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SOME BILATERAL GENERATING RELATIONS INVOLVING GEGENBAUER MATRIX POLYNOMIALS

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Abstract

In this paper, we derive three bilateral generating relations of Gegenbauer matrix polynomials $C_n^A(x)$ associated with hypergeometric matrix polynomials ${}_2F_1$, ${}_1F_2$ and ${}_3F_2$ by using series rearrangement technique.

1. Introduction

It is well known that special matrix functions appear in the study in statistics, theoretical physics , groups representation theory and number theory [1, 2, 5, 8, 12]. Jo'dar and Cortes [6] introduced and studied the hypergeometric matrix function $F(A, B; C; z)$ and the hypergeometric matrix differential equations, and they also presented the explicit closed form general solution of it in [7]. Moreover, some applications of matrix

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polynomials for Fourier series expansions are given by Defez and Jo'dar [3].

Throughout the paper, the symbol $C^{N\times N}$ stands for the set of all square complex matrices of common order N and $\sigma(A)$ stands for the set of all eigenvalues of $A \in C^{N\times N}$. The matrix $A \in C^{N\times N}$ is called *positive stable* if $\mathrm{Re}(\lambda) > 0$ for all $\lambda \in \sigma(A)$. If $A_0, A_1, \ldots, A_n$ are elements of $C^{N\times N}$ and $A_n \neq 0$, then we call

$$p_n(x) = A_n x^n + A_{n-1}x^{n-1} + \ldots\ldots\ldots + A_1 x + A_0$$

a matrix polynomial of degree n in x.

For any matrix p in $C^{N\times N}$ we will exploit the following relation due to Jo'dar and Corte's [6].

$$(1-x)^{-p} = \sum_{n\geq 0} \frac{1}{n!}(p)_n x^n, \quad |x| < 1.$$

Let $A \in C^{N\times N}$ such that $\mathrm{Re}(\lambda) > 0$ for every eigenvalue $\lambda \in \sigma(A)$, then the Gegenbauer matrix polynomials $C_n^A(x)$ is defined by Sayyed et al. [11] as follows:

$$C_n^A(x) = \sum_{k=0}^{[n/2]} \frac{(-1)^k (A)_{n-k} (2x)^{n-2k}}{k!\,(n-2k)!} \tag{1}$$

and

$$F = \left(1 - 2xt + t^2\right)^{-A} = \sum_{n=0}^{\infty} C_n^A(x) t^n, \tag{2}$$

where $C_n^A(x)$ is a Gegenbauer matrix polynomials of degree n in x, and satisfies the following relation.

$$C_n^A(-x) = (-1)^n C_n^A(x)$$

$$C_n^A(1) = \frac{(2A)_n}{n!}.$$

The main objective of this paper is to present some bilateral generating functions for Gegenbauer matrix polynomials.

2. Some Auxiliary Results

For the sake of clarity in the presentation we recall some generating relations for the Gegenbauer matrix polynomials and some notations which will be used throughout the next section. The following results are given, respectively, by Sayyed et al. [11], Defez and Jo'dar [4] and Defez and Jo'dar [3]:

1- $$C_n^A(x) = \frac{(2A)_n}{n!} F\left(\begin{matrix} -nI,\ 2A+nI; \\ A+\frac{1}{2}I; \end{matrix} \ \frac{1-x}{2} \right), \tag{3}$$

and

$$C_n^A(x) = (2A)_n \sum_{k=0}^{[n/2]} \left[\left(A + \frac{1}{2} I \right)_k \right]^{-1} \frac{\left(x^2-1\right)^k x^{n-2k}}{2^{2k} k!\,(n-2k)!}. \tag{4}$$

Also,

$$\frac{(-1)^k}{(n-k)!} I = \frac{(-n)_k}{n!} I = \frac{(-nI)_k}{n!}; \quad 0 \le k \le n, \tag{5}$$

$$\frac{1}{(n-k)!} I = \frac{(-nI)_k}{n!}; \quad 0 \le 2k \le n, \tag{6}$$

and

$$(-nI)_{2k} = 2^{-2k} \left(\frac{-n}{2} I \right)_k \left(\frac{-(n-1)}{2} I \right)_k. \tag{7}$$

