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LIE TRIPLE DERIVATIONS ON UPPER TRIANGULAR MATRICES

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Abstract

Let $\mathcal{C}$ be a commutative unital ring with a 2-torsion free unital $\mathcal{C}$-bimodule $\mathcal{M}$. Let $\mathcal{T}_n(\mathcal{C})$ and $\mathcal{T}_n(\mathcal{M})$ be the sets of all $n \times n$ upper triangular matrices over $\mathcal{C}$ and $\mathcal{M}$, respectively. We show that every Lie triple derivation $\delta : \mathcal{T}_n(\mathcal{C}) \to \mathcal{T}_n(\mathcal{M})$ is a sum of a derivation and a linear map having its range in the center of $\mathcal{T}_n(\mathcal{M})$.

1. Introduction

Let $\mathcal{C}$ be a commutative ring with unity. Let $\mathcal{A}$ be an algebra over $\mathcal{C}$ and $\mathcal{M}$ be an $\mathcal{A}$-bimodule. Recall that a $\mathcal{C}$-linear map d from $\mathcal{A}$ into $\mathcal{M}$ is called a *derivation*, if

$$d(ab) = d(a)b + ad(b), \quad \text{for all } a, b \in \mathcal{A}.$$

Let $[a, b] = ab - ba$ be denote the Lie product and $a \circ b = ab + ba$ be the Jordan product of elements $a, b \in \mathcal{A}$. Similarly, we set $[m, a] = ma - am$

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and $m \circ a = a \circ m = ma + am$, for all $a \in \mathcal{A}$ and $m \in \mathcal{M}$, respectively. Further, a $\mathcal{C}$-linear map $d : \mathcal{A} \to \mathcal{M}$ is called a *Lie derivation* (resp., a *Jordan derivation*), if it is a derivation for the Lie product (resp., Jordan product), i.e.,

$$d([a, b]) = [d(a), b] + [a, d(b)] \text{ (resp., } d(a \circ b) = d(a) \circ b + a \circ d(b)).$$

We say that a $\mathcal{C}$-linear map $d : \mathcal{A} \to \mathcal{M}$ is a *Lie triple derivation*, if

$$d([[a, b], c]) = [[d(a), b], c] + [[a, d(b)], c] + [[a, b], d(c)], \text{ for all } a, b, c \in \mathcal{A}.$$

Clearly, every Lie derivation is a Lie triple derivation.

Throughout this paper, $\mathcal{C}$ always denotes a commutative unital ring with a unital $\mathcal{C}$-bimodule $\mathcal{M}$. Let $\mathcal{T}_n(\mathcal{C})$ and $\mathcal{T}_n(\mathcal{M})$ be the sets of all $n \times n$ upper triangular matrices over $\mathcal{C}$ and $\mathcal{M}$. It is obvious that $\mathcal{T}_n(\mathcal{M})$ is a unital $\mathcal{T}_n(\mathcal{C})$-bimodule under the usual matrix addition and formal matrix multiplication. Also, $\mathcal{M}$ is 2-torsion free if and only if $\mathcal{T}_n(\mathcal{M})$ is 2-torsion free. Xu and Zhang [12] characterized left derivations and Jordan left derivations from $\mathcal{T}_n(\mathcal{C})$ into $\mathcal{T}_n(\mathcal{M})$.

In recent years, many significant researches have been done in Lie derivations and Lie triple derivations (see [2-11]). One of the earliest results on Lie derivations of associative rings is in Martindale [8], who proved that a Lie derivation of certain primitive ring is always a sum of a derivation and an additive map from the ring into its centroid. Cheung [4] gave sufficient conditions such that every Lie derivation on a triangular algebra $\mathcal{A}$ is a sum of a derivation on $\mathcal{A}$ and a map from $\mathcal{A}$ into its center. Benkovič [2] proved that every Lie derivation δ from $\mathcal{T}_n(\mathcal{C})$ into a 2-torsion free unital $\mathcal{T}_n(\mathcal{C})$-bimodule $\mathcal{M}$ is a sum of a derivation and a linear map having its range in the center of $\mathcal{M}$. Lu [7] showed that every Lie triple derivation from a nest algebra $\mathcal{A}$ into a weak* closed operator algebra containing $\mathcal{A}$ is a sum of derivation and a scalar multiple of the identity I.

