Journal of Statistics: Advances in Theory and Applications
Volume 14, Number 1, 2015, Pages 13-26
Available at http://scientificadvances.co.in
DOI: http://dx.doi.org/10.18642/jsata_7100121497

RELIABILITY TEST PLANS IN THE CASE OF DEPENDENT OBSERVATION BASED ON EXPONENTIAL FAILURE MODEL

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Abstract

This paper presents the structure of retentive production processes where successive observations are not independent. The concept of reliability test plan is enlarged for the lots form retentive processes. The operating characteristics (OC) function of the plan for exponential failure model have been derived and illustrated numerically. Tables are also presented to facilitate the operation and construction of the plan.

2010 Mathematics Subject Classification: 60-XX, 60EXXX.
Keywords and phrases: exponential distribution, retentive production processes, OC function.
Received April 11, 2015; Revised August 4, 2015

1. Introduction

The development of acceptance sampling based on life tests or reliability tests plan has attracted the attention of several research workers. Some references may be made to Epstein [5, 6], Gupta and Groll [8], Goode and Kao [7], Gupta [9], Kantam and Rosaiah [10], Kantam et al. [11], Rosaiah and Kantam [18], Rosaiah et al. [19], Rao et al. [16], Rao [17], Singh et al. [23], Aslam and Shahbaz [1], Khan and Islam [12], and Sriramachandran [24]. In general, the formulation of life tests plan consists of deriving the fraction defective from the lifetime distribution model and then to apply the notion of single sampling plan by attributes for acceptance or rejection of the lot.

It is well recognized by researchers and practitioners that a production process generally tends to lose some of its efficiency in service because of deterioration of machines and equipment as time elapses or due to erratic factors described, e.g., by Burr [3]. Therefore quality of products from the process decreases in time, and thereby, reflecting the quality of lots submitted for inspection. Duncan [4] remarked that "if a producer continues to submit to the consumer product from a process with a constant proportion defective, lot-after-lot, simple acceptance or rejection of lots submitted will not change the proportion defective the consumer will eventually receive, the consumer will receive the same proportion defective as was in the original process". Therefore, it becomes justified and necessary to study the performance characteristics of acceptance sampling plans in terms of process quality level.

Most of the acceptance sampling plans available in the literature assume (i) a constant probability of occurrence of a defective (or failure in the present context) and (ii) independence of successive observations. However, this is not always justified in many industrial processes. Barnard [2] has drawn attention to the fact that the production process that does not satisfy above assumptions, usually falls into one of the two categories (i) inert processes and (ii) retentive processes. Later on, Nath [14] developed control charts for fraction defective assuming that observations have been taken from the retentive production processes, and where the successive observations are not independent. In the other words, we have considered the situations where successive items produced depend on the nature of the preceding one, which is interesting and new approach for the development of present paper.

A few types of quality control models based on dependent observations have been studied by Singh and Singh [21, 22], and Rajarshi and Sampath Kumar [15]. Shankar and Gangeshwar [20] studied the cumulative sum control charts for retentive production processes. In this paper, an attempt has been made to develop the reliability (lifetime) test plans for lots from retentive production processes. The OC function of the plan have been derived for exponential failure model and illustrated numerically. Lastly, tables of minimum sample size, necessary for various acceptance number *c* and for various confidence level P^*, have been provided on the line of Gupta [9]. The effect of dependency on OC function has been studied graphically.

2. Structure of Retentive Production Processes

To model the situations where successive items produced depend on the nature of preceding one, Barnard [2] emphasized that production processes usually fall into one of the following two categories:

Inert processes 1. Which stay put at a given average level until some shock moves them to a new level, at which they again tend to stay put; and

Retentive processes 2. Which represent behaviour of a system, which moves under the influence of a shock, but which has a tendency to return to its old level.

