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# MULTI-EXPERT CONSENSUS IN STRATEGIC PORTFOLIO EVALUATION USING DISCRETE CHOQUET INTEGRAL FOR ANALYTICAL NETWORK PROCESS WEIGHTS

**VIVIANA VENTRE and ROBERTA MARTINO**

Department of Mathematics and Physics
University of Campania "L. Vanvitelli"
Viale A. Lincoln, 5 – 81100 Caserta
Italy
e-mail: viviana.ventre@unicampania.it
roberta.martino@unicampania.it

## Abstract

Customer profiling tools have been introduced in recent years to protect investors against the increasing complexity of financial instruments. Advisor experts have to combine measures of finance and behavioural finance studies in their evaluation. The need to integrate different competences and compare complex criteria naturally leads to the modelling of the portfolio selection problem as a group decision problem. The Analytic Network Process (ANP) is a

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useful technique that helps decision-makers evaluate the available alternatives and aggregate their professional knowledge. In this context, the Reaching Consensus Process (RCP) is crucial for the validity of choices due to the strong sensitivity of the ANP to the decision maker's judgement. However, the RCP may be affected by the behavioural characteristics of experts, the vulnerability of judgements and the uncertainty that characterises the decision-making context, leading to unbiased judgements that are not based on different expertise. Similarly, without the involvement of more experts, the process could be inefficient due to the sensitivity of the ANP decision-maker's judgement. In order to maximise the aggregation of experts' knowledge, to minimise the influence of the individual in quantifying the relationships between criteria and alternatives, and to preserve the impartiality of evaluations, the present paper discusses RCP through Choquet's discrete integral. This aggregation captures the non-linearity of preferences and includes the fuzziness of judgments. Thus, there is no need to reach a consensus when evaluating the criteria: each expert manages a single ANP whose synthesis are aggregated using Choquet's discrete integral. In this sense, the aggregation defines an unbiased consensus which mitigates the sensitivity of the ANP to the preferences of the decision-maker. The methodology is applied to a case study in which several experts evaluate investment alternatives of an individual, simultaneously incorporating behavioural criteria and Markets in Financial Instruments Directive (MiFID) client profiling criteria.

## 1. Introduction

The process of selecting a portfolio means the evaluation whereby the investor selects the best alternative that meets the set objectives. The financial decision-making process is characterised by high complexity, defined by a set of criteria and parameters in which uncertainty plays a key role. Classical financial theory assumes a rational decision maker, capable of coping with the complexity that characterises the decision-making environment. On the other hand, behavioural finance uses cognitive psychology to investigate the decision-making process behind individual choices in order to reduce the gap between expected and empirical behaviour (Joo and Durri [9]). The focus on the decision-maker's condition of limited rationality has motivated the intervention of regulatory instruments through regulations aimed at investor protection. In particular, the MiFID requires knowledge of the client to enable the

expert advisor to define the product that best meets both the individual's needs and her behavioural characteristics. The first work that unified the MiFID guidance, studies from the classical literature and results from behavioural finance used a network structure to simultaneously assess the influence of each criterion and support the elaborate decision-making process that experts are required to carry out (Martino and Ventre [17]). The authors proved how the implementation of the ANP (Saaty and Vargas [26]) can reduce the complexity of the context in which the choice of the best alternative may be difficult without a decision-support methodology. From a methodological point of view, the completeness of the proposed structure lies in having used a cluster for each class defined by the intersection of the regulatory context and the literature, thus including socio-demographic characteristics, experience and knowledge, financial situation, investment goals and tolerance to risk (Ganassin [8]). To these five classes, a cluster built on behavioural characteristics was added with a twofold function: to include among the criteria the client's personality traits (Pompian [20, 21]); to distinguish risk from uncertainty (Anderson et al. [2]) by including the latter through measures defined in the context of intertemporal choices (Samuelson [27, 28]). The inclusion of the weights, on the other hand, is achieved by a combination of three ways to maximise the effectiveness of the network: some weights are defined by an empirical collection of interviews; some comparisons are handled directly by the client; other weights, on the other hand, are the result of an evaluation between experts (Martino and Ventre [17]). With respect to expert evaluation, it is important to emphasise that given the complexity of the decision-making environment, the problem of portfolio selection cannot be considered to be completely satisfied by the use of multi-criteria models (Ballestero et al. [2]; Ehrgott et al. [6]; Xidonas and Psarras [35]): the requirement to compare criteria of a different nature determines the need to include the judgement of experts from different sectors. Therefore, the portfolio selection problem can also be modelled as Multi Criteria Group Decision Making (MCGDM). This is the reason why