Motivated by the study of Lie derivations and Lie triple derivations on (strictly) upper triangular matrices (see [2, 6, 11]), we consider Lie triple derivations from $\mathcal{T}_n(\mathcal{C})$ into a 2-torsion free unital $\mathcal{T}_n(\mathcal{C})$-bimodule $\mathcal{T}_n(\mathcal{M})$. We show that every Lie triple derivation from $\mathcal{T}_n(\mathcal{C})$ into $\mathcal{T}_n(\mathcal{M})$ is of the form $\delta = d + \tau$, where $d : \mathcal{T}_n(\mathcal{C}) \to \mathcal{T}_n(\mathcal{M})$ is a derivation and $\tau : \mathcal{T}_n(\mathcal{C}) \to Z(\mathcal{T}_n(\mathcal{M}))$ is a linear map sending commutators of $\mathcal{T}_n(\mathcal{C})$ into 0.

2. Main Results

By e_{ij} $(1 \leq i \leq j \leq n)$, we denote the matrix units of $\mathcal{T}_n(\mathcal{C})$. Set $\mathbf{1} = \sum_{i=1}^{n} e_{ii}$, where $\mathbf{1}$ is the unity of $\mathcal{T}_n(\mathcal{C})$. We remark that the algebra $\mathcal{T}_n(\mathcal{C})$ is a $\mathcal{C}$-module direct sum of the subalgebra $\mathcal{D}_n(\mathcal{C})$ of all diagonal matrices and the ideal $\mathcal{S}_n(\mathcal{C})$ of all strictly upper triangular matrices. Clearly, $\mathcal{S}_n(\mathcal{C})$ is generated by all commutators in $\mathcal{T}_n(\mathcal{C})$.

Lemma 2.1. *Let $\mathcal{M}$ be a unital $\mathcal{C}$-bimodule and $\delta : \mathcal{T}_n(\mathcal{C}) \to \mathcal{T}_n(\mathcal{M})$ be a Lie triple derivation. Then, for all $1 \leq i < j \leq n$, we have*

$$\delta(e_{ii}) = \sum_{k \neq i} e_{ii}\delta(e_{ii})e_{kk} + \sum_{k \neq i} e_{kk}\delta(e_{ii})e_{ii} + \sum_{k=1}^{n} e_{kk}\delta(e_{ii})e_{kk}, \tag{2.1}$$

and

$$\delta(e_{ij}) = \delta(e_{ii})e_{ij} + e_{ii}\delta(e_{ij})e_{jj} + e_{ij}\delta(e_{jj}) - e_{ij}\delta(e_{ii})e_{jj} - e_{ii}\delta(e_{jj})e_{ij}. \tag{2.2}$$

Proof. Let $i \neq j$. Then

$$\begin{aligned} 0 &= \delta([[e_{ii}, e_{jj}], e_{jj}]) \\ &= [[\delta(e_{ii}), e_{jj}], e_{jj}] + [[e_{ii}, \delta(e_{jj})], e_{jj}] + [[e_{ii}, e_{jj}], \delta(e_{jj})] \\ &= \delta(e_{ii})e_{jj} - 2e_{jj}\delta(e_{ii})e_{jj} + e_{jj}\delta(e_{ii}) + e_{ii}\delta(e_{jj})e_{jj} + e_{jj}\delta(e_{jj})e_{ii}. \end{aligned} \tag{2.3}$$

Multiplying (2.3) from the left by $e_{kk}(k \neq i,\ k \neq j)$ gives

$$e_{kk}\delta(e_{ii})e_{jj} = 0, \quad \text{for all } i \neq j,\ j \neq k,\ k \neq i, \tag{2.4}$$

and multiplying (2.3) from the left by e_{ii} gives

$$e_{ii}\delta(e_{ii})e_{jj} + e_{ii}\delta(e_{jj})e_{jj} = 0, \quad \text{for all } i \neq j. \tag{2.5}$$

