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# THE APPLICATION OF ZHANG-GRADIENT METHOD FOR ITERATIVE LEARNING CONTROL

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## Abstract

The novel sufficient conditions for nonlinear systems without and with time-delay, whose initial state are zero or not, are studied using the λ-norm, Zhang-gradient method and retarded Gronwall-like inequality. An examples is shown the effectiveness of the mentioned technique.

## 1. Introduction

Iterative learning control methodology, which is proposed by Arimoto et al. in 1984 (see [1]), utilizes the previous control information of studied systems. The repetitive behavior has been a major research area and a hot issue in recent years (see [2-15]).

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Recently, Zhang-gradient method has shown their powerful performance in solving online time-varying control problem (see [16-20]). The Zhang-gradient method is based on an indefinite error-monitoring function (see [20]), but the gradient dynamics method is usually designed from a norm or square-based energy function.

Stabilization problem of control systems has received some research results and have been reported in the literature [3, 11, 13, 15, 21-29]. However, only a few results combining with the iterative learning control items and Zhang-gradient method are available for nonlinear systems. In this paper, under the case that the initial state of the $k$-th iterative state vector $y_k(t)$ is the same with or different from the initial state of the $(k+1)$-th iterative state vector $y_{k+1}(t)$, the iterative learning controller of nonlinear systems is designed by using Zhang-gradient method and $\lambda$-norm.

## 2. Preliminaries

Throughout this paper, $\|x\| = (\sum_{i=1}^{n} x_i^2)^{1/2}$, is said to be the 2-norm for the vector $x = (x_1, x_2, \cdots, x_n)^T$, while the $\lambda$-norm for a function is defined as $\|\cdot\|_\lambda = \sup_{t\in[0,T]}\{e^{-\lambda t}\|\cdot\|\}$, where the superscript $T$ represents the transpose and $\lambda > 0$.

**Lemma 1** ([10, 30]). *Consider* $\sup_{t\in[0,T]}\left\{e^{-\lambda t}\int_0^t \|x(\tau)\|d\tau\right\} \le \frac{1}{\lambda}\|x(t)\|_\lambda$.

**Lemma 2** ([15, 31], **Retarded Gronwall-like inequality**). *Consider such an inequality*

$$u(t) \le a(t) + \sum_{i=1}^{n}\int_{b_i(t_0)}^{b_i(t)} f_i(t, s)w_i(u(s))ds, \quad t_0 \le t < t_1, \tag{1}$$

*and suppose that*

(1) *all* $w_i (i = 1, 2, \cdots, n)$ *are continuous and non-decreasing functions on* $[0, +\infty)$ *and positive on* $(0, +\infty)$ *such that* $w_1 \propto w_2 \propto \cdots \propto w_n$;

(2) $a(t)$ *is continuously differentiable in* $t$ *and non-negative on* $[t_0, t_1)$, *where* $t_0, t_1$ *are constants and* $t_0 < t_1$;

(3) *all* $b_i : [t_0, t_1) \to [t_0, t_1) (i = 1, 2, \cdots, n)$ *are continuously differentiable and non-decreasing such that* $b_i(t) \le t$ *on* $[t_0, t_1)$;

(4) *all* $f_i(t, s)$, $i = 1, 2, \cdots, n$, *are continuous and nonnegative functions on* $[t_0, t_1) \times [t_0, t_1)$. *Taking the notation* $W_i(s, s_0) := \int_{s_0}^{s} (1 / w_i(z)) dz$ *for* $s > 0$, *where* $s_0 > 0$ *is a given constant. It is denoted by* $W_i(s)$ *simply when there is no confusion. If* $u(t)$ *is a continuous and nonnegative function on* $[t_0, t_1)$ *satisfying* (1), *then*

$$u(t) \le W_n^{-1}[W_n(r_n(t)) + \int_{b_i(t_0)}^{b_i(t)} \max_{t_0 \le \tau < t} f_n(\tau, s) ds], \quad t_0 \le t < T,$$

*where* $r_n(t)$ *is determined recursively by*

$$r_1(t) := a(t_0) + \int_{t_0}^{t} |a'(s)| ds,$$

$$r_{i+1}(t) := W_i^{-1}[W_i(r_i(t)) + \int_{b_i(t_0)}^{b_i(t)} \max_{t_0 \le \tau < t} f_i(\tau, s) ds], \; i = 1, 2, \cdots, n-1;$$

$T < t_1$ *and* $T$ *is the largest number such that*

$$W_i(r_i(T)) + \int_{b_i(t_0)}^{b_i(T)} \max_{t_0 \le \tau < t} f_i(\tau, s) ds \le \int_{s_0}^{+\infty} \frac{dz}{w_i(z)}, \; i = 1, 2, \cdots, n.$$

## 3. Design of Iterative Learning Controller of Nonlinear Systems

In this section, we will take account of two cases about the iterative initial state vector $y_{k+1}(0) = y_k(0)$ and $y_{k+1}(0) \neq y_k(0)$, respectively, of nonlinear systems without and with time delay.