2- For $D \,\&\, C \in C^{r\times r}$, suppose that D is positively stable, $BC = CB$, and that

$C - D + KI$ and $C + KI$ are invertible for all nonnegative integers k. Then, for $|t| < 1$,

$$F(-nI, D; C; t) = (1-t)^n F\left(-nI, C-D; C; \frac{-t}{1-t}\right), \quad n = 0, 1, 2, 3, \ldots\ldots \quad (8)$$

3- For matrices $A(k, n)$ and $B(k, n)$ in $C^{N\times N}$, where $n \geq 0$, $k \geq 0$, the following relations are satisfied.

$$\sum_{n=0}^{\infty}\sum_{k=0}^{\infty} A(k, n) = \sum_{n=0}^{\infty}\sum_{k=0}^{[n/2]} A(k, n-2k) \quad (9)$$

and

$$\sum_{n=0}^{\infty}\sum_{k=0}^{\infty} B(k, n) = \sum_{n=0}^{\infty}\sum_{k=0}^{n} B(k, n-k). \quad (10)$$

3. Main Bilateral Generating Relations

In this section, we will study and derive some bilateral generating relations starting with replacing x by $\left(\frac{x-t}{\rho}\right)$ and t by y/ρ in Equation (2), we get:

$$\sum_{n=0}^{\infty} C_n^A\left(\frac{x-t}{\rho}\right)\rho^{-n} y^n = \left[1 - 2\left(\frac{x-t}{\rho^2}\right)y + (y/\rho)^2\right]^{-A}$$

$$\sum_{n=0}^{\infty} C_n^A\left(\frac{x-t}{\rho}\right)\rho^{-n-2A} y^n = \left[1 - 2x(t+y) + (t+y)^2\right]^{-A}$$

which by (2), yields

$$\sum_{n=0}^{\infty} C_n^A\left(\frac{x-t}{\rho}\right)\rho^{-n-2A} y^n = \sum_{n=0}^{\infty}\sum_{k=0}^{n} \frac{n!\, C_n^A(x) t^k y^{n-k}}{k!\,(n-k)!}.$$

By using (10) and, then equating the coefficients of y^n in the above, we find that

$$\rho^{-n-2A}C_n^A\left(\frac{x-t}{\rho}\right)=\sum_{k=0}^{\infty}\frac{(n+k)!C_{n+k}^A(x)t^k}{k!n!},\ \text{in which}\ \rho=\left(1-2xt+t^2\right)^{1/2}. \quad (11)$$

Now from (4), which by inserting (6) with the help of (7) yields

$$C_n^A(x)=\sum_{k=0}^{[n/2]}\frac{(2A)_n\left(x^2-1\right)^k x^{n-2k}(2)^{-2k}\left(-\frac{1}{2}nI\right)_k\left(-\frac{1}{2}n+\frac{1}{2}I\right)_k}{2^{2k}k!\,n!\left(A+\frac{1}{2}I\right)_k}$$

$$C_n^A(x)=\sum_{k=0}^{[n/2]}\frac{(2A)_n\left(x^2-1\right)^k x^{n-2k}\left(-\frac{1}{2}nI\right)_k\left(-\frac{1}{2}n+\frac{1}{2}I\right)_k}{2^{4k}k!\,n!\left(A+\frac{1}{2}I\right)_k}$$

$$=\frac{(2A)_n x^n}{n!}\,{}_2F_1\left[\begin{matrix}-\frac{1}{2}\mathrm{nI},\ \left(-\frac{1}{2}n+\frac{1}{2}\right)I;\\ A+\frac{1}{2}I;\end{matrix}\ \frac{x^2-1}{(4x)^2}\right]. \quad (12)$$

By applying (8), then (4) implies that

$$C_n^A(x)=\frac{(2A)_n}{n!}\left(\frac{1+x}{2}\right)^n{}_2F_1\left[\begin{matrix}-nI,\ \left(\left(\frac{1}{2}-n\right)I-A\right);\\ A+\frac{1}{2}I;\end{matrix}\ \frac{x-1}{x+1}\right]$$

$$=\sum_{k=0}^{n}\frac{(2A)_n(-nI)_k\left(-A-\left(n-\frac{1}{2}\right)I\right)_k(x+1)^{n-k}(x-1)^k}{2^n n!\,k!\left(A+\frac{1}{2}I\right)_k}. \quad (13)$$