In order to avoid excessive references, we first reproduce the findings of Nath [14] in brief, and then develop a life tests plan for exponential failure model. The retentive processes are usually, described by the Morkov chain having transition probability matrix

$$M = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}, \tag{2.1}$$

where $p_{11} + p_{12} = 1$ and $p_{21} + p_{22} = 1$.

The symbols p_{11}, p_{12}, $\ldots$ etc. are explained in Appendix 1. Nath [14] have further shown that, the probability of process producing a defective item at any stage irrespective of the initial stage is given by

$$\pi = \frac{p_{12}}{P_{21} + P_{12}}. \tag{2.2}$$

Now, in the present context of life testing, we assume that the lifetime of the products produced at initial stage follows an exponential distribution with parameter θ_1. Likewise, lifetime of products produced at second stage of production follows the exponential distribution with parameter θ_2. Suppose that we have a retentive production process operating in a random manner described by the transition probability matrix (2.1). The product of this process will be said to be of quality vector $v = [p_{12}, p_{22}]$. Further, let the lots of size N are made up of products of this process and submitted for inspection. More specifically, the probabilities p_{12} and p_{22} are defined as follows:

$$P_{12} = (1/\theta_1)\int_0^t \exp(-x/\theta_1)dx = 1 - \exp(-t/\theta_1), \tag{2.3}$$

and

$$\begin{aligned} P_{22} &= (1/\theta_2)\int_0^t \exp(-x/\theta_2)dx = 1 - \exp(-t/\theta_2) \\ &= 1 - \exp(-t/(g\theta_1)), \end{aligned} \tag{2.4}$$

where $\theta_2 = g\theta_1$ and $P_{21} = 1 - p_{22}$.

3. The Reliability Test Plan and OC Function

Assume that the lifetime of the products produced at both the stages of production follows exponential distribution. A common practice in life testing is to truncate the experiment at a per-assigned time t and note the number of failures. The decision to accept the specific mean life occurs, if and only if, the observed number of failures at the end of fixed time t does not exceed a given acceptance number c. (One can terminate the testing before time t is reached when the number of failures exceeds c, the decision in this case is to reject the lot.) The life test plan, thus, consists of

(1) the number of units on test;

(2) an acceptance number c such that lot is accepted, if c or fewer failures occur during the fixed time t; and

(3) the ratio $1/\theta_1$ and $g = \theta_2/\theta_1$, where θ_1 and θ_2 are the specified mean life defined above.

Nath [14] have shown that under the stabilized condition of the process, the constituents of the state vector at any stage become independent of the initial stage in which the process might have been. Consequently, the fraction non-conforming of the lot will follow a binomial distribution with fraction defective whose value lie between zero to one. The OC function of the plan may be written as

$$P_A = L(p_{12}, p_{22}) = \sum_{i=0}^{c} \binom{n}{i} \pi^i (1-\pi)^{n-i}, \tag{3.1}$$

where $\pi = \dfrac{p_{12}}{p_{21} + p_{12}}$ and $p_{21} = 1 - p_{22}$.

Under the stabilized condition of production, the ratio of mean life, i.e., $g = \theta_2/\theta_1$ is fixed. The probabilities p_{12} and p_{21} may be written as

$$p_{12} = 1 - \exp(-t/\theta_1) \quad \text{and} \quad p_{21} = \exp(-t/(g.\theta_1)),$$

The main objective of a life test experiment is to set a confidence (lower) limit on the mean life. It is then desired to establish a specified mean life with a given probability of at least P^*. Now, we are interested in obtaining sample size n, which satisfies the inequality

$$\sum_{i=0}^{c}\binom{n}{i}\pi^{i}(1-\pi)^{n-i} \leq 1-P^{*}. \tag{3.2}$$

Moreover, when the producer's risk is given, the minimal ratios of the true mean life to the specified mean life t / θ_2 can be determined with satisfies the following inequality:

$$\sum_{i=0}^{c}\binom{n}{i}\pi^{i}(1-\pi)^{n-i} \geq 1-\alpha. \tag{3.3}$$

Minimum ratio is

$$\sum_{i=0}^{c}\binom{n}{i}\pi^{i}(1-\pi)^{n-i} \geq 1-P^{*}. \tag{3.4}$$

4. Construction of the Table

The operating characteristic function of the proposed plan $(n, c, t / \theta_1, g)$ is given by (3.1). Now, under Poisson model, the OC function may be written as

$$P_A = \sum_{i=0}^{c}(e^{-\lambda}\lambda^{i} / i!\,), \text{ where } \lambda = n\pi. \tag{4.1}$$

