sine-cosine method is used to construct exact solutions of a generalized new coupled KP equation. Travelling wave solutions are determined. It is shown that the sine-cosine method provide a powerful mathematical tool for solving a great many nonlinear partial differential equations in mathematical physics.

1. Introduction

Recently, a new hierarchy of nonlinear evolution equations was derived by Qin [1] by using a finite-dimensional integrable system. An interesting equation in this hierarchy is a new coupled KdV equation

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$$\begin{cases} u_t = -u_{xxx} + \frac{3}{2}(uv)_x + \frac{3}{2}(vw)_x, \\ v_t = -v_{xxx} - 3(wu)_x, \\ w_t = -w_{xxx} - 3(uv)_x. \end{cases} \tag{1.1}$$

Later, this new coupled equation was investigated by Wu [6], by using matrix transformation and Lax pair. Most recently, in the sense of the KP equation, Wazwaz [5] has extend the new coupled KdV equation to the following new coupled KP equation:

$$\begin{cases} \left(u_t + u_{xxx} - \frac{3}{2}(uv)_x - \frac{3}{2}(vw)_x\right)_x + u_{yy} = 0, \\ (v_t + v_{xxx} + 3(wu)_x)_x + v_{yy} = 0, \\ (w_t + w_{xxx} + 3(uv)_x)_x + w_{yy} = 0, \end{cases} \tag{1.2}$$

and studied the new coupled KP equation by using the Hirota's bilinear method.

In this work, we aim to study the following generalized new coupled KP equation:

$$\begin{cases} (u_t + u_{xxx} - a(uv)_x - a(vw)_x)_x + u_{yy} = 0, \\ (v_t + v_{xxx} + b(wu)_x)_x + v_{yy} = 0, \\ (w_t + w_{xxx} + b(uv)_x)_x + w_{yy} = 0, \end{cases} \tag{1.3}$$

where $ab \neq 0$, a, b are arbitrary constants. When $a = \frac{3}{2}$ and $b = 3$, system (1.3) becomes (1.2). The sine-cosine method will be mainly used to back up our analysis. The sine-cosine method is direct and effective algebraic method for handling many nonlinear equations, where solitary wave solutions and triangular periodic solutions are generated.

2. The Method

In what follows, the methods will be reviewed briefly. Full details can be found in [2-4] and the references therein.

For both methods, we first use the wave variable $\xi = x + y - ct$ to carry a PDE in three independent variables

$$P(u, u_t, u_x, u_y, u_{xx}, u_{xy}, u_{yy}, \dots) = 0, \tag{2.1}$$

into an ODE

$$Q(u, u', u'', u''', \dots) = 0. \tag{2.2}$$

Equation (2.2) is then integrated as long as all terms contain derivatives where integration constants are considered zeroes.

The sine-cosine algorithm admits the use of the ansätz

$$\begin{cases} u(\xi) = \lambda_1 \cos^{\beta_1}(\mu\xi), \\ v(\xi) = \lambda_2 \cos^{\beta_2}(\mu\xi), \\ w(\xi) = \lambda_3 \cos^{\beta_3}(\mu\xi), \qquad |\mu\xi| < \dfrac{\pi}{2}, \\ 0, \qquad\qquad\qquad \text{otherwise}, \end{cases} \tag{2.3}$$

or in the form

$$\begin{cases} u(\xi) = \lambda_1 \sin^{\beta_1}(\mu\xi), \\ v(\xi) = \lambda_2 \sin^{\beta_2}(\mu\xi), \\ w(\xi) = \lambda_3 \sin^{\beta_3}(\mu\xi), \qquad |\mu\xi| < \pi, \\ 0, \qquad\qquad\qquad \text{otherwise}, \end{cases} \tag{2.4}$$

where λ, μ, and β_i, $i = 1, 2, 3$ are parameters that will be determined.

Substituting (2.3) or (2.4) into the reduced ODE obtained above after integrating (2.2) gives a trigonometric equation of cosine or sine terms.

The main task is to balance the exponents of the trigonometric functions cosine or sine. Collect all terms with same power in $\cos^k(\mu\xi)$ or $\sin^k(\mu\xi)$ and set to zero their coefficients to get a system of algebraic equations among the unknowns λ, μ, and β. The problem is now completely reduced to an algebraic one. Having determined λ, μ, and β by algebraic calculations or by using computerized symbolic calculations, the solutions proposed in (2.3) and (2.4) follow immediately. The algorithm described above certainly works well for a large class of very interesting nonlinear equations. The main advantage of the method is that, it is capable of greatly reducing the size of computational work

compared to existing techniques such as the pseudo spectral method, the inverse scattering method, Hirota's bilinear method, and the truncated Painlevé expansion.

