PANEL UNIT ROOT TESTS UNDER CROSS-SECTIONAL DEPENDENCE: AN OVERVIEW

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Abstract

The increasing availability of new datasets where the time-series dimension and the cross-section dimension are of the same order of magnitude asks for new techniques for the analysis of this peculiar kind of data. In the panel unit root test framework, two generations of tests have been developed: a first generation whose main limit is the assumption of cross-sectional independence across units; a second generation of tests that rejects the cross-sectional independence hypothesis. Although within this second generation of tests different approaches can be distinguished on the basis of the way in which the cross-sectional dependence is modelled the one that has encountered the most attention among researchers is the factor structure approach. This paper provides an updated overview of the main tests belonging to the second generation, and underlines the main issues which remain to be solved.

1. Introduction

Over the past decades the availability of several new panel data sets where the number of time series observations (T) and the number of
groups or individuals \( (N) \)
are of the same order of magnitude has increased the need for new econometric techniques which are able to deal with this kind of data.

Originally, the asymptotic properties of regression analysis in the panel context were derived under the hypothesis of weak stationarity for each individual timeseries (for a survey, see Hsiao [28]). Obviously, this was not a very restrictive assumption in the classical panel data case, where the time span is very short (usually 5 or 6 observations), but it could be a very critical assumption in the large panel data framework. In fact the presence of a unit root can significantly affect the asymptotic properties of time-series regression estimates and test statistics and can lead to spurious results, if there are not cointegrating relations among integrated variables, (i.e., Engle and Granger [21]).

Working in the large panel data context and developing appropriate test statistics require the preliminary solution of the problem of carrying out an asymptotic analysis, as both \( N \) and \( T \) go to infinity. Several theoretically interesting approaches have been developed considering how the two indexes go to infinity (Phillips and Moon [46]); however, from a practical point of view, sequential asymptotic results seem to be adequate in most cases. This procedure, consisting in letting \( T \) go to infinity first, and then \( N \) go to infinity second \((T, N \to \infty)_{\text{seq}}\) hereafter), is easy to implement, even though it can sometimes give misleading asymptotic results.

The first theoretical works on the non-stationary panel data focused on testing for unit roots in univariate panel, but since the work of Quah [48] and Breitung and Meyer [9], the interest on this topic has considerably increased.

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1 The cross-sectional units may be households, firms, regions, countries and so forth.

2 Other alternative approaches to the sequential limits are the diagonal-path limits, which consist of imposing restrictions on the relative rates at which \( N \) and \( T \) go to infinity, and the joint limits which allow both \( N \) and \( T \) to pass to infinity simultaneously without placing specific diagonal path restrictions on the divergence.
In general, the commonly used unit root tests as Augmented Dickey-Fuller (ADF) tests (Dickey and Fuller [19]) have non-standard limiting distributions that depends on which deterministic components are included in the regression equation\(^3\). In finite samples such tests based on a single time-series show little power in distinguishing the unit root from stationary alternatives, with highly persistent deviations from equilibrium. This problem seems to be particularly severe in the small sample case.

Moreover, standard unit root tests in the panel framework may show bad performances also because of the different null hypotheses that are tested in this case. When we consider the model

\[
A_{yt} = \rho_i y_{it-1} + u_{it} \quad i = 1, 2, \ldots, N \quad t = 1, 2, \ldots, T
\]

in the single equation case, we are interested in testing \(\rho_1 = 0\) against the alternative hypothesis \(\rho_1 < 0\) and we apply a unit root for the first time-series. Instead, in the panel data case, the hypothesis which we are interested in is

\[
H_0 : \rho_i = 0 \quad \text{against} \quad H_a : \rho_i < 0 \quad \text{for} \ i = 1, 2, \ldots, N.
\]

Referring to this situation, two generations of panel unit root tests have been developed.

The first generation includes Levin, Lin and Chu’s test [39]-LLC thereafter-, Im, Pesaran and Shin [30]-IPS thereafter-, the Fisher-type test proposed first by Maddala and Wu [42], later developed by Choi [14] and the Hadri test [24] for the null hypothesis of stationarity.\(^5\)

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\(^3\) The ADF test statistic converges to a function of Brownian motion (White, [54]) under very general conditions (Said and Dickey [49]).

