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APPLICATION OF HE'S VARIATIONAL ITERATION METHOD FOR SOLVING DAMPED FORCED OSCILLATOR PROBLEM

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Abstract

In this paper, we use the He's variational iteration method (VIM) to obtain the solution of damped forced oscillator problem as a second order ordinary differential equation. This method is based on Lagrange multiplier for identification of optimal value of parameter in a functional. Using this method, creates a sequence, which tends to the exact solution of problem. The method is capable of reducing the size of calculation, and easily overcomes the difficulty of the Adomian polynomials. The results reveal that He's variational iteration method is very effective for these types of equations.

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1. Introduction

Although, there are few phenomena in different fields of science occurring linearly, most of them occur nonlinearly. We know that except a limited number of these problems, most of them do not have precise analytical solutions, therefore, they have to be solved by using other approximate methods such as He's variational iteration method. The general form of the damped forced oscillator problem was described by a second order ordinary differential equation as the following:

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F(t), \tag{1}$$

with the initial conditions $x(0) = x_0$ and $x'(0) = x'_0$, in which m, b, and k are mass, the viscosity coefficient of the fluid, and the stiffness of the spring, respectively, and $F(t)$ is a time-dependent external force.

In this paper, we find the solution of damped forced oscillator problem by He's variational iteration method.

2. Basic Idea of VIM

To clarify the basic ideas of the variational iteration method, we consider the following differential equation:

$$L[u(t)] + N[u(t)] = g(t), \tag{2}$$

where L is a linear operator, N is a non-linear operator, and $g(t)$ is an inhomogeneous term. According to VIM, we can write down a correction functional as follows:

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda(Lu_n(\tau) + N\tilde{u}_n(\tau) - g(\tau))d\tau, \tag{3}$$

where λ is a general Lagrange multiplier [1-9], which can be identified optimally via the variational theory [8, 9, 10, 12]. The subscript n indicates the n-th approximation and $\tilde{u}_n$ is considered as a restricted variation, i.e., $\delta\tilde{u}_n = 0$. The variational iteration method proposed by He has been shown to solve effectively, easily, and accurately a large class of non-linear problems with approximations converging rapidly to accurate solutions.

3. Implementation of the Method

Consider the damped forced oscillator problem in following form:

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + 2x = e^{2t}\sin t, \tag{4}$$

with the initial conditions $x(0) = -\frac{4}{10}$, $x'(0) = -\frac{6}{10}$.

The exact solution of the equation is $x(t) = \frac{2}{10}e^{2t}(\sin t - 2\cos t)$. [11]

Its correction functional can be written down as follows:

$$x_{n+1}(t) = x_n(t) + \int_0^t \lambda(\tau)\left[\frac{d^2x_n(\tau)}{d\tau^2} - 2\frac{dx_n(\tau)}{d\tau} + 2x_n(\tau) - e^{2\tau}\sin(\tau)\right]d\tau. \tag{5}$$

Its stationary conditions are obtained as follows:

$$(1 - \lambda'(\tau))|_{\tau=t} = 0,$$

$$\lambda(\tau)|_{\tau=t} = 0,$$

$$\lambda''(\tau) = 0.$$

Lagrange multiplier can be identified as $\lambda(\tau) = \tau - t$, thus, we have:

$$x_{n+1}(t) = x_n(t) + \int_0^t (\tau - t)\left[\frac{d^2x_n(\tau)}{d\tau^2} - 2\frac{dx_n(\tau)}{d\tau} + 2x_n(\tau) - e^{2\tau}\sin(\tau)\right]d\tau.$$

Now, by using initial approximations and the above iteration formula, we can obtain the following iterations:

$$x_0(t) = -\frac{4}{10} - \frac{6}{10}t,$$

$$x_1(t) = -\frac{6}{25} - \frac{4}{10}t - \frac{2}{10}t^2 + \frac{2}{10}t^3 - \frac{4}{25}e^{2t}\cos(t),$$

$$x_2(t) = -\frac{88}{625} - \frac{32}{125}t - \frac{4}{25}t^2 + \frac{2}{15}t^4 - \frac{1}{50}t^5 - \frac{162}{625}e^{2t}\cos(t) + \frac{109}{625}e^{2t}\sin(t),$$

$$x_3(t) = -\frac{1264}{15625} - \frac{496}{3125}t - \frac{72}{625}t^2 - \frac{8}{375}t^3 + \frac{2}{75}t^4 + \frac{4}{75}t^5 - \frac{4986}{15625}e^{2t}\cos(t) + \cdots,$$

$$\vdots.$$

By continuing this manner, we obtain the solution of damped forced oscillator problem that the obtained solution is very closed to the exact solution.

4. Conclusion

In this paper, we present the application of He's variational iteration method to obtain the solution of damped forced oscillator problem. The variational iteration method is particularly suitable for solving this kind of problems. Some of the advantages of VIM are that, the initial solution can be freely chosen with some unknown parameters. Therefore, this method gives a powerful mathematical tool for linear problems. In final, notice that the software of Matlab has been used for computations in this paper.

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