3. Using the Sine-Cosine Method

In this section, we employ the sine-cosine method to the Equation (1.2).

Let $u(x, y, t) = u(\xi)$, $v(x, y, t) = v(\xi)$, $w(x, y, t) = w(\xi)$, $\xi = x + y - ct$. Then (1.2) becomes to

$$\begin{cases} [-cu' + u''' - a(uv)' - a(vw)']' + u'' = 0, \\ [-cv' + v''' + b(wu)']' + v'' = 0, \\ [-cw' + w''' + b(uv)']' + w'' = 0. \end{cases} \tag{3.1}$$

Integrating (3.1) twice with ξ, using the constants of integration to be zero, we find

$$\begin{cases} (1-c)u + u'' - av(u+w) = 0, \\ (1-c)v + v'' + bwu = 0, \\ (1-c)w + w'' + buv = 0. \end{cases} \tag{3.2}$$

Substituting (2.3) into (3.2) gives

$$\begin{cases} \lambda_1(1 - c - \mu^2\beta_1^2)\cos^{\beta_1}(\mu\xi) + \mu^2\lambda_1\beta_1(\beta_1 - 1)\cos^{\beta_1-2}(\mu\xi) \\ -a\lambda_2\cos^{\beta_2}(\mu\xi)[\lambda_1\cos^{\beta_1}(\mu\xi) + \lambda_3\cos^{\beta_3}(\mu\xi)] = 0, \\ \lambda_2(1 - c - \mu^2\beta_2^2)\cos^{\beta_2}(\mu\xi) + \mu^2\lambda_2\beta_2(\beta_2 - 1)\cos^{\beta_2-2}(\mu\xi) + b\lambda_1\lambda_3\cos^{\beta_1+\beta_3}(\mu\xi) = 0, \\ \lambda_3(1 - c - \mu^2\beta_3^2)\cos^{\beta_3}(\mu\xi) + \mu^2\lambda_3\beta_3(\beta_3 - 1)\cos^{\beta_3-2}(\mu\xi) + b\lambda_1\lambda_2\cos^{\beta_1+\beta_2}(\mu\xi) = 0. \end{cases} \tag{3.3}$$

Equation (3.3) is satisfied only if the following system of algebraic equations holds:

$$\beta_3 - 2 = \beta_1 + \beta_2,\ \beta_2 - 2 = \beta_1 + \beta_2 = \beta_2 + \beta_3,\ \beta_1 - 2 = \beta_1 + \beta_3 = \beta_2 + \beta_3,$$

$$1 - c = \mu^2\beta_1^2 = \mu^2\beta_2^2 = \mu^2\beta_3^2,\ \mu^2\lambda_1\beta_1(\beta_1 - 1) = a\lambda_2(\lambda_1 + \lambda_3),$$

$$(\beta_1 - 1)(\beta_2 - 1)(\beta_3 - 1) \neq 0,\ \mu^2\lambda_2\beta_2(\beta_2 - 1) + b\lambda_1\lambda_3 = 0,$$

$$\mu^2\lambda_3\beta_3(\beta_3 - 1) + b\lambda_1\lambda_2 = 0. \tag{3.4}$$

Solving the system (3.4) gives

Case 1.

$$\beta_1 = \beta_2 = \beta_3 = -2,\ \mu^2 = \frac{1-c}{4},\ \lambda_1 = \frac{3(c-1)}{2b},$$

$$\lambda_2 = \lambda_3 = \frac{3(c-1)(a \pm \sqrt{a^2 - 4ab})}{4ab},\ a^2 - 4ab \geq 0. \tag{3.5}$$

Case 2.

$$\beta_1 = \beta_2 = \beta_3 = -2,\ \mu^2 = \frac{1-c}{4},\ \lambda_1 = -\frac{3(c-1)}{2b},$$

$$\lambda_2 = -\lambda_3 = \frac{3(c-1)(a \pm \sqrt{a^2 + 4ab})}{-4ab},\ a^2 + 4ab \geq 0. \tag{3.6}$$