Hence by (2.4), we have

$$\delta(e_{ii}) = \mathbf{1}\delta(e_{ii})\mathbf{1} = \sum_{j,k=1}^{n} e_{jj}\delta(e_{ii})e_{kk} = \sum_{k \neq i} e_{ii}\delta(e_{ii})e_{kk}$$

$$+ \sum_{k \neq i} e_{kk}\delta(e_{ii})e_{ii} + \sum_{k=1}^{n} e_{kk}\delta(e_{ii})e_{kk}.$$

Next, let $1 \leq i < j \leq n$. Then

$$\begin{aligned}\delta(e_{ij}) &= \delta([[e_{ii}, e_{ij}], e_{jj}])\\ &= [[\delta(e_{ii}), e_{ij}], e_{jj}] + [[e_{ii}, \delta(e_{ij})], e_{jj}] + [[e_{ii}, e_{ij}], \delta(e_{jj})]\\ &= \delta(e_{ii})e_{ij} - e_{ij}\delta(e_{ii})e_{jj} - e_{jj}\delta(e_{ii})e_{ij} + e_{ii}\delta(e_{ij})e_{jj} + e_{jj}\delta(e_{ij})e_{ii}\\ &\quad + e_{ij}\delta(e_{jj}) - \delta(e_{jj})e_{ij}.\end{aligned} \tag{2.6}$$

For $m = (m_{ij}) \in \mathcal{T}_n(\mathcal{M})$, it is easy to show that

$$e_{jj}me_{ii} = m_{ji} = 0, \quad \text{for all } i < j. \tag{2.7}$$

Accordingly, it follows that

$$e_{jj}\delta(e_{ii})e_{ij} = e_{jj}\delta(e_{ii})e_{ii}e_{ij} = 0, \tag{2.8}$$

and

$$e_{jj}\delta(e_{ij})e_{ii} = 0. \tag{2.9}$$

By (2.1), it follows that

$$\delta(e_{jj})e_{ij} = e_{jj}\delta(e_{jj})e_{ij} + e_{ii}\delta(e_{jj})e_{ij} = e_{ii}\delta(e_{jj})e_{ij}. \tag{2.10}$$

Hence by (2.6)-(2.10), we have

$$\delta(e_{ij}) = \delta(e_{ii})e_{ij} + e_{ii}\delta(e_{ij})e_{jj} + e_{ij}\delta(e_{jj}) - e_{ij}\delta(e_{ii})e_{jj} - e_{ii}\delta(e_{jj})e_{ij}.$$

□

The main result of the paper states:

Theorem 2.2. *Let $\mathcal{M}$ be a 2-torsion free unital $\mathcal{C}$-bimodule. Then every Lie triple derivation $\delta : \mathcal{T}_n(\mathcal{C}) \to \mathcal{T}_n(\mathcal{M})$ is of the form $\delta = d + \tau$, where $d : \mathcal{T}_n(\mathcal{C}) \to \mathcal{T}_n(\mathcal{M})$ is a derivation and $\tau : \mathcal{T}_n(\mathcal{C}) \to Z(\mathcal{T}_n(\mathcal{M}))$ is a linear map sending commutators of $\mathcal{T}_n(\mathcal{C})$ into* 0.

Proof. Define a $\mathcal{C}$-linear map $\tau : \mathcal{T}_n(\mathcal{C}) \to \mathcal{T}_n(\mathcal{M})$ according to

$$\tau(e_{ii}) = \sum_{k=1}^{n} e_{kk}\delta(e_{ii})e_{kk}, \quad \text{for } i = 1, 2, \cdots, n, \tag{2.11}$$

and

$$\tau(e_{ij}) = 0, \quad \text{for all } 1 \leq i < j \leq n.$$

So τ sends all commutators of $\mathcal{T}_n(\mathcal{C})$ into 0. Let us prove that $\tau(\mathcal{T}_n(\mathcal{C})) \subseteq Z(\mathcal{T}_n(\mathcal{M}))$. It is enough to show that $[\tau(e_{ii}), e_{kl}] = 0$, for all $1 \leq i \leq n$ and $1 \leq k \leq l \leq n$. We consider four cases.