\(^4\) In the remainder of this paper, \(i\) and \(t\) are always assumed \(i = 1, 2, \ldots, N, t = 1, 2, \ldots, T\), where not differently specified.

\(^5\) For a survey on this first generation of tests, see Banerjee [6], Baltagi and Kao [5]. Table A.1 in Appendix briefly summaries in a tabular form the main characteristics of this first generation of tests.
The main limit of these tests is that they are all constructed under the assumption that the individual time-series in the panel are cross-sectionally independently distributed. This condition is needed in order to satisfy the Lindberg-Levy central limit theorem, and therefore obtain asymptotically normal distributed test statistics.

Nevertheless, this is a critical statement because a large amount of literature, (i.e., Backus and Kehoe [1]) has provided evidence on the strong co-movements between economic variables, and it has been recognized that the assumption of independence across members of the panel is rather restrictive, particularly in the context of cross-country regressions. Moreover, this cross-sectional correlation may affect the finite sample properties of panel unit root test (O’Connell [44])6. For instance, the limit distribution of the usual Wald type unit root tests based upon ordinary least squares (OLS) and generalized least squares (GLS) system estimators depend upon nuisance parameters defining correlations across individual units. Various attempts to eliminate the nuisance parameters in such systems have been proposed7; unfortunately, even when this procedure could partly deal with the problem, it is not appropriate if pair-wise cross-section covariances of the error terms vary across the individual series.

To overcome these difficulties, a second generation of tests rejecting the cross-sectional independence hypothesis has been proposed. Within this generation of tests, we can distinguish the contribution of Chang [12, 13], who proposed first imposition of few or no restrictions on the residual covariance matrix, and then the use of nonlinear instrumental variable methods or bootstrap approaches to solve the nuisance parameter problem due to cross-sectional dependence or the more recent contribution of Choi and Chue [16] who propose subsampling methods.

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6 The assumption of cross-sectional independence can lead to severe size distortions and low power of the tests when this hypothesis is not satisfied (Banerjee et al. [8], Breitung and Das [11]).

7 For example, the cross-sectionally de-meaning of the series before application of the panel unit root test. This is what has been done by Im et al. [29], who consider a simple form of cross-sectional correlation using time-specific effects.
Nevertheless, the most popular approach in this framework relies on the factor structure approach and includes contributions by Bai and Ng [3, 4], Phillips and Sul [47], Moon and Perron [43]), Choi [15] and Pesaran [45].

The attempt of the present work is to provide an overview of the recent developments in panel unit root tests literature and to underline the main issues which remain to be solved. This is fundamental for the econometric researcher, who wants to apply existing tests or to develop new and better tests. The paper is organized as follows. Section II reviews the second generation of panel unit root tests. Particular attention is made to the increasingly popular approach that deals with the problem of cross-sectional dependence by means of a factor structure representation. Other suitable alternative approaches are discussed before the end of the section with a comparison of the presented tests. Section III briefly provides a presentation of some simple tests belonging to the second generation which verify the null hypothesis of stationarity. Finally, some conclusions and an Appendix presenting in a tabular form the comparative characteristics of the discussed tests close the paper.

2. The Second Generation of Panel Unit Root Tests

As introduced previously, the first generation of tests was constructed under the assumption of cross-sectional independence between individual time-series in the panel. Nevertheless, this is a very strong hypothesis often disproved by empirical evidence. This is why, a new generation of panel unit root tests that explicitly consider the cross-section correlation between panel units has been proposed in the literature.

To build these kind of tests, a preliminary issue is the specification of the cross-sectional dependence. But since individual observations in a cross-section have no natural ordering this specification is not obvious. In

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8 See Table A.2. in Appendix for a summary of the main characteristics of this second generation of tests. The reader should note that this area of research and the linked literature are still under development, given the diversity of the potential cross-sectional correlations.
order to deal with this problem, various methods have been proposed and they can be organized in two main streams: 1) approaches modelling the cross-sectional dependence in the form of a low dimensional common factor model, (i.e., Choi [15], Bai and Ng [3], Moon and Perron [43], Phillips and Sul [47], Pesaran [45]), and 2) other more general approaches, (i.e., O’Connell [44], Maddala and Wu [42], Taylor and Sarno [53], Chang [12, 13], Breitung and Das [10], Choi and Chue [16]), often consisting in imposing few restrictions on the residual covariance matrix.