By using (5), then Equation (13), yields the Bateman generating relation

$$C_n^A(x)=\sum_{k=0}^{n}\frac{(2A)_n\left(A+\frac{1}{2}I\right)_n}{k!\,(n-k)!\left(A+\frac{1}{2}I\right)_k\left(A+\frac{1}{2}I\right)_{n-k}}\left(\frac{x-1}{2}\right)^k\left(\frac{x+1}{2}\right)^{n-k} \quad (14)$$

$$\sum_{n=0}^{\infty}\frac{C_n^A(x)t^n}{(2A)_n\left(A+\frac{1}{2}I\right)_n}=\sum_{n=0}^{\infty}\sum_{k=0}^{n}\frac{t^n}{k!\,(n-k)!\left(A+\frac{1}{2}I\right)_k\left(A+\frac{1}{2}I\right)_{n-k}}\left(\frac{x-1}{2}\right)^k\left(\frac{x+1}{2}\right)^{n-k}$$

$$= \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} \frac{t^{n+k}}{k!\, n!\left(A+\frac{1}{2}I\right)_k \left(A+\frac{1}{2}I\right)_n} \left(\frac{x-1}{2}\right)^k \left(\frac{x+1}{2}\right)^n$$

$$= {}_0F\left[\begin{matrix} - \\ A+\frac{1}{2}I \end{matrix}; \frac{t(x-1)}{2}\right] {}_0F_1\left[\begin{matrix} - \\ A+\frac{1}{2}I \end{matrix}; \frac{t(x+1)}{2}\right] \qquad (15)$$

For arbitrary γ & δ and using (15) we have:

$$\sum_{n=0}^{\infty} \frac{(\gamma)_n (\delta)_n C_n^A(x) t^n}{(2A)_n \left(A+\frac{1}{2}I\right)_n}$$

$$= \sum_{n=0}^{\infty}\sum_{k=0}^{n} \frac{(\gamma)_n (\delta)_n t^n}{k!\,(n-k)!\left(A+\frac{1}{2}I\right)_k \left(A+\frac{1}{2}I\right)_{n-k}} \left(\frac{x-1}{2}\right)^k \left(\frac{x+1}{2}\right)^{n-k}$$

$$= \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} \frac{(\gamma)_{n+k} (\delta)_{n+k}}{k!\, n!\left(A+\frac{1}{2}I\right)_k \left(A+\frac{1}{2}I\right)_n} \left(\frac{1}{2}t(x-1)\right)^k \left(\frac{1}{2}t(x+1)\right)^n$$

$$= F_4\left[\gamma,\, \delta;\, A+\frac{1}{2}I,\, A+\frac{1}{2}I;\, \left(\frac{1}{2}t(x-1)\right), \left(\frac{1}{2}t(x+1)\right)\right]. \qquad (16)$$

Lemma 1. *Let B, C and $D \in C^{r\times r}$ such that C and $(I - D + B + C)$ are positively stable, then*

$$F_4(B,\, C;\, D,\, I - B + C + D;\, -u/[(1-u)(1-\nu)], -\nu/[(1-u)(1-\nu)])$$

$$= {}_2F\left[\begin{matrix} B,\, C \\ D \end{matrix}; \frac{-u}{1-u}\right]_2 F\left[\begin{matrix} B,\, C \\ I - D + B + C \end{matrix}; \frac{-\nu}{1-\nu}\right]. \qquad (17)$$