**Theorem 1.** *Consider iterative learning control on the system*:

$$\dot{y}_k(t) = f(t, y_k(t)) + u_k(t), \tag{2}$$

*where* $y_k(t) \in R^n$ *is the k-th iterative state vector,* $f(t, y_k(t))$ *is the operator* $[0, T] \times R^n \to R^n$, *and satisfies*

$$\|f(t, y_{k+1}(t)) - f(t, y_k(t))\| \leq l_f \|y_{k+1}(t) - y_k(t)\|, \tag{3}$$

$u_k(t)$ *is the k-th iterative control input, T is a constant.*

(1) *When* $y_{k+1}(0) = y_k(0)$, *the iterative learning controller is designed as*

$$u_{k+1}(t) = u_k(t) + m\hbar(e_k(t)). \tag{4}$$

(2) *When* $y_{k+1}(0) \neq y_k(0)$, *the iterative learning controller is designed as*

$$u_{k+1}(t) = u_k(t) + m\hbar(e_k(t)) + \psi_{k,h}(t)(y_{k+1}(0) - y_k(0)), \tag{5}$$

*where* $e_k(t) = y_d(t) - y_k(t)$ *is Zhang function (i.e., error function),* $y_d(t)$ *is a desired output,* $\hbar(e_k(t))$ *is a monotonically increasing odd function,* $m < 0$ *is a constant, and* $\int_0^t \psi_{k,h}(s)ds = 1$. *Besides, according to the Zhang-gradient method* (*see* [32, 33]), *the formula can be used*

$$\dot{e}_k(t) = -\mu\hbar(e_k(t)), \quad \mu > 0. \tag{6}$$

*If there exist a constant* $\lambda > 0$ *such that*

$$\left(\frac{|\mu + m|}{\mu} + \frac{l_f}{\mu} \cdot \frac{|m|}{\lambda - l_f}\right) < 1, \tag{7}$$

*then the system* (2) *can guarantee that* $\|y_d(t) - y_k(t)\|$ *is bounded and* $y_k(t)$ *can track* $y_d(t)$ *on* $t \in [0, T]$, *i.e.,* $\lim_{k \to +\infty} y_k(t) = y_d(t)$.

**Proof.** From $e_k(t) = y_d(t) - y_k(t)$, we know that $e_{k+1}(t) = e_k(t) + y_k(t) - y_{k+1}(t)$.

So $\dot{e}_{k+1}(t) = \dot{e}_k(t) + \dot{y}_k(t) - \dot{y}_{k+1}(t)$.

According to (2), (3) and (6), we have

$$\begin{aligned} -\mu\hbar(e_{k+1}(t)) &= -\mu\hbar(e_k(t)) + f(t, y_k(t)) + u_k(t) - f(t, y_{k+1}(t)) - u_{k+1}(t) \\ &= -\mu\hbar(e_k(t)) - m\hbar(e_k(t)) + [f(t, y_k(t)) - f(t, y_{k+1}(t))] \\ &= -(\mu + m)\hbar(e_k(t)) + [f(t, y_k(t)) - f(t, y_{k+1}(t))], \end{aligned}$$

$$\hbar(e_{k+1}(t)) = \frac{(\mu + m)}{\mu}\hbar(e_k(t)) + \frac{1}{\mu}[f(t, y_{k+1}(t)) - f(t, y_k(t))],$$

$$\|\hbar(e_{k+1}(t))\| \le \frac{|\mu + m|}{\mu}\|\hbar(e_k(t))\| + \frac{l_f}{\mu}\|y_{k+1}(t) - y_k(t)\|.$$