Thus, given a number $P^{*}(0 < P^{*} < 1)$, t / θ_1, c, and "g", we want to find the smallest positive integer n such that

$$\sum_{i=0}^{c}(e^{-\lambda}\lambda^{i} / i!\,) < 1-P^{*}. \tag{4.2}$$

Since the probability of acceptance P_A can be shown to a function of $\lambda = n\pi$, for given values of t / θ_1, c, and g. Therefore, minimum sample size n satisfying (4.2) have been obtained for some selected values of t / θ_1 by using Newton's method of successive approximation. The minimum values of n have been obtained for $P^* = 0.95, 0.99$, $g = 0.99$, $0.75, 0.50$ and $t/\theta_1 = 1.0, 0.75, 0.50, 0.20, 0.10. 0.075, 0.05, 0.02, 0.01, 0.005$. These are given in Table 1. For a specified values of the producer's risk, say, 0.25 one may interested in knowing that value of t/θ_2 will insure a producer's risk less or equal to 0.25 for a given sampling plan. The value of t/θ_2 is smallest positive number for which the following equality holds:

$$\sum_{i=0}^{c} \binom{n}{i} \pi^i (1-\pi)^{n-i} \geq 0.75. \tag{4.3}$$

For example, for particular $n = 8$, $c = 2$ and $t/\theta_1 = 1$ the values of t/θ_2 comes 1.7061 and for other values $n = 10$, $c = 2$ and $t/\theta_1 = 0.75$ the value of t/θ_2 comes 2.075 using Equation (4.3).

Table 1. Values of minimum sample size $P^* = 0.95$

		t/θ_1									
g	***c***	**1.00**	**.75**	**.50**	**.20**	**.10**	**.075**	**.050**	**.020**	**.010**	**.005**
.90	**1**	7	8	11	25	49	65	96	239	476	950
	2	9	11	15	34	65	86	128	317	632	1261
	3	11	14	19	42	80	106	158	390	778	1553
	4	13	16	22	49	95	125	186	461	918	4834
	5	15	19	25	56	109	144	214	529	1055	2106
	6	18	21	29	64	123	162	241	596	1188	2373
	7	19	23	32	71	136	180	268	662	1319	2634
	8	21	26	35	78	150	198	294	727	1449	2892
	9	23	28	38	85	163	215	320	791	1576	3147
	10	25	30	41	91	176	232	345	854	1702	3399
.75	**1**	6	8	10	24	48	64	95	238	475	949
	2	8	10	14	32	64	85	127	315	630	1260
	3	10	13	17	40	79	104	156	389	776	1552
	4	12	15	21	47	93	123	184	459	916	1832
	5	14	17	24	54	107	142	212	527	1053	2104
	6	16	20	27	61	120	160	239	594	1186	2370
	7	18	22	30	68	134	177	265	659	1317	2631
	8	20	24	32	75	147	195	291	724	1445	2889
	9	22	26	36	82	160	212	316	787	1573	3143
	10	24	28	39	88	172	229	342	851	1699	3395
.50	**1**	5	6	9	22	45	61	92	234	472	946
	2	7	8	12	29	60	81	123	311	626	1256
	3	9	11	15	36	74	100	151	383	771	1546
	4	11	13	17	43	87	118	178	453	910	1826
	5	12	14	20	49	100	135	205	520	1046	2097
	6	14	16	22	55	113	152	231	586	1178	2362
	7	15	18	25	61	129	169	257	651	1308	2623
	8	17	20	27	67	138	186	282	714	1436	2879
	9	19	22	30	73	150	202	307	777	1562	3133
	10	20	24	32	79	162	219	331	839	1687	3384