Case A1. $k = l$. By (2.11), we have

$$\begin{aligned}[\tau(e_{ii}), e_{kk}] &= \tau(e_{ii})e_{kk} - e_{kk}\tau(e_{ii}) \\ &= e_{kk}\delta(e_{ii})e_{kk} - e_{kk}\delta(e_{ii})e_{kk} = 0.\end{aligned}$$

Case A2. $k < l$, $k \neq i$, and $l \neq i$. Then

$$\begin{aligned}0 &= \delta([[e_{kl}, e_{ii}], e_{ll}]) \\ &= [[\delta(e_{kl}), e_{ii}], e_{ll}] + [[e_{kl}, \delta(e_{ii})], e_{ll}] + [[e_{kl}, e_{ii}], \delta(e_{ll})] \\ &= -e_{ii}\delta(e_{kl})e_{ll} - e_{ll}\delta(e_{kl})e_{ii} + e_{kl}\delta(e_{ii})e_{ll} - \delta(e_{ii})e_{kl} + e_{ll}\delta(e_{ii})e_{kl}.\end{aligned} \tag{2.12}$$

Multiplying (2.12) from the left by e_{kk} gives

$$e_{kl}\delta(e_{ii})e_{ll} - e_{kk}\delta(e_{ii})e_{kl} = 0. \tag{2.13}$$

Applying (2.11) and (2.13), we have

$$[\tau(e_{ii}), e_{kl}] = \tau(e_{ii})e_{kl} - e_{kl}\tau(e_{ii}) = e_{kk}\delta(e_{ii})e_{kl} - e_{kl}\delta(e_{ii})e_{ll} = 0.$$

Case A3. $k < l$ and $l = i$. Then

$$\begin{aligned}\delta(e_{ki}) &= \delta([[e_{ki}, e_{ii}], e_{ii}]) \\ &= [[\delta(e_{ki}), e_{ii}], e_{ii}] + [[e_{ki}, \delta(e_{ii})], e_{ii}] + [[e_{ki}, e_{ii}], \delta(e_{ii})] \\ &= \delta(e_{ki})e_{ii} - e_{ii}\delta(e_{ki})e_{ii} - e_{ii}\delta(e_{ki})e_{ii} + e_{ii}\delta(e_{ki}) + e_{ki}\delta(e_{ii})e_{ii} \\ &\quad - \delta(e_{ii})e_{ki} + e_{ii}\delta(e_{ii})e_{ki} + e_{ki}\delta(e_{ii}) - \delta(e_{ii})e_{ki}. \end{aligned} \tag{2.14}$$

Multiplying (2.14) from the left by e_{kk} and from the right by e_{ii}, we have $2(e_{ki}\delta(e_{ii})e_{ii} - e_{kk}\delta(e_{ii})e_{ki}) = 0$. As $\mathcal{T}_n(\mathcal{M})$ is 2-torsion free, it follows that

$$e_{ki}\delta(e_{ii})e_{ii} - e_{kk}\delta(e_{ii})e_{ki} = 0. \tag{2.15}$$

Applying (2.11) and (2.15), we have

$$[\tau(e_{ii}), e_{ki}] = \tau(e_{ii})e_{ki} - e_{ki}\tau(e_{ii}) = e_{kk}\delta(e_{ii})e_{ki} - e_{ki}\delta(e_{ii})e_{ii} = 0.$$

Case A4. $k < l$ and $k = i$. Similar to the proof of Case A3, we may show that

$$[\tau(e_{ii}), e_{il}] = 0.$$

Hence $\tau(\mathcal{T}_n(\mathcal{C})) \subseteq Z(\mathcal{T}_n(\mathcal{M}))$.

