A MODEL FOR THE FORECASTING OF MONTHLY NIGERIAN BANK PRIME LENDING RATES: A SEASONAL BOX-JENKINS APPROACH

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Abstract

This work involves the fitting of a SARIMA model to the monthly prime lending rates of Nigerian banks from January 2006 to September 2014. Its time-plot shows a generally horizontal trend with a peak between 2009 and 2010. Evidence abounds for 12-monthly seasonality: The correlogram is sinusoidal-patterned with period 12 months and an inspection confirms the seasonality hypothesis. Hence it was necessary to difference the series seasonally once. The resultant series bears a lot of resemblance with the original series. The Augmented Dickey Fuller (ADF) test considers both series as non-stationary. A further but non-seasonal differencing of the series yields a series that is adjudged as stationary by the ADF test. Its time-plot shows a horizontal trend and no clear seasonality. However, its correlogram shows an evidence of stationarity and seasonality of period 12 months. Applying a new algorithm for subset SARIMA modelling, the SARIMA\((1, 1, 0) \times (1, 1, 0)_1\) model was fitted. It is shown to be more adequate that the corresponding additive model. It is also demonstrated to be multiplicative and not subset. Forecasting of the rates may therefore be based on it.
Keywords: prime lending rates, SARIMA modelling, subset SARIMA modelling, Nigerian banks.

1. Introduction

Bank prime lending rates are interest rates at which banks offer loans to the customers who have very good reputations with them. They are often big-time and loyal customers with a track record with them. These rates provide basis for determination of other interest rates. They are a significant cost of capital (Awoyemi and Jabar [4]). This paper aims at fitting a seasonal autoregressive integrated moving average (SARIMA) model to monthly Nigerian prime lending rates as published by the Central Bank of Nigeria (CBN) with a view to providing basis for forecasting of the rates.

SARIMA models were proposed by Box and Jenkins [6] for the modelling of seasonal time series. Many economic and financial time series are like that. The prime lending rates of monthly Nigerian banks have been shown herein to exhibit seasonality of period 12 months. SARIMA modelling is extensively applied of recent to model time series. Time series that have been modelled by SARIMA methods include savings deposit rates (Etuk et al. [10]), electricity demand (Makukule et al. [15]), crude oil exports (Ayinde and Abdulwahab [5]), meat exports (Paul et al. [20]), tourism patronage (Padhan [19]; Longanathan and Ibrahim [13]), aviation patronage (Box and Jenkins [6]), foreign exchange rates (Appiah and Adetunde [3]), fish catch (Prista et al. [21]), dengue numbers (Martinez et al. [16]), water demand (Mombeni et al. [17]). Others are inflation rates (Fannoh et al. [12]; Saz [22]), malaria incidence (Dan et al. [7]), cerebrospinal meningitis incidence (Abdelghani et al. [1]), tomato prices (Adanacioglu and Yercan [2]), cucumber prices (Luo et al. [14]), stock prices (Etuk [8]), unemployment rates (Etuk [9]), internally generated revenue (Etuk and Ojekudo [11]), and tuberculosis incidence (Moosazadeh et al. [18]). All these are to mention but a few.
2. Materials and Methods

2.1. Data

The data for this work are 105 monthly Nigerian bank prime lending rates from January 2006 to September 2014 published in the website of the CBN www.cenbank.org under the Money Market Indicators subsection of the statistics section.

2.2. Sarima models

A stationary time series $\{X_t\}$ is said to follow an autoregressive moving average model of order $p$ and $q$, denoted by ARMA$(p, q)$, if

$$X_t - \alpha_1 X_{t-1} - \alpha_2 X_{t-2} - \ldots - \alpha_p X_{t-p} = \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + \ldots + \beta_q \varepsilon_{t-q},$$

(1)

where the sequence $\{\varepsilon_t\}$ is a white noise process. The $\alpha$’s and $\beta$’s are constants such that the model is both stationary and invertible. Suppose the model (1) is put in the form

$$A(L)X_t = B(L)\varepsilon_t,$$

(2)

where $A(L) = 1 - \alpha_1 L - \alpha_2 L^2 - \ldots - \alpha_p L^p$ and $B(L) = 1 + \beta_1 L + \beta_2 L^2 + \ldots + \beta_q L^q$

and $L$ is the backward shift operator defined by $L^k X_t = X_{t-k}$. $A(L)$ is the autoregressive (AR) operator and $B(L)$ is the moving average (MA) operator. For stationarity and invertibility the zeros of these operators must lie outside the unit circle, respectively.

