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ON THE RECURSIONS OF TWO-DIMENSIONAL SUBSET AUTOREGRESSIONS

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Abstract

An innovative set of recursive algorithms for two-dimensional (2D) subset autoregressive (AR) models, including full order 2D ARs, is explored and developed by progressing further from the 2D full order AR recursions. After the initialisation is carried out by the direct method, the proposed recursions then allow us to develop successively all 2D subset ARs, of size κ (the number of lags with nonzero coefficient matrices) from 1 to K. Finally, the best subsets of each size with the minimum generalised residual power for that size are compared by employing a model selection criterion to find the optimum 2D subset AR.

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1. Introduction

Recent decades have witnessed a remarkable development in the statistical analysis of biomedical and environmental time-series signals [1-3, 5-7, 11]. In particular, recursive algorithms for fitting one-dimensional (1D) subset AR models have proven to be very successful in many biomedical and environmental time-series applications [8-10]. The use of the recursive method [5] for 2D AR fitting and spectrum analysis has been applied to biomedical signal processing, image processing, digitised radar processing, and seismic analysis. The choice of the model's order is an important feature of these analyses for reasons that will become obvious. Penm and Terrell [8] proposed a recursive algorithm to select the optimum 1D subset autoregression. Subsequently, Nakachi et al. [5] used a dimensional ordering approach to derive all their full-order algorithms for 2D full-order structures, which allow us to handle the forward and backward linear prediction problems in the 2D full-order context. This paper provides forward and backward space time recursions that recursively estimate 2D subset AR models, including 2D full-order AR models.

We have observed that model identification algorithms for the full-order 2D AR model have been developed [5]. However, if we simply set zero values to the coefficients of the missing lags and then use the proposed algorithm, it will *fail* as the properties of a 2D subset autoregression disappear in such circumstances. This justifies the need to develop an efficient model identification algorithm for fitting 2D subset AR models. In this paper, an order selection criterion such as the modified 2D Hannan information criterion [4] could be used at each order level with the proposed recursive algorithm to select the optimum 2D subset AR model.

Our algorithm can be applied to any subset issues (including full order) and is therefore applicable to a very wide class of 2D time-series filtering and prediction situations, such as forecasting of load problems in power systems as well as other 2D digitised data processing systems, where periodic signals arise. More importantly, if the underlying true 2D AR process has a *subset* structure, the suboptimal model specification (for instance, a full-order structure) can give rise to the identification of incorrect relationships among the various variables, thus producing inefficient estimates and inferior projections. Filtering, smoothing, and spectral estimation algorithms based on such a resulting 2D full-order model would lead to inaccurate representations.

2. Time-Update Recursions for Subset Autoregressions

We model the 2-D data sequence with $L \times K$ pixels as a 2D AR process:

$$x(n_1, n_2) + \sum_{(i, j) \in R} a_{i, j} x(n_1 - i, n_2 - j) = \varepsilon(n_1, n_2),$$

where $\varepsilon(n_1, n_2)$ is a zero-mean white 2D Gaussian process with variance σ^2, and $a_{i, j}$ are the model parameters to be estimated. The n_1 denotes the south direction, and the n_2 denotes the east direction. We associate the "past" of a 2D data with a region R, where

$$R = \{(i, j) | (i \geq 0, j > 0), (i \geq 1, j \leq 0)\}.$$

We consider a forward causal subset AR model with the deleted lags $i_1, i_2, \ldots, i_\eta$; and $j_1, j_2, \ldots, j_\rho$ of the form

$$x(n_1, n_2) + \sum_{(i, j) \in R} a_{i, j} x(n_1 - i, n_2 - j; I_\eta) = \varepsilon(n_1, n_2),$$

$$\{a_{i, j}(I_\eta) = 0, \text{ as } (i, j) \in I_\eta\}, \qquad (2.1)$$

where I_η represents an integer set with elements $(i_1, j_1), (i_2, j_2), \ldots, (i_\eta, j_\eta)$.