Now define $d : \mathcal{T}_n(\mathcal{C}) \to \mathcal{T}_n(\mathcal{M})$ by $d = \delta - \tau$. Clearly, d is a Lie triple derivation. By (2.1) and (2.11), we have

$$\begin{aligned} d(e_{ii}) &= \sum_{k \neq i} e_{ii}d(e_{ii})e_{kk} + \sum_{k \neq i} e_{kk}d(e_{ii})e_{ii} + \sum_{k=1}^{n} e_{kk}d(e_{ii})e_{kk} \\ &= \sum_{k \neq i} e_{ii}\delta(e_{ii})e_{kk} + \sum_{k \neq i} e_{kk}\delta(e_{ii})e_{ii}. \end{aligned} \tag{2.16}$$

Multiplying (2.16) from the left and from the right by e_{jj} gives

$$e_{jj}d(e_{ii})e_{jj} = 0, \quad \text{for all } 1 \leq i, \ j \leq n. \tag{2.17}$$

Since d is a Lie triple derivation, by Lemma 2.1 and (2.17), we have

$$d(e_{ii}) = \sum_{k \neq i} e_{ii}d(e_{ii})e_{kk} + \sum_{k \neq i} e_{kk}d(e_{ii})e_{ii}, \tag{2.18}$$

and

$$d(e_{ij}) = d(e_{ii})e_{ij} + e_{ii}d(e_{ij})e_{jj} + e_{ij}d(e_{jj}), \tag{2.19}$$

for all $1 \leq i < j \leq n$.

In the following, we show that d is a derivation. It is enough to check that $d(e_{ij}e_{kl}) = d(e_{ij})e_{kl} + e_{ij}d(e_{kl})$, for all $1 \leq i \leq j \leq n$ and $1 \leq k \leq l \leq n$. We consider five cases.

Case B1. $j \neq k$. Our goal is to show that $d(e_{ij})e_{kl} + e_{ij}d(e_{kl}) = 0$, for all $j \neq k$. Since d is a Lie triple derivation, it follows from the proof of (2.5) that $e_{ii}d(e_{ii})e_{jj} + e_{ii}d(e_{jj})e_{jj} = 0$, for all $i \neq j$. Hence by (2.19), we have

$$\begin{aligned} d(e_{ij})e_{kl} + e_{ij}d(e_{kl}) &= e_{ij}d(e_{jj})e_{kl} + e_{ij}d(e_{kk})e_{kl} \\ &= e_{ij}(e_{jj}d(e_{jj})e_{kk} + e_{jj}d(e_{kk})e_{kk})e_{kl} = 0. \end{aligned}$$

Case B2. $j = k$ and $i = l$. Then, we have to show that

$$d(e_{ii}) = d(e_{ii})e_{ii} + e_{ii}d(e_{ii}),$$

for all $1 \leq i \leq n$. This follows from (2.18).

Case B3. $j = k = i < l$. Our goal is to show that $d(e_{il}) = d(e_{ii})e_{il} + e_{ii}d(e_{il})$, for all $i < l$. It follows from (2.19) that

$$d(e_{il}) = d(e_{ii})e_{il} + e_{ii}d(e_{il})e_{ll} + e_{il}d(e_{ll}). \tag{2.20}$$

Multiplying (2.20) from the left by e_{ii} gives

$$e_{ii}d(e_{il}) = e_{ii}d(e_{ii})e_{il} + e_{ii}d(e_{il})e_{ll} + e_{il}d(e_{ll}). \tag{2.21}$$

From (2.17), we clearly have

$$e_{ii}d(e_{ii})e_{il} = e_{ii}d(e_{ii})e_{ii}e_{il} = 0. \tag{2.22}$$

Hence by (2.20)-(2.22), we have

$$d(e_{il}) = d(e_{ii})e_{il} + e_{ii}d(e_{il}).$$

Case B4. $i < l = j = k.$ Similar to the proof of Case B3, we may show that

$$d(e_{il}) = d(e_{il})e_{ll} + e_{il}d(e_{ll}).$$

Case B5. $i < j = k < l.$ Then, we have to show that $d(e_{il}) = d(e_{ij})e_{jl} + e_{ij}d(e_{jl}),$ for all $i < j < l.$ Since d is a Lie triple derivation, we have

$$\begin{aligned} d(e_{il}) &= d([[e_{ii}, e_{ij}], e_{jl}]) \\ &= [[d(e_{ii}), e_{ij}], e_{jl}] + [[e_{ii}, d(e_{ij})], e_{jl}] + [[e_{ii}, e_{ij}], d(e_{jl})] \\ &= d(e_{ii})e_{il} - e_{ij}d(e_{ii})e_{jl} - e_{jl}d(e_{ii})e_{ij} + e_{ii}d(e_{ij})e_{jl} + e_{jl}d(e_{ij})e_{ii} \\ &\quad + e_{ij}d(e_{jl}) - d(e_{jl})e_{ij}. \end{aligned} \tag{2.23}$$