Now, in $F_4(2.9)$ set $(A + (1/2)I) = I - (A + (1/2)I) + \gamma + \delta$ or $\delta = (2A - \gamma)$ and $-u/[(1-u)(1-\nu)] = [t(x-1)/2]$, $-\nu/[(1-u)(1-\nu)] = [t(x+1)/2]$ and using Lemma 1, we get Barafman's generating function

$$\sum_{n=0}^{\infty} \frac{(\gamma)_n (2A-\gamma)_n C_n^A(x) t^n}{(2A)_n \left(A+\frac{1}{2}I\right)_n} = {}_2F_1\left[\begin{matrix}\gamma,\ 2A-\gamma; \\ A+\frac{1}{2}I;\end{matrix}\ \frac{1+t-\sigma}{2}\right] {}_2F_1\left[\begin{matrix}\gamma,\ 2A-\gamma; \\ A+\frac{1}{2}I;\end{matrix}\ \frac{1-t-\sigma}{2}\right],$$

where $\rho = \left(1-2xt+t^2\right)^{1/2}$. (18)

Since $\sum_{n=0}^{\infty} C_n^A(x) t^n = \left(1-2xt+t^2\right)^{-A}$, then replacing x by $\frac{x-t}{\rho}$ and t by $\frac{\nu}{\rho}$ to get

$$\sum_{n=0}^{\infty} C_n^A\left(\frac{x-t}{\rho}\right)\left(\frac{\nu}{\rho}\right)^n = \left[1-2\left(\frac{x-t}{\rho}\right)\left(\frac{\nu}{\rho}\right)+\left(\frac{\nu}{\rho}\right)^2\right]^{-A}$$

or

$$\sum_{n=0}^{\infty} C_n^A\left(\frac{x-t}{\rho}\right)\rho^{-n-2A}\nu^n = \left[1-2x(t+\nu)+(t+\nu)^2\right]^{-A}$$

from (2), we have

$$\sum_{n=0}^{\infty} C_n^A\left(\frac{x-t}{\rho}\right)\rho^{-n-2A}\nu^n = \sum_{n=0}^{\infty} C_n^A(x)(t+\nu)^n$$

$$= \sum_{n=0}^{\infty}\sum_{k=0}^{n} \frac{n!\, C_n^A(x) t^k \nu^{n-k}}{k!\,(n-k)!}$$

$$= \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} \frac{(n+k)!\, C_{n+k}^A(x) t^k \nu^n}{k!\, n!}$$

by identification of the coefficients of ν^n, another form for the Gegenbauer matrix polynomials follows

$$\rho^{-n-2A} C_n^A\left(\frac{x-t}{\rho}\right) = \sum_{k=0}^{\infty} \frac{(n+k)!\, C_{n+k}^A(x) t^k}{k!\, n!}, \text{ where } \rho = \left(1-2xt+t^2\right)^{1/2}. \quad (19)$$

Corollary 1. *Let* D *and* $C \in C^{r\times r}$ *such that* D *is positively stable,* $BC = CB$, *and that* $C - D + KI$ *and* $C + KI$ *are invertible for all nonnegative integers* k.

Then, for $|t| < 1$,

$$F(-nI,\ D;\ C;\ t) = (1-t)^{C+n-D}F(C+nI,\ C-D;\ C;\ t).$$

Proof. By replacing $y = \dfrac{-t}{1-t}$ in the right hand side of (8), becomes

$$F(-nI,\ C-D;\ C;\ y) = (1-y)^{-C+D}F\left(C+nI,\ C-D;\ C;\ \frac{-y}{1-y}\right),$$

since $1 - y = (1-t)^{-1}$ and $-y/(1-y) = t$, then we have

$$F\left(-nI,\ C-D;\ C;\ \frac{-t}{1-t}\right) = (1-t)^{C-D}F(C+nI,\ C-D;\ C;\ t)$$

which combines with (8) to yield

$$F(-nI,\ D;\ C;\ t) = (1-t)^{C+n-D}F(C+nI,\ C-D;\ C;\ t).$$

Now, we consider the following cases

(i) for arbitrary γ and, using (4), one gets

$$\sum_{n=0}^{\infty}\frac{(\gamma)_n C_n^A(x)t^n}{(2A)_n} = \sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\frac{(\gamma)_{n+2k}x^n\left(x^2-1\right)^k t^{n+2k}}{2^{2k}k!\,n!\left(A+\frac{1}{2}I\right)_k}$$