Taking $\lambda$-norm, we have

$$\|\hbar(e_{k+1}(t))\|_\lambda \le \frac{|\mu + m|}{\mu}\|\hbar(e_k(t))\|_\lambda + \frac{l_f}{\mu}\|y_{k+1}(t) - y_k(t)\|_\lambda. \tag{8}$$

From iterative law (4), (5) and Lemma 1,

$$y_{k+1}(t) - y_k(t) = \int_0^t (\dot{y}_{k+1}(t) - \dot{y}_k(t))dt + (y_k(0) - y_{k+1}(0))$$

$$= \int_0^t (f(t,\ y_{k+1}(t)) - f(t,\ y_{k+1}(t)))dt$$

$$+ \int_0^t (u_{k+1}(t) - u_k(t))dt + (y_k(0) - y_{k+1}(0))$$

$$= \int_0^t (f(t,\ y_{k+1}(t)) - f(t,\ y_k(t)))dt + \int_0^t m\hbar(e_k(t))dt.$$

$$\|y_{k+1}(t) - y_k(t)\| \le \int_0^t l_f \|y_{k+1}(t) - y_k(t)\|dt + \int_0^t |m|\,\|\hbar(e_k(t))\|dt,$$

$$\|y_{k+1}(t) - y_k(t)\|_\lambda \le \frac{l_f}{\lambda}\|y_{k+1}(t) - y_k(t)\|_\lambda + \frac{|m|}{\lambda}\|\hbar(e_k(t))\|_\lambda,$$

$$\|y_{k+1}(t) - y_k(t)\|_\lambda \le \frac{|m|}{\lambda - l_f}\|\hbar(e_k(t))\|_\lambda, \tag{9}$$

where $\lambda > l_f$.

From (8) and (9), then

$$\|\hbar(e_{k+1}(t))\|_\lambda \le \frac{|\mu + m|}{\mu}\|\hbar(e_k(t))\|\lambda + \frac{l_f}{\mu}\cdot\frac{|m|}{\lambda - l_f}\|\hbar(e_k(t))\|_\lambda$$

$$= \left(\frac{|\mu + m|}{\mu} + \frac{l_f}{\mu}\cdot\frac{|m|}{\lambda - l_f}\right)\|\hbar(e_k(t))\|_\lambda. \tag{10}$$

When the condition (7) is true, we have $\lim_{k\to+\infty}\|\hbar(e_k(t))\|_\lambda = 0$. That $\hbar(e_k(t))$ is a monotonically increasing odd function implies $\lim_{k\to+\infty}\|e_k(t)\|_\lambda = 0$, i.e., $\lim_{k\to+\infty}\|e_k(t)\| = 0$.

**Theorem 2.** *Consider the following system*:

$$\dot{y}_k(t) = f(t,\ y_k(t)) + g(t,\ y_k(t-\tau)) + u_k(t), \tag{11}$$

*where* $y_k(t) \in R^n$ *is the k-th iterative state vector,* $f(t,\ y_k(t))$, $g(t,\ y_k(t-\tau))$ *are the operator* $[0,\ T] \times R^n \to R^n$, *and satisfies*

$$\|f(t,\ y_{k+1}(t)) - f(t,\ y_k(t))\| \le l_f \|y_{k+1}(t) - y_k(t)\|, \tag{12}$$

$$\|g(t,\ y_{k+1}(t-\tau)) - g(t,\ y_k(t-\tau))\| \le l_g \|y_{k+1}(t-\tau) - y_k(t-\tau)\|, \tag{13}$$

$u_k(t)$ *is the k-th iterative control input,* $\tau > 0$ *is time delay, and* $T$ *is a constant. When* $y_{k+1}(0) = y_k(0)$, *the iterative learning controller is designed as* (4). *When* $y_{k+1}(0) \neq y_k(0)$, *the iterative learning controller is designed as* (5). *Where* $e_k(t) = y_d(t) - y_k(t)$ *is Zhang function* (*i.e., error function*), $y_d(t)$ *is a desired output,* $\hbar(e_k(t))$ *is a monotonically increasing odd function,* $m < 0$ *is a constant, and* $\int_0^t \psi_{k,h}(\tau)d\tau = 1$. *Besides, according to the Zhang gradient method* (*see* [32, 33]), *the following Zhang gradient design formula can be used as* (6). *If there exist a constant* $\lambda > 0$ *and a continuous function* $\varphi(t) \neq 0$, $t \in [0,\ T]$, *such that*

$$\left( \frac{|\mu + m|}{\mu} + \frac{l_f}{\mu} \cdot \frac{e^{(l_f + l_g)t}|m|}{\lambda} + \frac{l_g}{\mu} \cdot \frac{e^{(l_f + l_g)(t-\tau)}|m|}{\lambda} \cdot \frac{\varphi(t-\tau)}{\varphi(t)} \right) < 1, \tag{14}$$

*then the system* (10) *can guarantee that* $\|y_d(t) - y_k(t)\|$ *is bounded and* $y_k(t)$ *can track* $y_d(t)$ *on* $t \in [0,\ T]$, *i.e.,* $\lim_{k \to +\infty} y_k(t) = y_d(t)$.