the authors considered two different experts in the evaluation. However, the problem is that different expertise and opinions can generate different ideas that make the Consensus Reaching Process (CRP) (Zhang et al. [40]), i.e., the group members' reaching a shared solution that satisfies all members, difficult. Among the various CPR techniques (Fernández et al. [7]; Wu et al. [36]), Ventre et al. [31] highlight the importance of the behavioural factor that must be included in order for the CPR to lead to the selection of the alternative that is simultaneously the most satisfactory for the group members from a personal point of view, and the most efficient from a portfolio selection problem-solving point of view. In order to maximise the outcome of the network, the present paper proposes a new approach to include expert judgements in evaluations without distorting individual opinions and preserving impartiality.

Specifically, instead of requiring the experts to agree during the weighting evaluation, the achievement of a common solution is constructed a posteriori through the use of an aggregation operator (Beliakov et al. [4]). Among the aggregation operators, Choquet's discrete integral (Choquet [5]) is among those most suited to our purposes for two main reasons: its definition is based on fuzzy logic (Zadeh [37, 38]) and is therefore particularly expressive with respect to the subjectivity of preferences and with respect to uncertainty; its construction allows us to aggregate input data by considering not only individual preferences but how these combines with each other and what their interactions are. In this sense, then, aggregation with Choquet's integral allows for the determining of an a posteriori consensus of individual network summaries, which is defined by the intersection of unconditional evaluations that combine with each other. This approach makes the network decision process proposed in (Martino and Ventre [17]), even more sophisticated. Application to the case study will clarify the methodology and highlight its concreteness and efficiency. The functionality of the approach is tested by a case study in which, assuming

two alternatives that equally satisfy an investment objective, the choice will be defined through the aggregation of three separate summaries of the alternatives generated by the ANP as shown in Martino and Ventre [17]. Therefore, the weights defined by the expert evaluation are independent of each other and only combined at the end. In this way, it is expected to increase the efficiency of the network, preserve individual competences and free the CRP from the behavioural aspect.

The paper is divided into the following: Section 2 will present the structure of the network and the measures used to construct the weights, from judgement to synthesis and from synthesis to aggregation; Section 3 presents the case study; Section 4 discusses future studies, decision-making implications of the approach and conclusions.

## 2. Materials and Method

### 2.1. Analytic network process applied to financial decision-making

A portfolio selection problem, because it is characterised by uncertain variables and the conditions defined by the objectives and limitations of the decision maker, can be described as a multi-criteria decision problem. Multi-criteria decision-making techniques have been widely used in recent literature for portfolio selection. In particular, ANP is a method of decision analysis that is particularly useful when the existing relationships between criteria, variables and alternatives that characterise the decision-making context must also be considered during the evaluation (Lami et al. [12]; Liang and Li [14]).

The implementation of an ANP can be summarised in the following steps:

(1) Creation of the network structure: during this phase, the clusters and nodes of the structure are identified. In particular, the node represents a single element of the decision context, which may be an alternative or a criterion, while the cluster is a grouping of nodes that can

be grouped together to simplify complex structures. The first phase also involves defining the relationships between nodes and clusters: nodes may be related to other nodes in the same cluster or to nodes in different clusters; clusters may be related to nodes in other clusters or different clusters. Figure 1 graphically represents the implementation of step (1).