The results (3.5) and (3.6) can be easily obtained, if we also use the sine method (2.4). Combining (3.5) and (3.6) with (2.3) and (2.4), the following periodic wave solutions:

$$\begin{cases} u_1 = \dfrac{3(c-1)}{2b}\sec^2\dfrac{1}{2}\sqrt{1-c}(x-ct), \\ v_1 = w_1 = \dfrac{3(c-1)(a \pm \sqrt{a^2 - 4ab})}{4ab}\sec^2\dfrac{1}{2}\sqrt{1-c}(x-ct), \\ |x-ct| < \dfrac{\pi}{\sqrt{1-c}},\ a^2 - 4ab \geq 0, \end{cases} \tag{3.7}$$

$$\begin{cases} u_2 = -\dfrac{3(c-1)}{2b}\sec^2\dfrac{1}{2}\sqrt{1-c}(x-ct), \\ v_2 = -w_2 = \dfrac{3(c-1)(a \pm \sqrt{a^2 + 4ab})}{-4ab}\sec^2\dfrac{1}{2}\sqrt{1-c}(x-ct), \\ |x-ct| < \dfrac{\pi}{\sqrt{1-c}},\ a^2 + 4ab \geq 0, \end{cases} \tag{3.8}$$

$$\begin{cases} u_3 = \dfrac{3(c-1)}{2b}\csc^2\dfrac{1}{2}\sqrt{1-c}(x-ct), \\ v_3 = w_3 = \dfrac{3(c-1)(a \pm \sqrt{a^2 - 4ab})}{4ab}\csc^2\dfrac{1}{2}\sqrt{1-c}(x-ct), \\ |x-ct| < \dfrac{\pi}{\sqrt{1-c}},\ a^2 - 4ab \geq 0; \end{cases} \tag{3.9}$$

and

$$\begin{cases} u_4 = -\dfrac{3(c-1)}{2b}\csc^2\dfrac{1}{2}\sqrt{1-c}(x-ct), \\ v_4 = -w_4 = \dfrac{3(c-1)(a \pm \sqrt{a^2+4ab})}{-4ab}\csc^2\dfrac{1}{2}\sqrt{1-c}(x-ct), \\ |x-ct| < \dfrac{\pi}{\sqrt{1-c}},\ a^2+4ab \geq 0, \end{cases} \tag{3.10}$$

are readily obtained.

However, for $1-c<0$, we obtain the following solitary wave solutions:

$$\begin{cases} u_5 = \dfrac{3(c-1)}{2b}\operatorname{sech}^2\dfrac{1}{2}\sqrt{c-1}(x-ct), \\ v_5 = w_5 = \dfrac{3(c-1)(a \pm \sqrt{a^2-4ab})}{4ab}\operatorname{sech}^2\dfrac{1}{2}\sqrt{c-1}(x-ct),\ a^2-4ab \geq 0, \end{cases} \tag{3.11}$$

$$\begin{cases} u_6 = -\dfrac{3(c-1)}{2b}\operatorname{sech}^2\dfrac{1}{2}\sqrt{c-1}(x-ct), \\ v_6 = -w_6 = \dfrac{3(c-1)(a \pm \sqrt{a^2+4ab})}{-4ab}\operatorname{sech}^2\dfrac{1}{2}\sqrt{c-1}(x-ct),\ a^2+4ab \geq 0; \end{cases} \tag{3.12}$$

and solitary patterns wave solutions

$$\begin{cases} u_7 = -\dfrac{3(c-1)}{2b}\operatorname{csch}^2\dfrac{1}{2}\sqrt{c-1}(x-ct), \\ v_7 = w_7 = \dfrac{3(c-1)(a \pm \sqrt{a^2-4ab})}{-4ab}\operatorname{csch}^2\dfrac{1}{2}\sqrt{c-1}(x-ct),\ a^2-4ab \geq 0, \end{cases} \tag{3.13}$$

and

$$\begin{cases} u_8 = \dfrac{3(c-1)}{2b}\operatorname{csch}^2\dfrac{1}{2}\sqrt{c-1}(x-ct), \\ v_8 = -w_8 = \dfrac{3(c-1)(a \pm \sqrt{a^2+4ab})}{4ab}\operatorname{csch}^2\dfrac{1}{2}\sqrt{c-1}(x-ct),\ a^2+4ab \geq 0. \end{cases} \tag{3.14}$$

4. Discussion

In this paper, we used the sine-cosine method to study a generalized new coupled KP equation. As a result, we obtained eight kinds of exact solutions including solitary wave solutions, solitary wave patterns solutions, and periodic wave solutions. The method provided solitary wave solutions and periodic wave solutions. Moreover, the obtained results in this work clearly demonstrate the reliability of the method that were used.

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