Table 1. (Continued) Values of minimum sample size $P^* = 0.99$

		t / θ_1									
g	***c***	**1.00**	**.75**	**.50**	**.20**	**.10**	**.075**	**.050**	**.020**	**.010**	**.005**
.90	**1**	10	12	16	35	69	91	135	334	666	1330
	2	12	15	20	45	87	115	171	423	843	1684
	3	15	18	24	54	104	137	204	506	1008	2012
	4	17	21	28	62	120	159	236	584	1164	2325
	5	19	23	32	71	136	180	267	660	1315	2626
	6	22	26	35	78	151	200	297	734	1462	2919
	7	24	29	39	86	166	219	326	806	1606	3206
	8	26	31	42	94	181	238	354	876	1747	3487
	9	28	34	46	101	195	257	383	946	1885	3763
	10	30	36	49	109	209	276	410	1015	2022	4036
.75	**1**	9	11	15	34	67	89	133	333	664	1328
	2	11	14	19	43	85	113	169	421	842	1682
	3	14	17	23	52	102	135	202	503	1006	2010
	4	16	19	26	60	118	156	234	582	1162	2322
	5	18	22	30	68	133	177	264	657	1313	2623
	6	20	24	33	76	148	197	294	731	1459	2916
	7	22	27	36	83	163	216	322	802	1602	3202
	8	24	29	40	90	177	235	351	873	1743	3483
	9	26	31	43	98	191	253	379	942	1881	3759
	10	28	34	46	105	205	272	406	1010	2017	4032
.50	**1**	8	9	12	31	63	85	129	328	660	1324
	2	10	11	16	39	80	108	164	416	836	1677
	3	12	14	19	47	96	129	196	497	999	2004
	4	14	16	22	54	111	149	226	574	1154	2315
	5	15	18	25	61	125	169	256	659	1304	2615
	6	17	20	28	68	139	188	284	721	1449	2906
	7	19	22	30	75	153	206	312	792	1592	3191
	8	21	24	33	81	167	224	340	861	1731	3471
	9	22	26	36	88	180	242	367	930	1869	3747
	10	24	28	38	94	193	260	393	997	2004	4018

5. Illustration and Discussion of the Result

Assume that lifetime follows exponential distribution at both the stage of production. Suppose the experimenter is interested in establishing the true unknown average life at least $\theta_1 = 10,000$ hours for the initial stage with confidence $P^* = 0.99$ and $g = \theta_2 / \theta_1 = 0.75$. It is desired to stop the experiment at $t = 1,000$ hours. Then, for acceptance number $c = 2$, the required n is the entry in Table 1 corresponding to the values of $t / \theta_1 = 1,000 / 10,000 = 0.1$, $P^* = 0.99$, and $g = 0.75$. This number is $n = 85$. Thus, 85 units have been put on the test. If during 1000 hours, no more than 2 failures out of 85 are observed, experimenter can assert with a confidence level of 0.99 that the average life at the initial stage is at least 10000 hours.

Now, following Wang [25], Medhi [13], and Rajarshi and Sampath Kumar [15], it can be shown that the serial correlation co-efficient of the Markov chain given as $\delta = 1 - (p_{12} + p_{21})$. Consequently, we have $\pi = p_{12} / (1 - \delta)$. In order to study the effect of δ on OC function, we consider the following plan $n = 30$ and $c = 2$. For some chosen values of t / θ_1 and for $\delta = 0.00, +0.50, -0.50$, the values of OC function have been worked out and results are shown in Table 2. That dependency has no effect on the OC functions when $t / \theta_1 < 0.01$. For a visual comparison, OC curves have been drawn in Figure 1. The figure shows how the OC curve for a plan varies with the δ. For the case of positive serial correlation the plan is tightened up, and thereby, lowering the OC curves (probability of acceptance). However, for negative serial correlation coefficient the plan becomes, more lax and the effect is to raise the OC curve. It is further seen from the Figure 1. that the dependency distorts the OC by causing an increase in procedure’s risk for positive serial correlation and an increase in consumer’s risk for negative serial correlation.