These two streams are presented in the following sub-sections.

2.1. The factor structure approach

The factor structure approach assumes a common factor representation in which an observed series is written as a linear combination of (unobserved) common and idiosyncratic components. This model is estimated and conditioned out before constructing the panel unit root tests. The advantages of this procedure are the possibility to model the cross-sectional dependence by allowing the common factors to have differential effects on different cross-section units as well as the reduction of the number of required unobserved common factors. Among the tests adopting a factor structure approach, we will describe in the following only those developed by Pesaran [45], Bai and Ng [2] and Moon and Perron [43].

2.1.1. Pesaran [45] test

Pesaran [45], presents a new and simple procedure for testing unit roots in dynamic panels subject to possibly cross-sectionally dependent as

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9 Phillips and Sul [47] and Choi [15] propose test procedures that are very similar to that of Moon and Perron [43]. Nevertheless, since respect to the Moon and Perron approach the Phillips and Sul’s one is more restrictive in term of the common components specification, and that of Choi shows worst performance in term of size and power, they are not discussed here.
well as serially correlated errors and proposes a cross-sectionally augmented version of the IPS standardized \( t \)-test (Im et al. [30]).

In this approach, the observations \( y_{it} \) are supposed to be generated according to a simple dynamic linear heterogeneous panel data model

\[
y_{it} = (1 - \rho_i)\mu_i + \rho_i y_{it-1} + u_{it},
\]

(2.1.1.1)

where \( \mu_i \) is a deterministic component, the initial values \( y_{i0} \) are given and the disturbances follow a one-factor structure

\[
u_{it} = \lambda_i f_t + \varepsilon_{it}
\]

(2.1.1.2)

in which

- the idiosyncratic shocks, \( \varepsilon_{it} \) are independently distributed both across \( i \) and \( t \) with mean zero, variance \( \sigma_i^2 \) and finite fourth-order moment;

- the unobserved common factor \( f_t \) is serially uncorrelated with mean zero, constant variance \( \sigma_f^2 \) and finite fourth-order moment. Without loss of generality, \( \sigma_f^2 \) is set equal to one;

- the variables \( \varepsilon_{it}, f_t \) and \( \lambda_i \) are assumed to be independently distributed for all \( i \).

The assumptions made about \( \varepsilon_{it} \) and \( f_t \) imply serial uncorrelation for the \( u_{it} \)\(^{10}\). Equations (2.1.1.1) and (2.1.1.2) can be more conveniently written as

\[
\Delta y_{it} = (1 - \rho_i)\mu_i - (1 - \rho_i) y_{it-1} + \lambda_i f_t + \varepsilon_{it},
\]

(2.1.1.3)

being \( \Delta y_{it} = y_{it} - y_{it-1} \). Pesaran [45] considers the following unit root hypothesis

\(^{10}\) This assumption as well as the assumption that \( K = 1 \) (there is only one common factor) could be relaxed. Pesaran [45] explicitly considers this situation in the follow of his work.
against the possibly heterogeneous alternatives

\[ H_1 : \begin{cases} \rho_i < 1 & \text{for } i = 1, \ldots, N_1, \\ \rho_i = 1 & \text{for } i = N_1, \ldots, N, \end{cases} \]

where the fraction of the stationary processes is such that \( N_1 / N \to \kappa \), as \( N \to \infty \) with \( 0 < \kappa \leq 1 \).

Instead of basing the unit root tests on deviations from the estimated common factors, Pesaran [45] proposes a test based on standard unit root statistics in a Cross-sectionally Augmented DF (CADF thereafter) regression - that is a DF (or ADF) regression which is augmented with the cross-section averages of lagged levels and first-differences of the individual series

\[ \Delta y_{it} = a_i + b_i y_{it-1} + c_i \bar{y}_{t-1} + d_i \Delta \bar{y}_t + e_{it}, \tag{2.1.1.4} \]

where \( e_{it} \) is the regression error and \( \bar{y}_t = N^{-1} \sum_{j=1}^{N} y_{jt} \) and \( \Delta \bar{y}_t = N^{-1} \sum_{j=1}^{N} \Delta y_{jt} \) are included into Equation (2.1.1.4) as a proxy for the unobserved common factor \( f_t \).