Further, we consider a backward causal subset AR model of the form

$$x(n_1, n_2) + \sum_{(i, j) \in R} b_{i, j} x(n_1 + i, n_2 + j; L_\eta) = \bar{\varepsilon}(n_1, n_2),$$

$$\{b_{i, j}(L_\eta) = 0, \text{ as } (i, j) \in L_\eta\}, \qquad (2.2)$$

where L_η represents an integer set with elements $(i^1, j^1), (i^2, j^2), \ldots,$ (i^η, j^η). A reciprocal integer pair for a forward causal subset AR model and a backward causal subset AR model is a pair of (2.1) and (2.2). Figure 1 shows causal forward lags and causal backward lags of $x(n_1, n_2)$. Figure 2 illustrates the reciprocal integer pairs of all subset AR processes up to and including maximal lag length of (1,1). In this paper, numerals represent particular lags in a forward causal AR and underlined numerals represent such lags in a backward causal AR.

$x(n_1 - 1, n_2 - 1)$	$x(n_1 - 1, n_2)$	$x(n_1, n_2 + 1)$
$x(n_1, n_2 - 1)$	$x(n_1, n_2)$	$x(n_1, n_2 + 1)$
$x(n_1 + 1, n_2 - 1)$	$x(n_1 + 1, n_2)$	$x(n_1 + 1, n_2 + 1)$

: causal forward lags : causal backward lags

Figure 1. Causal forward lags and causal backward lags of $x(n_1, n_2)$.

Forward AR	Backward AR	Forward AR	Backward AR
(0 1)	$(\underline{0}\ \underline{-1})$	(0 1)(1 –1)	$(\underline{0}\ \underline{-1})(\underline{-1}\ \underline{1})$
(1– 1)	$(\underline{-1}\ \underline{1})$	(0 1)(1 0)	$(\underline{-1}\ \underline{1})\ (\underline{-1}\ \underline{0})$
(1 0)	$(\underline{-1}\ \underline{0})$	(0 1)(1 1)	$(\underline{-1}\ \underline{0})(\underline{-1}\ \underline{-1})$
(1 1)	$(\underline{-1}\ \underline{-1})$	(1 –1)(1 0)	$(\underline{0}\ \underline{-1})(\underline{-1}\ \underline{0})$
		(1 –1) (1 1)	$(\underline{-1}\ \underline{1})(\underline{-1}\ \underline{-1})$
		(1 0) (1 1)	$(\underline{0}\ \underline{-1})(\underline{-1}\ \underline{-1})$

Forward AR	Backward AR	Forward AR	Backward AR
(0 1) (1 –1) (1 0)	$(\underline{0}\ \underline{-1})\ (\underline{-1}\ \underline{1})\ (\underline{-1}\ \underline{0})$	(0 1) (1 –1) (1 0) (1 1)	$(\underline{0}\ \underline{-1})\ (\underline{-1}\ \underline{1})\ (\underline{-1}\ \underline{0})\ (\underline{-1}\ \underline{-1})$
(0 1) (1 –1) (1 1)	$(\underline{-1}\ \underline{1})\ (\underline{-1}\ \underline{0})\ (\underline{-1}\ \underline{-1})$		
(0 1) (1 0) (1 1)	$(\underline{0}\ \underline{-1})\ (\underline{-1}\ \underline{0})\ (\underline{-1}\ \underline{-1})$		
(1 –1) (1 0) (1 1)	$(\underline{0}\ \underline{-1})\ (\underline{-1}\ \underline{1})\ (\underline{-1}\ \underline{-1})$		

Figure 2. Reciprocal integer pairs of subset AR models up to and including maximal lag length of (1,1). In this paper, numerals represent particular lags in a forward causal AR and underlined numerals represent such lags in a backward causal AR.

In 2D radar signal processing $[xx]$, X-ray digital imaging $[xx]$ and biomedical digitized picture processing $[xx]$, the data are considered to be updated in a helicoidal fashion when data have arrived at the end of the row and restarted at the beginning of the next row. Here, we introduce a linear scanning index n, where $n = n_2L + n_1$.