**Proof.** From $e_k(t) = y_d(t) - y_k(t)$, we know that

$$e_{k+1}(t) = e_k(t) + y_k(t) - y_{k+1}(t).$$

So $\dot{e}_{k+1}(t) = \dot{e}_k(t) + \dot{y}_k(t) - \dot{y}_{k+1}(t)$, that is,

$$\begin{aligned}
-\mu\hbar(e_{k+1}(t)) &= -\mu\hbar(e_k(t)) + f(t, y_k(t)) + g(t, y_k(t-\tau)) + u_k(t) \\
&\quad - f(t, y_{k+1}(t)) - g(t, y_{k+1}(t-\tau)) - u_{k+1}(t) \\
&= -\mu\hbar(e_k(t)) - m\hbar(e_k(t)) + [f(t, y_k(t)) - f(t, y_{k+1}(t))] \\
&\quad + [g(t, y_k(t-\tau)) - g(t, y_{k+1}(t-\tau))] \\
&= -(\mu + m)\hbar(e_k(t)) + [f(t, y_k(t)) - f(t, y_{k+1}(t))] \\
&\quad + [g(t, y_k(t-\tau)) - g(t, y_{k+1}(t-\tau))],
\end{aligned}$$

$$\begin{aligned}
\hbar(e_{k+1}(t)) &= \frac{(\mu + m)}{\mu}\hbar(e_k(t)) + \frac{1}{\mu}[f(t, y_{k+1}(t)) - f(t, y_k(t))] \\
&\quad + \frac{1}{\mu}[g(t, y_{k+1}(t-\tau)) - g(t, y_k(t-\tau))],
\end{aligned}$$

$$\begin{aligned}
\|\hbar(e_{k+1}(t))\| &\le \frac{|\mu + m|}{\mu}\|\hbar(e_k(t))\| + \frac{l_f}{\mu}\|y_{k+1}(t) - y_k(t)\| \\
&\quad + \frac{l_g}{\mu}\|y_{k+1}(t-\tau) - y_k(t-\tau)\|.
\end{aligned}$$

Taking $\lambda$-norm, we have

$$\begin{aligned}
\|\hbar(e_{k+1}(t))\|_\lambda &\le \frac{|\mu + m|}{\mu}\|\hbar(e_k(t))\|_\lambda + \frac{l_f}{\mu}\|y_{k+1}(t) - y_k(t)\|_\lambda \\
&\quad + \frac{l_g}{\mu}\|y_{k+1}(t-\tau) - y_k(t-\tau)\|_\lambda. \quad (15)
\end{aligned}$$

$$\begin{aligned}
y_{k+1}(t) - y_k(t) &= \int_0^t (\dot{y}_{k+1}(t) - \dot{y}_k(t))dt + (y_k(0) - y_{k+1}(0)) \\
&= \int_0^t (\dot{y}_{k+1}(t) - \dot{y}_k(t))dt
\end{aligned}$$

$$= \int_0^t (f(t,\ y_{k+1}(t)) - f(t,\ y_k(t)))dt + \int_0^t (u_{k+1}(t) - u_k(t))dt$$

$$+ \int_0^t (g(s,\ y_{k+1}(s-\tau)) - g(s,\ y_k(s-\tau)))ds$$

$$= \int_0^t (f(t,\ y_{k+1}(t)) - f(t,\ y_k(t)))dt + \int_0^t m\hbar(e_k(t))dt$$

$$+ \int_0^t (g(s,\ y_{k+1}(s-\tau)) - g(s,\ y_k(s-\tau)))ds.$$

$$\|y_{k+1}(t) - y_k(t)\| \le \int_0^t \|f(s,\ y_{k+1}(s)) - f(s,\ y_k(s))\|ds + \int_0^t |m|\,\|\hbar(e_k(s))\|ds$$