Suppose $\{X_t\}$ is non-stationary. Box and Jenkins [6] proposed that differencing of the series to an appropriate degree $d$ could render it stationary. Suppose the $d$-th difference, $\{\nabla^d X_t\}$, of $\{X_t\}$ is stationary and follow an ARMA$(p, q)$ model. Then $\{X_t\}$ is said to follow an autoregressive integrated moving average model of order $p, d, q$.
designated ARIMA($p, d, q$). Here $\nabla = 1 - L$. Before and after differencing of a time series stationarity shall be tested by Augmented Dickey Fuller (ADF) test.

If moreover $\{X_t\}$ is seasonal of period $s$, Box and Jenkins [6] further proposed that it could be modelled by

$$A(L)\Phi(L^s)\nabla^d\nabla^D_s X_t = B(L)\Theta(L^s)\epsilon_t,$$

where $\Phi(L)$ and $\Theta(L)$ are the seasonal AR and MA operators, respectively, defined by $\Phi(L) = 1 + \phi_1 L + \phi_2 L^2 + \ldots + \phi_p L^p$ and $\Theta(L) = 1 + \theta_1 L + \theta_2 L^2 + \ldots + \theta_Q L^Q$ and the $\phi$’s and the $\theta$’s constants such that the entire model (3) is both stationary and invertible. The symbol $\nabla_s$ is the seasonal difference operator defined by $\nabla_s = 1 - L^s$. The model is called a *multiplicative seasonal autoregressive integrated moving average model of order $p, d, q, P, D, Q$, and $s$ and denoted by SARIMA $(p, d, q)\times (P, D, Q)_s$.

### 2.3. Subset sarima modelling

Suhartono [23] proposed a subset SARIMA modelling algorithm thus:

Fit the SARIMA$(0, 0, 1)\times (0, 0, 1)$ model

$$X_t = \beta_1 \epsilon_{t-1} + \beta_s \epsilon_{t-s} + \beta_{s+1} \epsilon_{t-s-1}.$$  

If $\beta_{s+1} = 0$, then the model is said to be *additive* but if $\beta_{s+1} \neq 0$ and $\beta_{s+1} = \beta_1 \beta_s$, the model is *multiplicative*. Otherwise, it is said to be *subset*.

Etuk and Ojekudo [11] using AR and MA duality arguments, proposed that the above SARIMA$(0, 0, 1)\times (0, 0, 1)$ model could be replaced by the dual SARIMA$(1, 0, 0)\times (1, 0, 0)$ model. They therefore proposed the following algorithm:
Fit the SARIMA(1, 0, 0) × (1, 0, 0) model

\[ X_t + \alpha_1 X_{t-1} + \alpha_s X_{t-s} + \alpha_{s+1} X_{t-s-1} = \varepsilon_t. \] (5)

If \( \alpha_{s+1} = 0 \), the model is said to be additive. If \( \alpha_{s+1} \neq 0 \) and if \( \alpha_{s+1} = \alpha_1 \alpha_s \), then the model is said to be multiplicative. Otherwise, it is said to be subset.

Assuming \( \alpha_{s+1} \neq 0 \), to test the null hypothesis H0 that the model is multiplicative, against the alternative hypothesis H1 that the model is subset, we proposed the statistic

\[ T = (\alpha_{s+1} - \alpha_1 \alpha_s) / S.E.(\alpha_{s+1}), \] (6)

where \( S.E.(\alpha_{s+1}) \) is the standard error of \( \alpha_{s+1} \). Clearly under H0, \( T \) is distributed as a \( t \)-variable with \( n - 1 \) degrees of freedom, where \( n \) is the sample size.

2.4. Statistical software

Eviews 7 was used for the analytical work of this write-up. The parametric estimation using this software is based on the least error sum of squares principle.

3. Results and Discussion

For the sake of this write-up, the 105-point realization analyzed is called NPLR. Its time plot in Figure 1 shows an overall flat secular trend with a hunch between 2009 and 2010. Its correlogram in Figure 2 has a sinusoidal pattern of period 12 months attesting to a 12-monthly seasonal nature. For further confirmation of a yearly seasonality, an inspection of NPLR shows that annual minimums tend to lie in the second and third quarters of the year, whereas the maximums tend to lie in the first and the fourth quarters of the year.
Twelve-monthly differencing of NPLR yields the series SDNPLR, which exhibits similar patterns as NPLR (see Figure 3). The ADF test considers both NPLR and SDNPLR as non-stationary. A non-seasonal differencing of SDNPLR yields a series DSDNPLR, which has a horizontal trend (see Figure 4) and a correlogram with the autocorrelation function (ACF) showing stationarity and confirming the hypothesised seasonality of 12 months periodicity.