$$= \sum_{k=0}^{\infty}\frac{((1/2)\gamma I)_k(((1/2)\gamma+(1/2))I)_k x^n\left(x^2-1\right)^k t^{2k}}{k!\,(A+(1/2)I)_k(1-xt)^{\gamma+2k}},$$

which gives another hypergeometric matrix representation in the form:

$$= (1-xt)^{-\gamma}{}_2F_1\left[\begin{matrix}\frac{1}{2}\gamma I,\ \left(\frac{1}{2}\gamma+\frac{1}{2}\right)I;\\ A+\frac{1}{2}I;\end{matrix}\ \frac{t^2\left(x^2-1\right)}{(1-xt)^2}\right]. \tag{20}$$

(ii) From (20) replace x by $\left(\frac{x-t}{\rho}\right)$ and t by $-ty/\rho$, where $\rho = \left(1-2xt+t^2\right)^{1/2}$, we get

$$\rho^{2\gamma}\left(\rho^2+xyt-yt^2\right)^{-\gamma}\sum_{n=0}^{\infty}\frac{((\gamma/2)I)_n\left(\left(\frac{\gamma+1}{2}\right)I\right)(ty)^{2n}\left(x^2-1\right)^n}{\left(A+\frac{1}{2}I\right)_n n!\left(\rho+xyt-yt^2\right)^{2n}}$$

$$=\sum_{n=0}\frac{(\gamma)_n}{(2A)_n}C_n^A\left(\frac{x-t}{\rho}\right)\left(\frac{ty}{\rho}\right)^n \tag{21}$$

with application of (8), then (21) implies that

$$\rho^{2\gamma}\left(\rho^2+xyt-yt^2\right)^{-\gamma}{}_2F_1\left[\begin{matrix}(\gamma/2)I,\ \left(\frac{\gamma+1}{2}\right)I;\\ A+\frac{1}{2}I;\end{matrix}\frac{(yt)^2\left(x^2-1\right)}{\left(\rho+xyt-yt^2\right)^2}\right]$$

$$=\sum_{n=0}^{\infty}\frac{(-1)^n(\gamma)_n y^n\rho^{2A}}{(2A)_n}\sum_{k=0}^{\infty}\frac{(k+n)!}{k!\,n!}C_{k+n}^A(x)t^{k+n}. \tag{22}$$

Now, the main results for bilateral generating relations are given by:

(1) By using the identity 10 in the right hand side of (22), we get

$$\rho^{2\gamma}\left(\rho^2+xyt-yt^2\right)^{-\gamma}{}_2F_1\left[\begin{matrix}(\gamma/2)I,\ \left(\frac{\gamma+1}{2}\right)I;\\ A+\frac{1}{2}I;\end{matrix}\frac{(yt)^2\left(x^2-1\right)}{\left(\rho+xyt-yt^2\right)^2}\right]$$

$$=\sum_{k=0}^{\infty}C_k^A(x)t^k\sum_{n=0}^{k}\frac{(-k)_n(\gamma)_n y^n\rho^{2A}}{(2A)_n n!}$$

$$\rho^{2\gamma-2A}\left(\rho^2+xyt-yt^2\right)^{-\gamma}{}_2F_1\left[\begin{matrix}(\gamma/2)I,\ \left(\frac{\gamma+1}{2}\right)I;\\ A+\frac{1}{2}I;\end{matrix}\frac{(yt)^2\left(x^2-1\right)}{\left(\rho+xyt-yt^2\right)^2}\right]$$

$$=\sum_{k=0}^{\infty}C_k^A(x)\,{}_2F_1\left[\begin{matrix}-k,\ \gamma;\\ 2A;\end{matrix}\ y\right]t^k. \tag{23}$$

For special case, setting $y = 1$ and using the Corollary (1), we get the following generating relation:

$$\sum_{k=0}^{\infty} C_k^A(x)\frac{(2A-\gamma)t^k}{(2A)_k}$$

$$= (1-xt)^{\gamma-2A}{}_2F_1\left[\begin{matrix} A-(\gamma/2)I,\ \left(\frac{I-\gamma I+2A}{2}\right); \\ A+\frac{1}{2}I; \end{matrix}\ \frac{t^2\left(x^2-1\right)}{(1-xt)^2}\right].$$