Following $[xx]$, we define an i-th "channel" as the i-th row in the 2D support region. Let $y_\alpha(n)$ be the i-th channel data at position "n" such as

$$y_\alpha(n) = x(n + (\alpha - 1)L).$$

The index $\boldsymbol{\lambda}$ of 2D region R denotes the number of its element. In this case, a one-to-one mapping $\boldsymbol{\beta}$ between 2D region R and an integer set $\boldsymbol{\lambda} = \{1, 2, \ldots, \lambda\}$ are defined by

$$\boldsymbol{\beta}\textbf{:}\ R \to \boldsymbol{\lambda};\ I_\eta \to I_s;\ \text{and}\ L_\eta \to M_s. \tag{2.3}$$

For a specific $\boldsymbol{\beta}$, we introduce the notation

$$(m;\ n) - \delta = (m-i;\ n-j),\ (i,\ j) \in R, \quad \delta = 1, 2, \ldots, \lambda.$$

The forward and backward prediction errors are now defined by

$$e_{q,(m,n)-(\lambda+1)}(I_s) = [I|A'_{q,\lambda}(I_s)]Y_{q,(m,n)-(\lambda+1)}(L_s);$$

$$\bar{e}_{q,(m,n)-(\lambda+1)}(I_s) = [A'_{q,\lambda}(I_s)|I]Y_{q,(m,n)-(\lambda+1)}(M_s),$$

where I_s is used in (2.3), and represents an integer set with elements $i_1, i_2, \ldots, i_s, 1 \le i_1 \le i_2 \ldots \le i_s \le q-1$;

M_s is used in (2.3), and represents an integer set with elements $m_1, m_2, \ldots, m_s, m_j = q - i_j,\ j = 1, 2, \cdots, s$; and

L_s represents an integer set with elements $1_j,\ j = 1, \cdots, s,$ and $1_j = i_j + 1$.

As q is the estimated order of the actual scheme, we denote the resulting estimates of a_k and b_k by $a_{q,\lambda}(k)$ and $b_{q,\lambda}(k)$, respectively, where λ is the sample size under examination. Then, we define:

$$Y'_{q,(m,n)-i} = [y'(i),\ y'(i-1), \cdots,\ y'(i-q)],$$

where

$$y(j) = 0 \quad \text{for} \quad j \le 0,$$

$$A'_{q,\lambda} = [a_{q,\lambda}(1), \cdots,\ a_{q,\lambda}(q)],$$

$$B'_{q,\lambda} = [b_{q,\lambda}(q), \cdots,\ b_{q,\lambda}(1)].$$

Thus, for a forward AR $(q,\ I_s)$ fitted by this sample set, we have

$$R_{q,\lambda}(I_s) = \sum_{i=1}^{\lambda} Y_{q,(m,n)-i}(L_s) Y'_{q,(m,n)-i}(L_s),$$

where

$$Y_{q,(m,n)-i}(L_s) = \left[\frac{y(i)}{Y_{q-1,(m,n)-i-1}(I_s)}\right] = \left[\frac{Y_{q-1,(m,n)-i}(L_s)}{y(i-q)}\right],$$

$$R_{q,\lambda}(I_s)\left[\frac{I}{A_{q,\lambda}(I_s)}\right] = \left[\frac{V_{q,\lambda}(I_s)}{0}\right],$$

$$V_{q,\lambda}(I_s) = \sum_{t=1}^{\lambda} \varepsilon_q(t,\ I_s)\varepsilon'_q(t,\ I_s),$$

where $Y_{q,(m,n)-i}(L_s)$ and $Y_{q-1,(m,n)-i}(L_s)$ are formed by removing the $(1_1), \cdots, (1_s)$-th row block of $Y_{q,(m,n)-i}$ and $Y_{q-1,(m,n)-i}$, respectively, and $Y_{q-1,(m,n)-i-1}(I_s)$ is formed by removing the $(i_1), \cdots, (i_s)$-th row block of $Y_{q-1,(m,n)-i-1}$. Now, we can define

$$C_{q,\lambda}(I_s) = \sum_{i=1}^{\lambda} Y_{q-1,(m,n)-i-1}(I_s) Y'_{q-1,(m,n)-i-1}(I_s),$$

$$W_{q,\lambda}(I_s) = C^{-1}_{q,\lambda}(I_s) Y_{q-1,(m,n)-\lambda}(I_s),$$

and

$$\alpha_{q,\lambda}(I_s) = 1 + Y'_{q-1,(m,n)-\lambda}(I_s) W_{q,\lambda}(I_s).$$