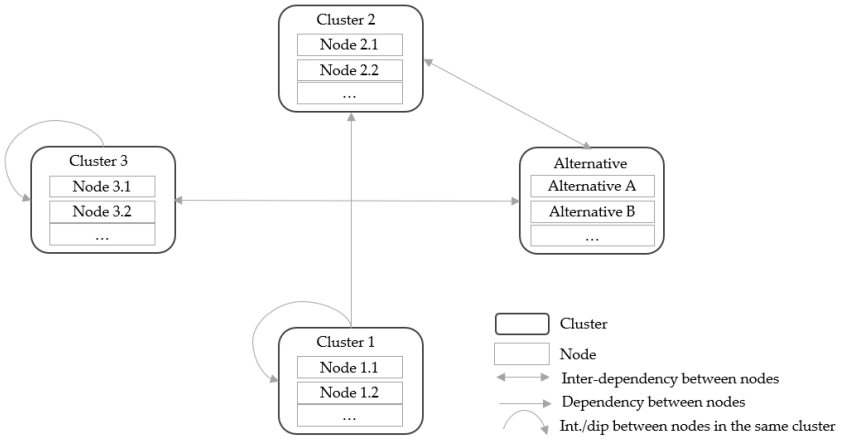


**Figure 1.** Generic structure of ANP.

(2) Evaluations and comparisons: after constructing the network, the weights are assigned to define the relative importance of the nodes and clusters with respect to the alternatives and with respect to the relationships and interrelations defined by the structure.

(3) Comparison matrices: the previous comparisons and evaluations are entered into a matrix with as many columns as the number of clusters in the structure. The resulting matrix, called a supermatrix, encloses the relationships that characterise the network. Therefore, the null element corresponds to the absence of relationships. The supermatrices of the process, whose structure is shown in Figure 2, are: the unweighted supermatrix containing the priorities of the previous step; the weighted supermatrix obtained by the product of the unweighted supermatrix by the matrix expressing the weight of the clusters; the limit supermatrix defined as $\lim_{k \to +\infty} W^k$, where $W$ is the weighted supermatrix and contains the final priority vector.

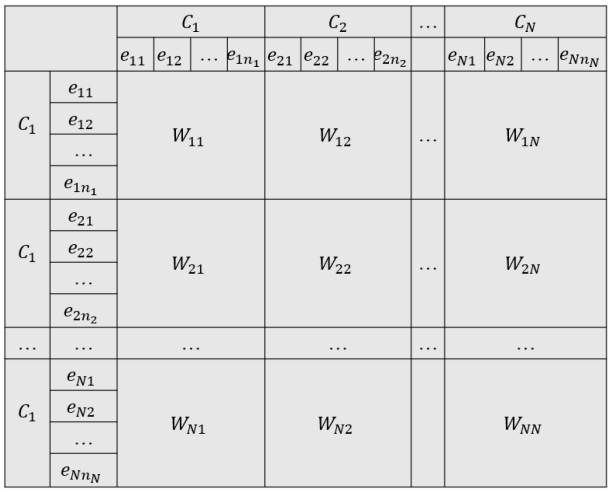


**Figure 2.** Generic supermatrix obtained in an ANP.

(4) Evaluation of alternatives and choice of the best alternative: the last step consists of summarising all evaluations to obtain a score for each alternative.

From a practical point of view, the implementation of the ANP method is supported by the SuperDecisions software (https://www.superdecisions.com/), which simplifies the calculation and visualisation of results. In particular, SuperDecisions allows the entry of weights in three different modes: the direct mode allows the direct entry of the weight of the network element; the graphical mode allows the entry of the weight with respect to the length of a bar, where the entire length equals a weight of 1; the verbal method includes the use of words corresponding to Saaty's scale to express preferences; the matrix method, allows a pairwise comparison of the network elements and finally, the questionnaire method guides the evaluation of the network elements through questions such as "how much is element $i$ more satisfactory/important/preferred than element $j$ with respect to the node/cluster?" (Adams and Saaty [1]).

The structure implemented to support decision-making in the context of behavioural finance refers to the five classes shown in Table 1, with respect to the indications of the regulatory environment and research in the literature (Ganassin [8]).

**Table 1.** Indications on the questionnaire crossing regulatory framework and literature

| **Regulatory Framework** | |
|---|---|
| **Classes** | **Variables** |
| Socio-demographic characteristics | Not planned |
| Experience and knowledge | • Profession<br>• Education<br>• Nature, volume, and frequency of financial transactions by the client<br>• Services, transactions, and financial products with which the client is familiar with |
| Financial situation | • Regular income information<br>• Regular financial commitments<br>• Information on investments, assets, and movable property |
| Investment goals | • Desired retention time of the investment<br>• Investment goals<br>• Preferences and risk profile |
| Tolerance to risk | • Not expected, included in the investment objectives |