(2) Replacing x by $\left(\frac{x-t}{\rho}\right)$ and t by $-ty/\rho$ in Equation (15), where $\rho = \left(1-2xt+t^2\right)^{1/2}$, we get

$$\sum_{n=0}^{\infty}\frac{(-ty)^n\left(\frac{x-t-\rho}{2\rho^2}\right)^n}{\left(A+\frac{1}{2}I\right)_n n!}\sum_{n=0}^{\infty}\frac{(-ty)^k\left(\frac{x-t+\rho}{2\rho^2}\right)^k}{\left(A+\frac{1}{2}I\right)_k k!}$$

$$= \sum_{n=0}^{\infty}\frac{1}{(2A)_n\left(A+\frac{1}{2}I\right)_n}C_n^A\left(\frac{x-t}{\rho}\right)(-ty)^n\rho^{-n}$$

$${}_0F_1\left[\begin{matrix}-\\ A+\frac{1}{2}I;\end{matrix};\ \frac{-yt(x-t-\rho)}{2\rho^2}\right]{}_0F_1\left[\begin{matrix}-\\ A+\frac{1}{2}I;\end{matrix};\ \frac{-yt(x-t+\rho)}{2\rho^2}\right].$$

From (10), it follows that

$$\rho^{-2A}{}_0F_1\left[\begin{matrix}-\\ A+\frac{1}{2}I;;\end{matrix}\ \frac{-yt(x-t-\rho)}{2\rho^2}\right]{}_0F_1\left[\begin{matrix}-\\ A+\frac{1}{2}I;\end{matrix};\ \frac{-yt(x-t+\rho)}{2\rho^2}\right]$$

$$= \sum_{k=0}^{\infty}C_k^A(x)\ {}_1F_2\left[\begin{matrix}-k;\\ 2A,\ A+\frac{1}{2}I;\end{matrix}\ y\right]t^k. \tag{24}$$

(3) By replacing x by $\left(\frac{x-t}{\rho}\right)$ and t by $-ty/\rho$ in Equation (18), where $\rho = \left(1-2xt+t^2\right)^{1/2}$ and, then using the right hand side of relation (8), yields

$$\sum_{n=0}^{\infty}\frac{(\gamma)_n(2A-\gamma)_n}{\left(A+\frac{1}{2}I\right)_n}\left(\frac{\rho-ty-\rho^2}{2\rho}\right)^n\sum_{k=0}^{\infty}\frac{(\gamma)_k(2A-\gamma)_k}{\left(A+\frac{1}{2}I\right)_k}\left(\frac{\rho+ty-\rho^2}{2\rho}\right)^k$$

$$=\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\frac{(\gamma)_n(2A-\gamma)_n(k+n)!}{(2A)_n k!\,n!\left(A+\frac{1}{2}I\right)_n}C_{k+n}^{A}(x)t^k, \tag{25}$$

which can be write in the form

$$\sum_{k=0}^{\infty}C_k^A(x)\ {}_3F_2\left[\begin{matrix}-m,\ \gamma,\ 2A-\gamma;\\ 2A,\ A+\frac{1}{2}I;\end{matrix}\ y\right]t^k$$

$$=\rho^{-2A}\,{}_2F_1\left[\begin{matrix}\gamma,\ 2A-\gamma\\ A+\frac{1}{2}I;\end{matrix}\ ;\ \frac{(\rho+yt-\rho z)}{2\rho}\right]{}_2F_1\left[\begin{matrix}\gamma,\ 2A-\gamma\\ A+\frac{1}{2}I;\end{matrix}\ ;\ \frac{(\rho-yt-\rho z)}{2\rho}\right], \tag{26}$$

where

$$\rho=\left(1-2xt+t^2\right)^{1/2},\ \text{and}$$

$$z=\sqrt{\rho^2+2ty(x-t)+t^2y^2}\ /\ \rho\ \ or\ \sqrt{1+\left(2ty(x-t)+t^2y^2\right)/\left(1-2xt+t^2\right)}.$$

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