In addition, for the corresponding backward AR $(q,\ M_s)$, we will have

$$R_{q,\lambda}(M_s) = \sum_{i=1}^{\lambda}\left[\frac{Y_{q-1,(m,n)-i}(M_s)}{y(i-q)}\right]\left[Y'_{q-1,(m,n)-i}(M_s) \middle| y'(i-q)\right],$$

$$D_{q,\lambda}(M_s) = \sum_{i=1}^{\lambda} Y_{q-1,(m,n)-i}(M_s) Y'_{q-1,(m,n)-i}(M_s),$$

$$\overline{V}_{q,\lambda}(M_s) = \sum_{t=1}^{\lambda} \overline{\varepsilon}_q(t, M_s)\overline{\varepsilon}'_q(t, M_s),$$

and

$$R_{q,\lambda}(M_s)\left[\frac{B_{q,\lambda}(M_s)}{I}\right] = \left[\frac{0}{\overline{V}_{q,\lambda}(M_s)}\right],$$

where $B_{q,\lambda}(M_s)$ and $Y_{q-1,(m,n)-i}(M_s)$ are formed by removing the $(q+1-m_1), \cdots, (q+1-m_s)$-th row block of $B_{q,\lambda}$ and $Y_{q-1,(m,n)-i}$, respectively. We note $q+1-m_j = 1+i_j = 1_j$, and thus we can identify that $Y_{q-1,(m,n)-i}(M_s) = Y_{q-1,(m,n)-i}(L_s)$ and $R_{q,\lambda}(M_s) = R_{q,\lambda}(I_s)$.

Next, we consider a forward AR $(q+1, L_s)$ model using the sample $\{y(1), \cdots, y(\lambda+1)\}$, and we have

$$R_{q+1,\lambda+1}(L_s) = \sum_{i=1}^{\lambda+1}\left[\frac{y(i)}{Y_{q,(m,n)-i-1}(L_s)}\right]\left[y'(i) \middle| Y'_{q,(m,n)-i-1}(L_s)\right],$$

$$C_{q+1,\lambda+1}(L_s) = \sum_{i=1}^{\lambda+1} Y_{q,(m,n)-i-1}(L_s) Y'_{q,(m,n)-i-1}(L_s),$$

$$W_{q+1,\lambda+1}(L_s) = C^{-1}_{q+1,\lambda+1}(L_s) Y_{q,(m,n)-\lambda+1}(L_s),$$

and

$$\alpha_{q+1,\lambda+1}(L_s) = 1 + Y'_{q,(m,n)-\lambda+1}(L_s) W_{q+1,\lambda+1}(L_s).$$

Again, we consider a forward AR (q, L_s) model fitted from this sample set, and analogously, we have

$$W_{q,\lambda+1}(L_s) = C^{-1}_{q,\lambda+1}(L_s) Y_{q-1,(m,n)-\lambda+1}(L_s),$$

and

$$\alpha_{q,\lambda+1}(L_s) = 1 + Y'_{q-1,(m,n)-\lambda+1}(L_s) W_{q,\lambda+1}(L_s).$$

We also consider a backward AR (q, M_s) model fitted by this sample, and we have

$$R_{q,\lambda+1}(M_s) = \sum_{i=1}^{\lambda+1} \left[\frac{Y_{q-1,(m,n)-i}(M_s)}{y(i-q)}\right] \left[Y'_{q-1,(m,n)-i}(M_s) | y'(i-q)\right],$$

and

$$R_{q,\lambda+1}(M_s) \left[\frac{B_{q,\lambda+1}(M_s)}{I}\right] = \left[\frac{0}{\overline{V}_{q,\lambda+1}(M_s)}\right].$$

The above expressions lead to the following partitions:

$$C_{q+1,\lambda+1}(L_s) = \begin{bmatrix} \sum_{i=1}^{\lambda} y(i) y'(i) & \vdots & \sum_{i=1}^{\lambda} y(i) Y'_{q-1,(m,n)-i-1}(I_s) \\ \cdots & \vdots & \cdots \\ \sum_{i=1}^{\lambda} Y_{q-1,(m,n)-i-1}(I_s) y'(i) & \vdots & C_{q,\lambda}(I_s) \end{bmatrix} \tag{2.4}$$

$$= \begin{bmatrix} D_{q,\lambda}(M_s) & \vdots & \sum_{i=1}^{\lambda} Y_{q-1,(m,n)-i}(M_s) y'(i-q) \\ \cdots & \vdots & \cdots \\ \sum_{i=1}^{\lambda} y(i-q) Y'_{q-1,(m,n)-i}(M_s) & \vdots & \sum_{i=1}^{\lambda} y(i-q) y'(i-q) \end{bmatrix}. \tag{2.5}$$