**Table 1.** (Continued)

| Literature | |
|---|---|
| **Classes** | **Variables** |
| Socio-demographic characteristics | • Civil status |
| | • Family status and dependents |
| | • Gender |
| | • Age |
| | • Profession |
| | • Education |
| Experience and knowledge | • Previous experience in investments and positive or negative outcomes |
| | • Knowledge of the functioning of financial markets and of certain terminologies such as trade-offs, risk return, diversification portfolio… |
| | • Knowledge of financial products |
| | • Overconfidence and optimism |
| Financial situation | • Regular income information |
| | • Regular financial commitments |
| | • Information on investments, assets and movable property |
| | • Other financial commitments, including projected commitments and expectations of changes in regular expenditures |
| Investment goals | • Desired retention time of the investment |
| | • Investment goals |
| | • Preference towards time |
| | • Liquidity needs |
| Tolerance to risk | • Amount of investment best in relation to wealth or income |
| | • Attitude toward risk (objective risk) |
| | • Emotional ability to take on risks (subjective risk) |
| | • Awareness of losses |

Martino and Ventre [17] added to these indications some clusters dedicated to the behavioural characteristics of the individual: behavioural attitudes were included by considering Kersey's temperament theory (Keirsey [10]); how the individual interfaces with uncertainty, on the other hand, was included by adding classes defined on measures of intertemporal choice theory to the structure (Ventre et al. [32]; Ventre et al. [30]; Ventre and Martino [29]).

The structure used for the case study and implemented in the Superdecisions software is shown in Figure 3.

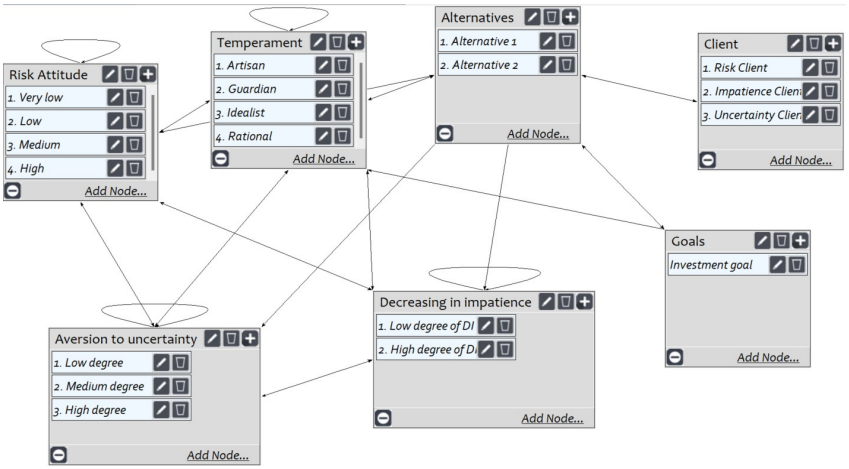


**Figure 3.** ANP implementation in Martino and Ventre [17].

The connections and interconnections presented in the structure of Figure 3 are motivated by various works (Ventre et al. [33]; Ventre et al. [34]; Martino and Ventre [19]; Martino et al. [18]).

### 2.2. Uncertainty management in behavioural finance

Intertemporal choices are decisions whose consequences are distributed over time and, since the future is uncertain by definition, such choices are chosen under uncertainty. Mathematical modelling of intertemporal choices has a strong descriptive character with respect to

the individual's attitude to interface with uncertain events, such as the choice to buy a car, to book a trip or to select a portfolio for an investment. The mathematical model of reference is the Discounted Utility Model (Samuelson [27, 28]), whereby the future utility of an asset is obtained by multiplying the utility of the asset as if it were perceived at the time of choice by the discount function calculated at the time the asset is perceived. Let's fixed $X = \{x_1, \ldots, x_n\}$ the set of the outcome, $T = \{t_1, \ldots, t_n\}$ the set of the time instants an intertemporal perspective is an *n-pla* defined as $(x_0, t_0; \ldots; x_n, t_n)$: if the DM accepts the prospect, she will receive the value $x_i$ at the moment $t_i$. The utility of the intertemporal prospect is:

$$U(x_0, t_0; \ldots; x_n, t_n) = \sum_{i=0}^{n} f(t_i)U(x_i), \tag{1}$$

where $f : [0, +\infty) \to \mathbb{R}$ is the discount function, whose task is to reduce the present utility of the good $U(x_i)$ according to how the individual perceives the indeterminacy of the future. The psychological mechanisms behind this decrease are gathered by the discount factor and the degree of impatience, respectively defined as $\rho(t) = -\frac{f'(t)}{f(t)}$ and $1 - \frac{f(t_2)}{f(t_1)}$ with $t_2 < t_1$: the first expresses the rate at which the discount function decreases in the standard period (Read [24]); the second expresses the amount of money one is willing to lose in order to receive a unit of money immediately (Rambaud and Torrecillas [23]). The assumptions of the model are that the discount function has an exponential trend, in which the discount rate and the degree of impatience are constant over time, in line with the assumption of a perfectly rational decision maker. Over time, empirical evidence has proven that individuals do not discount exponentially over time, so research has turned to defining functions that better express actual behaviour. In this context, a hyperbolic discount has greater descriptive power. Discounting of a hyperbolic nature is

characterised by a discount rate that decreases over time and a non-constant degree of impatience (Prelec [22]). In this regard, previous studies on intertemporal choice have tried to reduce the gap between expected and empirical behaviour by exploiting findings from cognitive psychology and behavioural finance. In particular, some studies in the literature have described the anomalies (Loewenstein and Prelec [15]) of the Discounted Utility Model with respect to the degree to which impatience decreases over time. In this context, anomalous preferences, i.e., those that do not respect the trend predicted by the exponential model, are the expression of an inconsistency in the decision-making process due to the emotional pulsations of the decision-maker (Ventre et al. [32]). The degree to which impatience decreases can be determined empirically through the hyperbolic factor (Rohde [25]). This measure allows a classification based on this dynamic to be introduced into the network, defined in Figure 3 as 'Decreasing in impatience'. The second measure referred to by the network in Ventre and Martino [29] is uncertainty aversion, the definition of which is based on the concept of subjective time. In fact, although the assumption of objective time, which corresponds to chronological time, is indisputable, the behavioural description of decision-making dynamics must include that each individual perceives time passing differently (Zimbardo and Boyd [39]). The maximum degree of distance between exponential and hyperbolic preferences defines the classification in the Uncertainty Aversion cluster in Figure 3.

### 2.3. Discrete Choquet integral for the consensus process

Aggregation functions are mathematical tools that generate a single output data from a set of input data (Beliakov et al. [4]). Such operators are widely used in decision theory as they allow a single representative value to be obtained by simplifying the data.

**Definition 1.** An aggregation function is a function of $n > 1$ arguments with $n \in \mathbb{N}$, such that $f : [0, 1]^n \to [0, 1]$ with the following properties:

(i) $f(0, 0, \ldots, 0) = 0$ and $f(1, 1, \ldots, 1) = 1$;

(ii) $\forall \boldsymbol{x}, \boldsymbol{y} \in [0, 1]^n$, $\boldsymbol{x} \leq \boldsymbol{y}$ implies $f(\boldsymbol{x}) \leq f(\boldsymbol{y})$.

In general, the range $[0, 1]$ can be replaced with any generic range $[a, b] \subset \mathbb{R}$ and the typo of aggregation operators is chosen according to the purpose and characteristics of the data and aggregation operators based on fuzzy measures are particularly effective in describing uncertainty. Fuzzy theory was introduced by Zadeh [37] and is based on more vague definitions to increase the expressive power of classical set theory. In practice, membership or non-membership of a fuzzy set is expressed through a degree of membership, resulting in a description that is closer to real sets and the way individuals make their evaluations. The applications of fuzzy theory are wide-ranging since, unlike traditional methods, it is able to handle information that cannot be represented in a strictly quantitative manner due to its subjective and qualitative nature. Applications extend to various fields, in particular fuzzy theory is effective in decision support systems (Kickert [11]): uncertainty and lack of data are common in real-life problems and the language with which decision-makers express their opinions is not always accurate. Even when using the same words, two decision-makers might have slightly different concepts in mind (Mardani et al. [16]; Leondes [13]).

**Definition 2.** Let $N = \{1, \ldots, n\}$. A discrete fuzzy measure is a set function $v : 2^n \to [0, 1]$ which is monotonic ($v(A) \leq v(B)$ if $A \subset B$) and $v(\emptyset) = 0$, $v(N) = 1$.

The Choquet integral, introduced by Choquet in 1954 [5], is a crucial operator for data aggregation. It is based on the notion of a fuzzy measure and this aspect gives considerable flexibility in aggregating weights, allowing the complexity of the relationships between the input data to be modelled more accurately.