Table 2. Values of OC function for the plan

Let $n = 30,\ c = 2$

t / θ_1	δ		
	0.50	**- 0.50**	**0.00**
0.01	0.9985	0.9989	0.9967
0.02	0.9956	0.9928	0.9788
0.03	0.7397	0.9791	0.9420
0.05	0.4289	0.9272	0.8214
0.07	0.2089	0.8485	0.6688
0.10	0.0569	0.7038	0.4464
0.12	0.0214	0.6035	0.3250
0.15	0.0043	0.4636	0.1916
0.20	0.0000	0.2800	0.0717
0.30	0.0000	0.0886	0.0079
0.40	0.0000	0.0256	0.0000

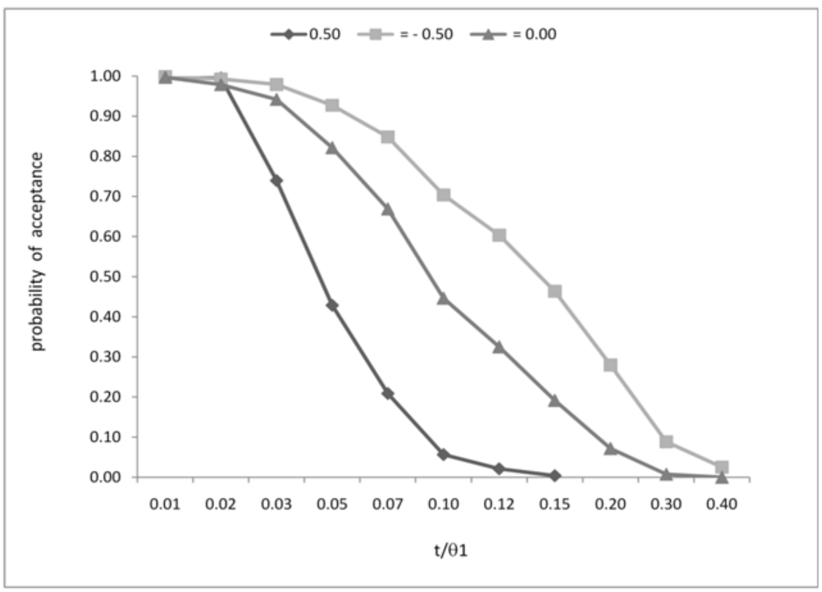

Figure 1. OC curve.

6. Conclusion

Life tests plan possesses wide potential applicability in industry insuring a higher standard of quality attainment. Here, we have introduced life tests plan for lots from retentive production processes. For practical utility of plan, the minimum sample sizes necessary to assure the specified mean life has been tabulated for exponential failure model. In the other words, tables are provided here which tailor-made, handy and ready-made use to the industrial shop-floor condition. The concept of this article may be of assistance in the development of plans for other failure models.

References

[1] M. Aslam and M. Q. Shahbaz, Economic reliability tests plans using the generalized exponential distribution, Journal of Statistics 14 (2007), 52-59.

[2] G. A. Barnard, New methods of quality control, Journal of Royal Statistical Society A 126(2) (1963), 255-258.

[3] I. W. Burr, Statistical Quality Control Method, Marcel Dekker, New York, pp. 82, 1976.

[4] A. J. Duncan, Quality Control and Industrial Statistics (4th Edition) Richard D. Irwin, Homewood, Illinois. 2, 1974.

[5] B. Epstein, Truncated life test in exponential case, Annals Mathematical Statistics 25 (1954), 555-564.