$$+ \int_0^t \|g(s,\ y_{k+1}(s-\tau)) - g(s,\ y_k(s-\tau))\|ds$$

$$\le l_f \int_0^t \|y_{k+1}(s) - y_k(s)\|ds + \int_0^t |m|\,\|\hbar(e_k(s))\|ds$$

$$+ l_g \int_0^t \|y_{k+1}(s-\tau) - y_k(s-\tau)\|ds.$$

We use Lemma 2. Taking $t_0 = 0$, $b_1(t) = t$, $b_2(t) = t - \tau$, $a(t) = \int_0^t |m|\,\|\hbar(e_k(s))\|ds$,

$f_1(t, s) = f_2(t, s) = 1$, $W_1(s) = \frac{1}{l_f}\int_1^s \frac{dz}{z} = \frac{1}{l_f}\ln s$, and $W_2(s) = \frac{1}{l_g}\int_1^s \frac{dz}{z} = \frac{1}{l_g}\ln s$,

then $r_1(t) = \int_0^t |m|\,\|\hbar(e_k(s))\|ds$, $r_2(t) = r_1(t)e^{tl_f}$. So we have

$$\|y_{k+1}(t) - y_k(t)\| \le e^{(l_f + l_g)t} \int_0^t |m| \, \|\hbar(e_k(s))\| ds,$$

$$\|y_{k+1}(t) - y_k(t)\|_\lambda \le \frac{e^{(l_f + l_g)t}|m|}{\lambda} \|\hbar(e_k(t))\|_\lambda, \tag{16}$$

$$\|y_{k+1}(t-\tau) - y_k(t-\tau)\|_\lambda \le \frac{e^{(l_f + l_g)(t-\tau)}|m|}{\lambda} \|\hbar(e_k(t-\tau))\|_\lambda. \tag{17}$$

From (15)-(17) we obtain

$$\|\hbar(e_{k+1}(t))\|_\lambda \le \left( \frac{|\mu + m|}{\mu} + \frac{l_f}{\mu} \cdot \frac{e^{(l_f + l_g)t}|m|}{\lambda} \right) \|\hbar(e_k(t))\|_\lambda$$

$$+ \left( \frac{l_g}{\mu} \cdot \frac{e^{(l_f + l_g)(t-\tau)}|m|}{\lambda} \right) \|\hbar(e_k(t-\tau))\|_\lambda. \tag{18}$$

It is easy to know that the solution of the equation

$$x_{k+1}(t) = \alpha x_k(t) + \beta x_k(t-\tau)$$

is

$$x_k(t) = c\varphi(t) \left( \alpha + \beta \frac{\varphi(t-\tau)}{\varphi(t)} \right)^k, \tag{19}$$

where $\alpha$, $\beta$ are given constants, $c$ is an arbitrary constant, $\varphi(t)$ is an arbitrary function and satisfies that $\frac{\varphi(t-\tau)}{\varphi(t)}$ is a constant.

From (19) and the condition (14), we have $\lim_{k \to +\infty} \|\hbar(e_k(t))\|_\lambda = 0$. That $\hbar(e_k(t))$ is a monotonically increasing odd function implies $\lim_{k \to +\infty} \|e_k(t)\|_\lambda = 0$, i.e., $\lim_{k \to +\infty} \|e_k(t)\| = 0$.

## 4. Example

For further illustration, we consider the following system:

$$\dot{y}_k(t) = 0.8y_k(t) \cdot \sin t - 0.5(|y_k(t-1)+1| - |y_k(t-1)-1|) + u_k(t),$$

$$u_{k+1}(t) = u_k(t) + me_k(t) - \psi_{k,h}(t)(y_k(0) - y_{k+1}(0)).$$

Taking $m = -1.5$, $\mu = 2$, $l_f = 1.6$, $l_g = 2$, $\varphi(t) = 0.2e^{1.2t}$,

$$\psi_{k,h}(t) = \begin{cases} \pi\cos(\pi t), & t \in [0, 1], \\ 0, & t \in (1, 2]. \end{cases}$$

From the above example, it can be easily proved that the condition (13) of Theorem 2 is satisfied.

## 5. Conclusion

In this paper, considering the iterative learning control problem for nonlinear systems without and with time delays, and combining with ZD method, the novel controllers, which can guarantee the robust convergence of the tracking error, are designed.

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