Matrix inversion for a partitioned matrix is used on both (2.4) and (2.5) to obtain the following equations:

$$W_{q+1,\lambda+1}(L_s) = \begin{bmatrix} 0 \\ W_{q,\lambda}(I_s) \end{bmatrix} + \begin{bmatrix} I \\ A_{q,\lambda}(I_s) \end{bmatrix} V_{q,\lambda}^{-1}(I_s) e_{q,\lambda+1}(I_s),$$

$$\alpha_{q+1,\lambda+1}(L_s) = \alpha_{q,\lambda}(I_s) + e'_{q,(m,n)-(\lambda+1)}(I_s) V_{q,\lambda}^{-1}(I_s) e_{q,(m,n)-(\lambda+1)}(I_s),$$

where

$$e_{q,(m,n)-(\lambda+1)}(I_s) = \left[I \middle| A'_{q,\lambda}(I_s)\right] Y_{q,(m,n)-(\lambda+1)}(L_s),$$

and the space recursive equations of $W_{q,\lambda+1}(L_s)$ and $\alpha_{q,\lambda+1}(L_s)$:

$$W_{q+1,\lambda+1}(L_s) = \begin{bmatrix} W_{q,\lambda+1}(L_s) \\ 0 \end{bmatrix} + \begin{bmatrix} B_{q,\lambda}(M_s) \\ I \end{bmatrix} \overline{V}_{q,\lambda}^{-1}(M_s)\bar{e}_{q,(m,n)-(\lambda+1)}(M_s),$$

$$\alpha_{q+1,\lambda+1}(L_s) = \alpha_{q,\lambda+1}(L_s)$$

$$+ \bar{e}'_{q,(m,n)-(\lambda+1)}(M_s)V_{q,\lambda}^{-1}(M_s)\bar{e}_{q,(m,n)-(\lambda+1)}(M_s),$$

where

$$\bar{e}_{q,(m,n)-(\lambda+1)}(M_s) = [B'_{q,\lambda}(M_s)|I]\, Y_{q,(m,n)-(\lambda+1)}(M_s).$$

In addition, the overall forward time update algorithm is summarised in Table 1.

Table 1

Forward space time recursions from λ to $\lambda + 1$

$$e_{q,(m,n)-(\lambda+1)}(I_s) = \left[I \middle| A'_{q,\lambda}(I_s)\right] Y_{q,\lambda+1}(L_s), \tag{2.6a}$$

$$\varepsilon_{q,(m,n)-(\lambda+1)}(I_s) = e_{q,(m,n)-(\lambda+1)}(I_s)\alpha^{-1}_{q,\lambda}(I_s), \tag{2.6b}$$

$$A_{q,\lambda+1}(I_s) = A_{q,\lambda}(I_s) - W_{q,\lambda}(I_s)\varepsilon'_{q,(m,n)-(\lambda+1)}(I_s), \tag{2.6c}$$

$$V_{q,\lambda+1}(I_s) = V_{q,\lambda}(I_s) + \varepsilon_{q,(m,n)-(\lambda+1)}(I_s)e'_{q,(m,n)-(\lambda+1)}(I_s), \tag{2.6d}$$

$$W_{q+1,\lambda+1}(L_s) = \begin{bmatrix} 0 \\ W_{q,\lambda}(I_s) \end{bmatrix} + \begin{bmatrix} I \\ A_{q,\lambda}(I_s) \end{bmatrix} V^{-1}_{q,\lambda}(I_s)e_{q,(m,n)-(\lambda+1)}(I_s), \tag{2.6e}$$

partition

$$W_{q+1,\lambda+1}(L_s) = \left[\frac{D}{d}\right],$$

$$\bar{e}_{q,(m,n)-(\lambda+1)}(M_s) = \overline{V}_{q,\lambda}(M_s)d, \tag{2.6f}$$

$$W_{q,\lambda+1}(L_s) = D - B_{q,\lambda}(M_s)d, \tag{2.6g}$$

$$\alpha_{q+1,\lambda+1}(L_s) = \alpha_{q,\lambda}(I_s) + e'_{q,(m,n)-(\lambda+1)}(I_s)V^{-1}_{q,\lambda}(I_s)e_{q,(m,n)-(\lambda+1)}(I_s), \tag{2.6h}$$