**Definition 3.** The discrete Choquet integral with respect to a fuzzy measure $v$ is given by

$$C_v(x) = \sum_{i=1}^{n} x_{(i)}\left[v\left(\{x_j \geq x_{(i)}\}\right) - v\left(\{x_j \geq x_{i+1}\}\right)\right],$$

where the input vector $\boldsymbol{x}$ is ordered in a non-decreasing permutation $\boldsymbol{x}_{\uparrow} = \left(x_{(1)}, \ldots, x_{(n)}\right)$ and $x_{(n+1)} = \infty$ by convention.

Definition 3 could be rewritten as

$$C_v(x) = \sum_{i=1}^{n} \left[x_{(i)} - x_{(i-1)}\right] v(H_i),$$

where $x_{(0)} = 0$ by convention and $H_i = \{(i), \ldots, (n)\}$ is the subset of indices on the $n - i + 1$ largest components of $\boldsymbol{x}$.

From a conceptual point of view, this integral combines the characteristics of the input data while considering the interactions between them. This implies that a single input data may have little relevance when considered in isolation but acquires significance when evaluated together with other input data. In practice, the Choquet integral aggregates initial values according to their group membership among all inputs. This makes the Choquet integral particularly suitable for situations in which the distance or relationship between the input data is important, such as in the case of reaching consensus among experts in a group.

## 3. Consensus in Portfolio Evaluation using Discrete Choquet Integral: A Case Study

A number of experts equal to $n = 3$ were invited to answer the decision problem proposed in Martino and Ventre [17]. Specifically, the aim is to estimate the best investment alternative for a client whose goal is to accumulate €50,000 in five years. Table 2 summarises the available alternatives.

**Table 2.** Decision problem: Goal and alternatives

| Goal | Alternative 1 | Alternative 2 |
|---|---|---|
| To finance an entrepreneurial project by accumulating capital of EUR 50, 000.00 within five years | Allocate 70% in money market funds (low risk) and the remaining 30% in medium-term bonds (medium risk) | Allocate 50% of assets into stocks (moderate risk), 30% into intermediate-term bonds (moderate risk) and 20% into money market funds (moderate risk) |

The selection of experts was conducted by looking for volunteers with the following characteristics:

- Knowledge of behavioural finance theory and research;
- Ability to develop customised strategies and manage long-term planning;
- Awareness of an ethical approach without conflicts of interest;
- Understanding of multi-criteria structures.

It is important to note that since our model does not require experts to reach consensus in a straightforward manner, behavioural characteristics of the experts themselves such as the capacity for dialogue, mutual respect, mediation and flexibility of ideas or opinions were glossed over (Ventre et al. [31]). The experts therefore compiled the network independently, avoiding any confrontation and exchange of opinions. The summaries obtained for each alternative are shown in Table 3.

**Table 3**. Synthesis of alternatives in SuperDecisions for each expert

| | Alternative 1 | Alternative 2 |
|---|---|---|
| Expert 1 | 0.55 | 0.45 |
| Expert 2 | 0.28 | 0.72 |
| Expert 3 | 0.63 | 0.37 |

To construct the consensus, consider the vector $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$, whose components are respectively the experts' summaries of the two alternatives, i.e., $\boldsymbol{a}_1 = (0.55, 0.28, 0.63)$ and $\boldsymbol{a}_2 = (0.45, 0.72, 0.37)$. To calculate the Choquet integral as in Definition 3:

(1) Fix the indices of the components of the vector $\boldsymbol{a}_1$ and of the vector $\boldsymbol{a}_2$. So:

$$\boldsymbol{a}_1 = ((0.55, 1), (0.28, 2), (0.63, 3)),$$

$$\boldsymbol{a}_2 = ((0.45, 1), (0.72, 2), (0.37, 3)).$$

(2) The defined vectors must be ordered non-decreasingly:

$$\boldsymbol{a}_{1\uparrow} = ((0.28, 2), (0.55, 1), (0.63, 3))$$

$$\boldsymbol{a}_{2\uparrow} = ((0.37, 3), (0.45, 1), (0.72, 2)).$$

(3) Fix $T_i = \{1, 2, 3\}$ and $S_i = \{0\}$, $i = 1, 2, 3$.