[6] B. Epstein, Statistical life test acceptance procedures, Technometrics 2(4) (1960), 435-446.

[7] H. P. Goode and J. H. K. Kao, Sampling plans based on Weibull distribution, Proceedings of Seventh National Symposium on Reliability and Quality Control, Philadelphia, Pennsylvania (1961), 24-40.

[8] S. Gupta and P. A. Groll, Gamma distribution in acceptance sampling based on life tests, Journal of American Statistical Association 56 (1961), 942-970.

[9] S. S. Gupta, Life test sampling plans for normal and lognormal distributions, Technometrics 4(2) (1962), 155-175.

[10] R. R. L. Kantam and K. Rosaiah, Half logistic distribution in acceptance sampling based on life tests, IAPQR Transaction 23(2) (1998), 117-125.

[11] R. R. L. Kantam, K. Rosaiah and R. G. Srinivasa, Acceptance sampling based on life tests log-logistic model, Journal of Applied Statistics 28 (2001), 121-128.

[12] M. A. Khan and H. M. Islam, On the robustness of reliability characteristic of alpha distributed lifetimes, Economic Quality Control 25(2) (2010), 173-181.

[13] J. Medhi, Stochastic Process, New Age International P Ltd. Publishers (2nd Edition), New Delhi, 83-86, 1999.

[14] P. Nath, Control chart for fraction defective in case of dependent observations, UDC 57 (1976), 69-71.

[15] M. B. Rajarshi and K. V. S. Sampath, Dodge's CSP-1 for Morkov dependent process using probability sampling, Journal of the Indian Statistical Association 21 (1983), 99-111.

[16] G. S. Rao, M. E. Chitany and R. R. L. Kantam, Reliability test plans for Marshall-Olkin extended exponential distribution, Applied Mathematical Science 3(55) (2009), 2745-2755.

[17] G. S. Rao, Double acceptance sampling plans based on truncated life tests for the Marshall-Olkin extended exponential distribution, Australian Journal of Statistics 40(3) (2011), 169-176.

[18] K. Rosaiah and R. R. L. Kantam, Acceptance sampling based on the inverse Rayleigh distribution, Economic Quality Control 20 (2005), 277- 286.

[19] K. Rosaiah, R. R. L. Kantam and and K. Santosh, Reliability test plans for exponentiated log-logistic distribution, Economic Quality Control 21(2) (2006), 165-175.

[20] G. Shankar and D. K. Gangeshwer, Cumulative sum control charts for retentive production processes, Assam Statistical Review 19(2) (2005), 117-128.

[21] H. R. Singh and J. R. Singh, Variable sampling plan under second order autoregressive model, IAPQR Transaction 7(2) (1982), 97-104.

[22] H. R. Singh and J. R. Singh, Variable sampling plan under first order autoregressive model, Journal of National Academy Mathematics 4 (1986), 73-80.

[23] B. Singh, K. K. Sharma and D. Tyagi, Acceptance sampling plan based on truncated life tests for compound Rayleigh distribution, Journal of Reliability and Statistical Studies 6(2) (2013), 1-15.

[24] G. V. Sriramachandran, Double acceptance sampling plan for time truncated life tests based on Burr type XII distribution, Applied Mathematical Sciences 7(67) (2013), 3345-3354.

[25] Y. H. Wang, On the limit of Markov-Bernoulli distribution, Jour. Appl. Probability 18 (1981), 937-942.

Appendix

The following symbols (Nath [14]) are customarily used in defining transition matrix of retentive process:

p_{11} = Conditional probability of production of a conforming item, given the preceding was conforming.

p_{12} = Conditional probability of production of a non-conforming (defective) item, given the preceding was conforming.

p_{21} = Conditional probability of production of a conforming item, given the preceding was defective.

p_{22} = Conditional probability of production of a defective item, given the preceding was defective.

π = Probability of process producing a defective item when observations are dependent, irrespective of initial stage.

■