Figure 1. NPLR.
Figure 2. Correlogram of NPLR.
Figure 3. SDNPLR.

Figure 4. DSDNPLR.
Figure 5. Correlogram of DSDNPLR.
Application of the algorithm (5) yields the series estimated in Table 1 as

\[ X_t - .4537X_{t-1} + .5234X_{t-12} - .1801X_{t-13} = \varepsilon_t. \]  
(\(\pm .1023\) \(\pm .0794\) \(\pm .0974\))

The last coefficient is non-significant at 0.05 level of significance. This makes suggestive the additive model estimated in Table 2 as

\[ X_t - .3201X_{t-1} + .4901X_{t-12} = \varepsilon_t, \]
(\(\pm .0827\) \(\pm .0780\))

which is inferior to (7) on all counts. The \(t\) test statistic value based on Equation (6) for the model (7) is equal to 0.5890, which is not significant. Hence the model (7) is not subset but multiplicative. This multiplicative model is observed to be adequate on the following grounds:

1. Its residuals are mostly uncorrelated (see Figure 7).

2. The actual and fitted values of the series agree very closely (see Figure 6).
Table 1. Estimation of the Sarima $(1, 1, 0) \times (1, 1, 0)_2$ model

Dependent Variable: DSDNPLR
Method: Least Squares
Date: 11/28/14  Time: 13:15
Sample (adjusted): 2008M03 2014M09
Included observations: 79 after adjustments
Convergence achieved after 3 iterations

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
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<td>AR(1)</td>
<td>0.453689</td>
<td>0.102320</td>
<td>4.434001</td>
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<td>AR(12)</td>
<td>–0.523412</td>
<td>0.079445</td>
<td>–6.588385</td>
<td>0.0000</td>
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<td>AR(13)</td>
<td>0.180074</td>
<td>0.097395</td>
<td>1.848909</td>
<td>0.0684</td>
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R-squared  0.528307  Mean dependent var  0.021899
Adjusted R-squared  0.515894  S. D. dependent var  0.653358
S. E. of regression  0.454592  Akaike info criterion  1.298401
Sum squared resid  15.70567  Schwartz criterion  1.388380
Log likelihood  –48.28682  Hannan-Quinn criter.  1.334449
Durbin-Watson stat  1.900657

Inverted AR Roots 

| .93 – .24i | .93 + .24i | .68 + .67i | .68 – .67i |
| .34 | .25 + .91i | .25 – .91i | –.24 + .91i |
| –.24 – .91i | –.66 – .67i | –.66 + .67i | –.91 + .24i |
| –.91 – .24i |
**Table 2. Estimation of the additive sarima model**

Dependent Variable: DSDNPLR

Method: Least Squares

Date: 11/28/14  Time: 13:19

Sample (adjusted): 2008M02 2014M09

Included observations: 80 after adjustments

Convergence achieved after 2 iterations

<table>
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<tr>
<td>R-squared</td>
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<td>0.022500</td>
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<tr>
<td>Adjusted R-squared</td>
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<td>0.649232</td>
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<td>S. E. of regression</td>
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<td>1.321449</td>
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<tr>
<td>Sum squared resid</td>
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<td>1.345325</td>
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<td>Log likelihood</td>
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<tr>
<td>Durbin-Watson stat</td>
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<tr>
<td>Inverted AR Roots</td>
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<td>.94 + .24i</td>
<td>.70 – .66i</td>
<td>.70 + .66i</td>
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<tr>
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<td>.27 – .91i</td>
<td>.27 + .91i</td>
<td>–.22 + .91i</td>
<td>–.22 – .91i</td>
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<tr>
<td></td>
<td>–.64 – .66i</td>
<td>–.64 + .66i</td>
<td>–.89 + .24i</td>
<td>–.89 – .24i</td>
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Figure 7. Correlogram of the residuals of the \( \text{Sarima}(1, 1, 0) \times (1, 1, 0)_12 \) model.

4. Conclusion

It may be concluded that monthly Nigerian Prime Lending rates follow a multiplicative SARIMA model (7). Forecasting of the series may be based on the model.
References


www.ircmj.com/?page=article_id=11779