(i) $S_1 = 0 + [0.28 - 0]v\{(1, 2, 3)\}$, $S_2 = 0 + [0.37 - 0]v\{(1, 2, 3)\}$

(ii) $T_1 = \{1, 2, 3\} \setminus \{2\}$ and $T_2 = \{1, 2, 3\} \setminus \{3\}$.

(i) $S_1 = [0.55 - 0.28]v\{(1, 3)\}, + [0.28 - 0]v\{(1, 2, 3)\}$,

$$S_2 = [0.45 - 0.37]v\{(1, 2)\} + [0.37 - 0]v\{(1, 2, 3)\};$$

(ii) $T_1 = \{1, 3\} \setminus \{1\}$ and $T_2 = \{1, 2\} \setminus \{1\}$;

(i) $S_1 = [0.63 - 0.55]v\{(3)\} + [0.55 - 0.28]v\{(1, 3)\}, + [0.28 - 0]v\{(1, 2, 3)\}$,

$$S_2 = [0.72 - 0.45]v\{(2)\} + [0.45 - 0.37]v\{(1, 2)\} + [0.37 - 0]v\{(1, 2, 3)\};$$

(4) As a fuzzy measure, let's consider a measure in which the cardinality of the whole is important rather than the order of its elements since the way in which the experts' judgements were ordered has no methodological motivation but is related to the timing with which these summaries were delivered. Fix $v(A) = \left(\frac{|A|}{n}\right)^2$ as the discrete fuzzy measure (Beliakov et al. [4]). Therefore:

$$v\{(1,\ 2,\ 3)\} = 1,$$

$$v\{(1,\ 3)\} = v\{(1,\ 2)\} = 0.45,$$

$$v\{(2)\} = v\{(3)\} = 0.11.$$

(5) Substituting the measurements obtained in step (4):

$$C_v(\boldsymbol{a}_1) = 0.41,$$

$$C_v(\boldsymbol{a}_2) = 0.44.$$

## 4. Discussion and Conclusion

The present paper analyses the consensus building process for a group of experts facing a financial decision problem in the context of behavioural finance. In particular, by exploiting the structure defined and implemented in Martino and Ventre [17], an attempt was made to overcome the limitation of ANP defined by the subjectivity of the expert decision maker who performs the evaluations necessary for the synthesis of alternatives. The increase in the number of experts inevitably defines a more complex and difficult consensus reaching process. To cope with this problem, in this work consensus was built after the expert evaluations. The summaries of alternatives obtained from each expert network were aggregated through Choquet's discrete integral. The strength of this methodology relates to two aspects: each expert has the possibility to make her evaluations independently and unconditionally, so that the behavioural characteristics of openness and flexibility necessary to reach

a common point with the group can be neglected in the selection of experts; aggregation with Choquet's integral includes in the consensus both the uncertainty and subjectivity associated with human nature, and the interaction between the data and thus between the judgments of the decision-makers. In fact, among the aggregation operators, Choquet's integral is the one that lends itself best to our purposes, being defined on a fuzzy measure and being structured in such a way as to give importance to the way the data interact. In this sense, the key feature of this aggregation is that an input data may not be significant on its own but may become so when related to other input data. The indicated properties were revealed in the application of the case study, in which the three alternative scores were aggregated according to a fixed fuzzy measure. With respect to the results obtained, as can be seen in Table 3, alternative 1 was preferred 2 out of 3 times, but from Choquet's aggregation, alternative 2 has a higher score, with 0.44 compared to 0.41. In this way, it is possible to see how the data interacted to indicate that the consensus could fall on alternative 2, whereby the only expert who preferred it has a score of 0.72, the highest of all evaluations. The consensus built through Choquet's integral allows for the improvement of the network methodology, whose structural limitation is precisely the subjectivity of the evaluation (Martino and Ventre [17]). With a consensus constructed a posteriori in this way, it will be possible to increase the number of experts without increasing the complexity of the methodology and the difficulty in reaching common ground during individual evaluations. The effectiveness of this approach is evidenced by how the consensus falls on the alternative that has a more powerful interaction of scores while being preferred by only one out of three experts. Possible future studies may include the introduction of other aggregation operators or other fuzzy measures, investigating which among them may improve the performance of the methodology to support the decision-making process when the decision-making context requires the intervention of several experts.

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