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ADAPTIVE SYNCHRONIZATION OF LU'S SYSTEM

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Abstract

In this paper, we study Lu's system, and we study the stability of equilibrium point of Lu's system. Then, we also study chaos synchronization of Lu's system by using adaptive control methods.

1. Introduction

Beginning with [11], synchronization of chaotic systems becomes a popular research topic [4], [5], [6], [8], [17]. Two important notions are full (or complete) synchronization, (i.e., when all trajectories of the synchronized systems asymptotically converge to each other [3], [8], [10]) and partial synchronization (when it is necessary to ensure convergence only for part of systems variables [16], [17]). If the models of systems to be synchronized contain parametric uncertainties and adaptation techniques are to be designed, then the problem is called adaptive synchronization problem [3], [7]. Besides, chaos in control systems and controlling chaos in dynamical systems have both attracted increasing

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attention in recent years (see in [2], [3], [9], [10], [12], [13], [14], and references therein). A chaotic system has complex dynamical behaviors that possess some special features, such as being extremely sensitive to tiny variations of initial conditions, having bounded trajectories in the phase space. Controlling chaos has focused on the nonlinear systems such as a Lu's system.

Lu's system was first introduced in [1], which is described by

$$\begin{aligned} \dot{x} &= a(y - x), \\ \dot{y} &= -xz + cy, \\ \dot{z} &= xy - bz, \end{aligned} \tag{1.1}$$

where x, y, z are state variables and a, b, c are positive constants.

The objective of this paper is as follow. We investigate adaptive synchronization for Lu's system, when the parameters of the drive system are fully unknown and different with those of the response system.

2. Synchronization of the Lu's System

Consider two nonlinear systems:

$$\dot{x} = f(t, x), \tag{$*$}$$

$$\dot{y} = g(t, y) + u(t, x, y), \tag{$**$}$$

where
$x, y \in R^n$, $f, g \in C^r[R^+ \times R^n, R^n]$, $u \in C^r[R^+ \times R^n \times R^n]$, $r \geq 1$, R^+ is the set of nonnegative real numbers. Assume that $(*)$ is the drive system, $(**)$ is the response system, and $u(t, x, y)$ is the control vector.

Definition 2.1. Response system and drive system are said to be *synchronic*, if for any initial conditions $x(t_0)$, $y(t_0) \in R^n$,

$$\lim_{t \to +\infty} \|x(t) - y(t)\| = 0.$$

In this section, we consider adaptive synchronization for Lu's systems. This approach can synchronize the chaotic systems, when the parameters of the drive system are fully unknown and different with those of the response system. Assume that, there are two Lu's systems such that the drive system (with the subscript 1) is to control the response system (with the subscript 2). The drive and response system are given, respectively, by

$$\begin{aligned} \dot{x}_1 &= a(y_1 - x_1), \\ \dot{y}_1 &= -x_1 z_1 + c y_1, \\ \dot{z}_1 &= x_1 y_1 - b z_1, \end{aligned} \tag{2.1}$$

where the parameters a, b, c are unknown or uncertain, and

$$\begin{aligned} \dot{x}_2 &= a_1(y_2 - x_2) - u_1, \\ \dot{y}_2 &= -x_2 z_2 + c_1 y_2 - u_2, \\ \dot{z}_2 &= x_2 y_2 - b_1 z_2 - u_3, \end{aligned} \tag{2.2}$$

where a_1, b_1, c_1 are parameters of the response system, which need to be estimated, and $u = [u_1, u_2, u_3]^T$ is the controller, we introduced in (2.2). We choose

$$\begin{aligned} u_1 &= k_1 e_x - a(x_2 - x_1), \\ u_2 &= k_2 e_y + c(y_2 - y_1), \\ u_3 &= k_3 e_z - b(z_2 - z_1), \end{aligned} \tag{2.3}$$

where e_x, e_y, e_z are the error states, which are defined as follows:

$$\begin{aligned} e_x &= x_2 - x_1, \\ e_y &= y_2 - y_1, \\ e_z &= z_2 - z_1, \end{aligned}$$

and

$$\begin{aligned} \dot{a}_1 &= f_a = -\gamma(-x_2 e_x - y_2 e_x), \\ \dot{b}_1 &= f_b = \theta y_2 e_z, \\ \dot{c}_1 &= f_c = -\delta x_2 e_z, \end{aligned} \tag{2.4}$$

where $k_1, k_2, k_3 \geq 0$ and γ, θ, δ are positive real constants.

Theorem 2.1. *Let $k_1, k_2, k_3 \geq 0$ be properly chosen so that the following matrix inequality holds,*

$$P = \begin{pmatrix} k_1 - a & -\frac{1}{2}(a+1) & 0 \\ -\frac{1}{2}(a+1) & k_2 + 1 & -\frac{1}{2}(1-b) \\ 0 & -\frac{1}{2}(1-b) & k_3 \end{pmatrix} > 0, \tag{2.5}$$

or k_1, k_2, k_3 can be chosen so that the following inequalities holds,

(i) $A = k_1 - a > 0$;

(ii) $B = A(k_2 + 1) - \frac{1}{4}(a+1)^2 > 0$;

(iii) $C = A(k_2 + 1)k_3 - A\frac{1}{4}(1-b)^2 - \frac{k_3}{4}(a+1) > 0$,

then the two Lu's systems (2.1) *and* (2.2) *can be synchronized under the adaptive controls* (2.3) *and* (2.4).

Proof. It is easy to see from (2.1) and (2.2), that the error system is

$$\begin{aligned} \dot{e}_x &= a_1(y_2 - x_2) - a(y_1 - x_1) - u_1, \\ \dot{e}_y &= -x_2 z_2 + c_1 y_2 + x_1 z_1 - c y_1 - u_2, \\ \dot{e}_z &= x_2 y_2 - b_1 z_2 - x_1 y_1 + b z_1 - u_3. \end{aligned} \tag{2.6}$$

Let $e_a = a_1 - a$, $e_b = b_1 - b$, and $e_c = c_1 - c$. Choose the Lyapunov function as follows:

$$V(t) = \frac{1}{2}\left(e_x^2 + e_y^2 + e_z^2 + \frac{1}{\gamma}e_a^2 + \frac{1}{\theta}e_b^2 + \frac{1}{\delta}e_c^2\right).$$