From (2.7) and (2.19), we have

$$e_{jl}d(e_{ij})e_{ii} = 0 = e_{jl}d(e_{ii})e_{ij}, \tag{2.24}$$

and

$$d(e_{jl})e_{ij} = e_{jl}d(e_{ll})e_{ij} = 0. \tag{2.25}$$

From (2.17), we have

$$e_{ij}d(e_{ii})e_{jl} = 0. \tag{2.26}$$

Hence by (2.23)-(2.26), we have

$$d(e_{il}) = d(e_{ii})e_{il} + e_{ii}d(e_{ij})e_{jl} + e_{ij}d(e_{jl}). \tag{2.27}$$

It follows from the result of Case B3 that $d(e_{ij}) = d(e_{ii})e_{ij} + e_{ii}d(e_{ij}),$ which implies that

$$d(e_{ij})e_{jl} = d(e_{ii})e_{il} + e_{ii}d(e_{ij})e_{jl}. \tag{2.28}$$

Hence by (2.27) and (2.28), we have

$$d(e_{il}) = d(e_{ij})e_{jl} + e_{ij}d(e_{jl}).$$

The proof is complete. □

The following are some corollaries. The first corollary is a special case of [2, Theorem 2.2] and the second corollary is a special case of [1, Theorem 1.1].

Corollary 2.3. *Let* $\mathcal{T}_n(\mathcal{M})$ *be a* 2*-torsion free unital* $\mathcal{T}_n(\mathcal{C})$*-bimodule. Then every Lie derivation* $\delta : \mathcal{T}_n(\mathcal{C}) \to \mathcal{T}_n(\mathcal{M})$ *is of the form* $\delta = d + \tau$, *where* $d : \mathcal{T}_n(\mathcal{C}) \to \mathcal{T}_n(\mathcal{M})$ *is a derivation and* $\tau : \mathcal{T}_n(\mathcal{C}) \to Z(\mathcal{T}_n(\mathcal{M}))$ *is a linear map such that* $\tau([\mathcal{T}_n(\mathcal{C}), \mathcal{T}_n(\mathcal{C})]) = 0$.

Proof. Every Lie derivation is a Lie triple derivation and so the result follows from the main theorem. □

Corollary 2.4. *Let* $\mathcal{T}_n(\mathcal{M})$ *be a* 2*-torsion free unital* $\mathcal{T}_n(\mathcal{C})$*-bimodule. Then every Jordan derivation* $\delta : \mathcal{T}_n(\mathcal{C}) \to \mathcal{T}_n(\mathcal{M})$ *is a derivation.*

Proof. The well-known formula

$$[[a, b], c] = a \circ (b \circ c) - b \circ (a \circ c), \quad \text{for all } a, b, c \in \mathcal{A},$$

implies that every Jordan derivation is also a Lie triple derivation. Hence, according to the main theorem, we have $\delta = d + \tau$, where d is a derivation and $\tau : \mathcal{T}_n(\mathcal{C}) \to Z(\mathcal{T}_n(\mathcal{M}))$ is a linear map sending commutators of $\mathcal{T}_n(\mathcal{C})$ into 0. We must prove that $\tau = 0$. Since $\tau(\mathcal{S}_n(\mathcal{C})) = 0$, it is enough to see that τ vanishes on $\mathcal{D}_n(\mathcal{C})$. Since d is also a Jordan derivation, it follows that $\tau = \delta - d$ is a Jordan derivation. Now, by using that $\tau(e_{ii})$ lies in the center of $\mathcal{T}_n(\mathcal{M})$, we see that $\tau(e_{ii}) = \tau(e_{ii}^2) = 2\tau(e_{ii})e_{ii}$, for all $i = 1, 2, \cdots, n$. Multiplying this identity by e_{ii}, we get $\tau(e_{ii}) = 2\tau(e_{ii})e_{ii} = 0$. Therefore $\tau(\mathcal{D}_n(\mathcal{C})) = 0$. □

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