$$\alpha_{q,\lambda+1}(L_s) = \alpha_{q+1,\lambda+1}(L_s) - \bar{e}'_{q,(m,n)-(\lambda+1)}(M_s)\,d, \tag{2.6i}$$

$$\bar{\varepsilon}_{q,(m,n)-(\lambda+1)}(M_s) = \bar{e}_{q,(m,n)-(\lambda+1)}(M_s)\left[\alpha_{q,\lambda+1}(L_s)\right]^{-1}, \tag{2.6j}$$

$$\overline{V}_{q,\lambda+1}(M_s) = \overline{V}_{q,\lambda}(M_s) + \bar{\varepsilon}_{q,(m,n)-(\lambda+1)}(M_s)\bar{e}'_{q,(m,n)-(\lambda+1)}(M_s), \tag{2.6k}$$

$$B_{q,\lambda+1}(M_s) = B_{q,\lambda}(M_s) - W_{q,\lambda+1}(L_s)\bar{\varepsilon}'_{q,(m,n)-(\lambda+1)}(M_s), \tag{2.6l}$$

We observe that, if there is a consecutive set of k deleted lags beginning at lag 1 in the forward AR (p, I_s) model and fitted using the sample $\{y(1), \cdots, y(\lambda+1)\}$, we have

$$W_{q,\lambda+1}(I_s) = W_{q-k,\lambda+1-k}(I_k),$$

and

$$\alpha_{q,\lambda+1}(I_s) = \alpha_{q-k,\lambda+1-k}(I_k), \tag{2.6m}$$

where I_s contains $i_1, \cdots, i_k, \cdots, i_s$, and $i_j = j$, $j = 1, 2, \cdots, k$, and I_k contains $i_{k+1} - k, \cdots, i_s - k$.

When we consider a backward AR (q, N_s) model fitted from the sample $\{y(1), \cdots, y(\lambda)\}$, we have

$$D_{q,\lambda}(N_s) = \sum_{i=1}^{\lambda} Y_{q-1,(m,n)-i}(N_s) Y'_{q-1,(m,n)-i}(N_s),$$

and

$$W^*_{q,\lambda}(N_s) = D^{-1}_{q,\lambda}(N_s) Y_{q-1,(m,n)-\lambda}(N_s).$$

Further, we define

$$\alpha^*_{q,\lambda}(N_s) = 1 - Y'_{q-1,(m,n)-\lambda}(N_s) W^*_{q,\lambda}(N_s),$$

$$\alpha^*_{q+1,\lambda+1}(N_s) = 1 - Y'_{q,(m,n)-\lambda+1}(M_s) W^*_{q+1,\lambda+1}(M_s),$$

$$\alpha^*_{q,\lambda+1}(M_s) = 1 - Y'_{q-1,(m,n)-\lambda+1}(M_s) W^*_{q,\lambda+1}(M_s).$$

Next, we consider a backward AR $(q+1, N_s)$ model fitted from the sample $\{y(1), \cdots, y(\lambda+1)\}$, where n_j of N_s has the relation $n_j = m_j + 1$. We have

$$D_{q+1,\lambda+1}(N_s) = \sum_{i=1}^{\lambda+1} Y_{q,(m,n)-i}(M_s) Y'_{q,(m,n)-i}(M_s),$$

and

$$W^*_{q+1,\lambda+1}(N_s) = D^{-1}_{q+1,\lambda+1}(N_s) Y_{q,(m,n)-\lambda+1}(M_s).$$

Using the analogous relations for deriving the λ to $\lambda+1$ overall forward time update algorithm, we can have the $\lambda+1$ to λ overall backward time update algorithm that is summarised in Table 2.

Table 2

Backward space time recursions from $\lambda+1$ to λ

$$\bar{\varepsilon}_{q,(m,n)-(\lambda+1)}(M_s) = [B'_{q,\lambda+1}(M_s)|I]Y_{q,\lambda+1}(M_s), \tag{2.7a}$$

$$\bar{e}_{q,(m,n)-(\lambda+1)}(M_s) = \bar{\varepsilon}_{q,(m,n)-(\lambda+1)}(M_s)\alpha^{*-1}_{q,\lambda+1}(M_s), \tag{2.7b}$$

$$B_{q,\lambda}(M_s) = B_{q,\lambda+1}(M_s) + W^*_{q,\lambda+1}(M_s)\bar{e}'_{q,(m,n)-(\lambda+1)}(M_s), \tag{2.7c}$$