Then, the differentiation of V along trajectories of (2.6) is

$$\begin{aligned}
\dot{V} &= e_x\dot{e}_x + e_y\dot{e}_y + e_z\dot{e}_z + \frac{1}{\gamma}e_a\dot{e}_a + \frac{1}{\theta}e_b\dot{e}_b + \frac{1}{\delta}e_c\dot{e}_c \\
&= e_x[a_1(y_2 - x_2) - a(y_1 - x_1) - u_1] + e_y[-x_2z_2 + c_1y_2 + x_1z_1 - cy_1 - u_2] \\
&\quad + e_z[x_2y_2 - b_1z_2 - x_1y_1 + bz_1 - u_3] + \frac{1}{\gamma}e_af_a + \frac{1}{\theta}e_bf_b + \frac{1}{\delta}e_cf_c \\
&= e_x[a_1y_2 - ay_1 + ay_2 - ay_2] - e_x[a_1x_2 - ax_1 + ax_2 - ax_2] \\
&\quad + e_y[x_1z_1 - x_2z_2 + x_2z_1 - x_2z_1] + e_y[c_1y_2 - cy_1 + cy_2 - cy_2] \\
&\quad + e_z[x_2y_2 - x_1y_1 + x_1y_2 - x_1y_2] + e_z[bz_1 - b_1z_2 + b_1z_1 - b_1z_1] \\
&\quad - e_xu_1 - e_yu_2 - e_zu_3 + \frac{1}{\gamma}e_af_a + \frac{1}{\theta}e_bf_b + \frac{1}{\delta}e_cf_c \\
&= y_2e_ae_x + ae_xe_y - e_ae_x - (x_2 - x_1)e_x + x_2e_ae_x + e_x^2e_xe_y \\
&\quad - e_y^2 + e_ye_z - y_2e_be_z - be_ye_z + x_2e_ce_z + c(x_2 - x_1)e_z - k_1e_x^2 \\
&\quad + (x_2 - x_1)e_x - k_2e_y^2 - k_3e_z^2 - c(x_2 - x_1)e_z + \frac{1}{\gamma}e_af_a + \frac{1}{\theta}e_bf_b + \frac{1}{\delta}e_cf_c \\
&= -(k_1 - a)e_x^2 - (k_2 + 1)e_y^2 - k_3e_z^2 + (a + 1)e_xe_y + (1 - b)e_ye_z \\
&\quad + e_a\left[\frac{1}{\gamma}f_a + y_2e_x + x_2e_x - e_x\right] + e_b\left[\frac{1}{\theta}f_b - y_2e_z\right] + e_c\left[\frac{1}{\delta}f_c + x_2e_z\right] \\
&\le -(k_1 - a)e_x^2 - (k_2 + 1)e_y^2 - k_3e_z^2 + (a + 1)|e_xe_y| + (1 - b)|e_ye_z| \\
&= -e^TPe,
\end{aligned}$$

where $e = \left[|e_x|\,|e_y|\,|e_z|\right]^T$ and P is as in (2.5). Since, $V(t)$ is positive definite and $\dot{V}(t)$ is negative semidefinite, it follows that e_x, e_y, e_z, a_1, b_1, $c_1 \in L_\infty$. From $\dot{V}(t) \le -e^TPe$, we can easily show that the square of e_x, e_y, e_z are integrable with respect to t, namely, e_x, e_y, $e_z \in L_2$. From

(2.6), for any initial conditions, we have $\dot{e}_x(t), \dot{e}_y(t), \dot{e}_z(t) \in L_\infty$. By the well-known Barbalat's lemma, we conclude that $(e_x, e_y, e_z) \to (0, 0, 0)$ as $t \to +\infty$. Therefore, in the closed-loop system, $x_2(t) \to x_1(t)$, $y_2(t) \to y_1(t)$, $z_2(t) \to z_1(t)$ as $t \to +\infty$. This implies that the two Lu's systems have synchronized under the adaptive controls (2.3) and (2.4). □

Numerical Simulations

The numerical simulations are carried out using the fourth-order Runge-Kutta method. The initial states are $x_1(0) = 0.65$, $y_1(0) = 0$, $z_1(0) = 0$ for the drive system, and $x_2(0) = 0.2$, $y_2(0) = 0.1$, $z_2(0) = 0.1$ for the response system. The parameters of the drive system are $a = 5$, $b = 10$, and $c = 0.5$. The control parameters are chosen as follows $k_1 = 5$, $k_2 = 50$, $k_3 = 21$, which satisfy (2.5). Choose $\gamma = \theta = \delta = 1$. The initial values of the parameters a_1, b_1, c_1 are all chosen to be 0. As shown in Figure 1, the response system synchronizes with the drive system. The changing parameters of a_1, b_1, c_1 are shown in Figures 2-4.

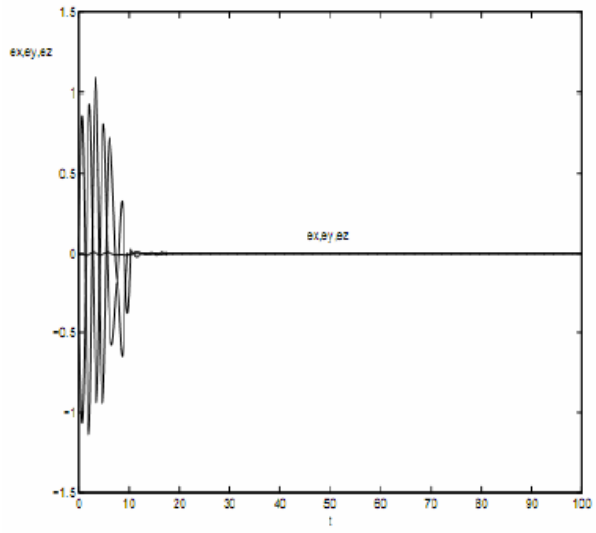

Figure 1. Synchronization error (e_x, e_y, e_z) for systems (2.1) and (2.2) with time t.

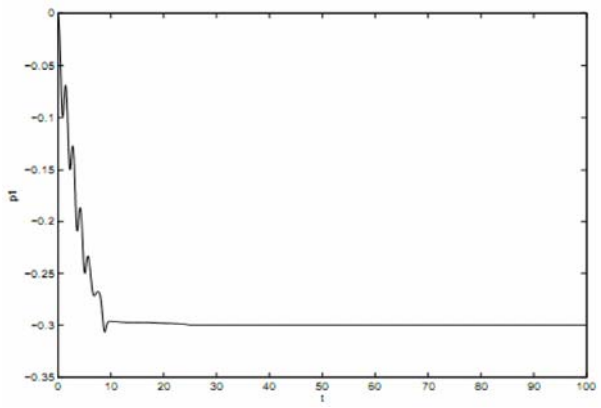

Figure 2. Changing parameter a_1 of the system (2.2) with time t.

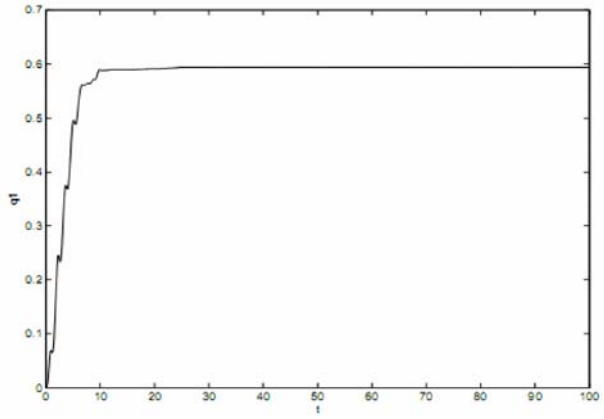

Figure 3. Changing parameter b_1 of the system (2.2) with time t.

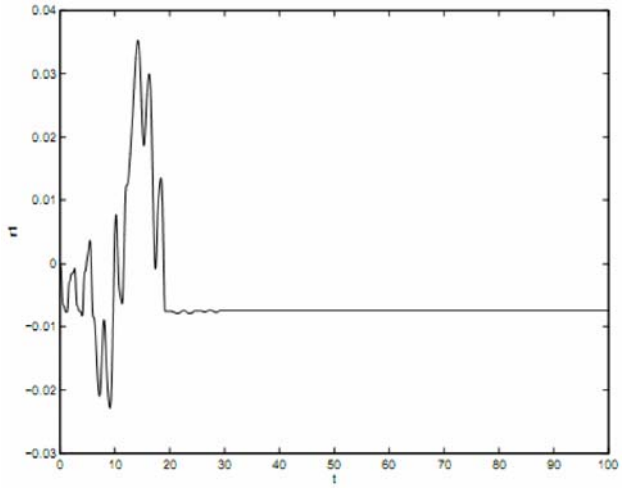

Figure 4. Changing parameter c_1 of the system (2.2) with time t.

3. Conclusions

In this paper, we give sufficient conditions for stability of equilibrium points of synchronization of two Lu's systems using adaptive control, which control the chaotic behavior of Lu's system to its equilibrium points. Numerical simulations are also given to verify results that we obtained.

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