$$\overline{V}_{q,\lambda}(M_s) = \overline{V}_{q,\lambda+1}(M_s) - \bar{\varepsilon}_{q,(m,n)-(\lambda+1)}(M_s)\bar{e}'_{q,(m,n)-(\lambda+1)}(M_s), \tag{2.7d}$$

$$W^*_{q+1,\lambda+1}(N_s) = \begin{bmatrix} W^*_{q,\lambda+1}(M_s) \\ 0 \end{bmatrix} + \begin{bmatrix} B_{q,\lambda+1}(M_s) \\ I \end{bmatrix}\overline{V}^{-1}_{q,\lambda+1}(M_s)\bar{\varepsilon}_{q,(m,n)-(\lambda+1)}(M_s), \tag{2.7e}$$

partition

$$W^*_{q+1,\lambda+1}(N_s) = \left[\frac{d^*}{D^*}\right],$$

$$\varepsilon_{q,(m,n)-(\lambda+1)}(I_s) = V_{q,\lambda+1}(I_s)d^*, \tag{2.7f}$$

$$W^*_{q,\lambda}(N_s) = D^* - A_{q,\lambda+1}(I_s)d^*, \tag{2.7g}$$

$$\alpha^*_{q+1,\lambda+1}(N_s) = \alpha^*_{q,\lambda+1}(M_s) - \bar{\varepsilon}^*_{q,(m,n)-(\lambda+1)}(M_s) \times \overline{V}^{-1}_{q,\lambda+1}(M_s)\bar{\varepsilon}_{q,(m,n)-(\lambda+1)}(M_s), \tag{2.7h}$$

$$\alpha^*_{q,\lambda}(N_s) = \alpha^*_{q+1,\lambda+1}(N_s) + \varepsilon'_{q,(m,n)-(\lambda+1)}(I_s)d^*, \tag{2.7i}$$

$$e_{q,(m,n)-(\lambda+1)}(I_s) = \varepsilon_{q,(m,n)-(\lambda+1)}(I_s)\left[\alpha^*_{q,\lambda}(N_s)\right]^{-1}, \tag{2.7j}$$

$$V_{q,\lambda}(I_s) = V_{q,\lambda+1}(I_s) - \varepsilon_{q,(m,n)-(\lambda+1)}(I_s)e'_{q,(m,n)-(\lambda+1)}(I_s), \tag{2.7k}$$

$$A_{q,\lambda}(I_s) = A_{q,\lambda+1}(I_s) + W^*_{q,\lambda}(N_s)e'_{q,(m,n)-(\lambda+1)}(I_s). \tag{2.7l}$$

Note that if there is a consecutive set of k deleted leads beginning at lead 1 in the backward AR (p, M_s) model fitted using the sample $\{y(1), \cdots, y(\lambda)\}$, we have

$$W^*_{q,\lambda}(M_s) = W^*_{q-k,\lambda}(M_K),$$

and

$$\alpha^*_{q,\lambda}(M_s) = \alpha^*_{q-k,\lambda}(M_K), \tag{2.7m}$$

where M_s contains $m_1, \cdots, m_k, \cdots, m_s$, and $m_j = j$, $j = 1, 2, \cdots, k$, and M_k contains $m_{k+1} - k, \cdots, m_s - k$.

Furthermore, a model selection criterion for the 2D case as suggested by Hannan and Deistler [4], could be modified to use at each time instant with the proposed method to select the optimal 2D subset autoregressive model among all candidate 2D autoregressive models, including 2D full-order models. This model selection criterion is defined by: $|G| + [2 \log\log \lambda / \lambda]S$, where S is the number of functionally independent parameters, and $\hat{G}$ is the estimate of $V_{q,\lambda}$ for the autoregression.

3. Conclusion

We have developed an effective recursive algorithm for fitting 2D subset autoregressions, including 2D full-order models. The optimum model is found by employing the proposed recursive procedures in conjunction with model selection criteria. Compared to the conventional procedure of including every lag in the model, this method has the advantage of considering all possible 2D ARs, and so establishing the number of nonzero parameters, which characterise the system. To implement this procedure effectively, we have found necessary, however, to select a maximum pair of lag length (M, N), which exceeds the true pair of lag length.

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