Let \( CADF_i \) be the ADF statistic for the \( i \)-th cross-sectional unit given by the, \( t \)-ratio of the OLS estimate \( \hat{b}_i \) of \( b_i \) in the CADF regression (2.1.1.4).

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11 This is a natural extension of the DF approach in order to deal with residual serial correlation, where lagged changes of the series are used to filter out the time-series dependence when \( T \) is sufficiently large.

12 This approximation is applicable if \( \overline{\lambda} = N^{-1} \sum_{j=1}^{N} \lambda_j \) and \( \overline{\lambda} \neq 0 \) for a fixed \( N \) and as \( N \to \infty \). Moreover, Equation (2.1.1.4) is valid for serially uncorrelated \( u_{it} \). For a more general case, lagged values of \( \Delta y_{it} \), but also of \( \Delta \bar{y}_t \) need to be included in the estimation (Pesaran [45]).
Individual \textit{CADF} statistics are used to develop a modified version of the \textit{IPS} \textit{t-bar} test denoted \textit{Cross-sectionally Augmented IPS} (\textit{CIPS} thereafter) that simultaneously take account of cross-section dependence and residual serial correlation

\[
\text{CIPS} = \frac{1}{N} \sum_{i=1}^{N} \text{CADF}_i,
\]

The asymptotic null distributions of the single \textit{CADF} statistics are similar and do not depend on the factor loadings. Unfortunately, \textit{CADF} statistics are correlated because of their dependence on the common factor. Then, even if \textit{CIPS} statistics can be built, it is not possible to apply standard central limit theorems to them. Moreover, in contrast to the results obtained by Im et al. [30] under cross-sectional independence\textsuperscript{13}, the distribution of the \textit{CIPS} statistic is shown to be nonstandard even for large \(N\).

Pesaran also considers a truncated version of \textit{CADF} (\textit{CADF}*) to avoid excessive influence of extreme outcomes that could arise for small \(T\) samples. The results for \textit{CADF} and \textit{CIPS} are valid also for \textit{CADF}* and the related \textit{CIPS}* and, even if it is not normal, the null asymptotic distribution of \textit{CIPS}* statistic exists and is free of nuisance parameter.

Pesaran proposes simulated critical values of \textit{CIPS} for various sample sizes and three different specifications of deterministic components, (i.e., models without intercept or trend, models with individual-specific intercepts and models with incidental linear trends) and analyzes the small sample properties of his tests by Monte Carlo experiments. In this way, \textit{CADF} tests-but mostly the truncated version of the \textit{CIPS} test-show satisfactory properties for small \(N\) and \(T\) as well, even if the power of the tests critically depends on the sample sizes and on whether the model contains linear time trends.

\textsuperscript{13} In this case, a standardized average of individual ADF statistics was normally distributed for large \(N\).
Following Maddala and Wu [42] or Choi [14], Pesaran also proposes Fisher-type tests based on the significant levels of individual CADF statistics. In this case as well, the statistics do not have standard distributions because of the previous reasons.

Pesaran [45] also extends his approach to serially correlated residuals. For an AR($p$) error specification, the relevant individual CADF statistics can be computed from a $p$-th order cross-section/time-series augmented regression

$$
\Delta y_{it} = \alpha_i + \rho_i y_{it-1} + c_i \bar{y}_{t-1} + \sum_{j=0}^{p} d_{ij} \Delta \bar{y}_{t-j} + \sum_{j=0}^{p} \beta_{ij} \Delta y_{t-j} + \mu_{it}.
$$

Finally, note that both Pesaran’s [45] CADF and CIPS tests are designed for testing for unit roots when cross-sectional dependence is due to a single common factor, but the CIPS test has better power properties than the individual CADF tests, and therefore it should be preferred.

### 2.1.2. Moon and Perron [43] tests

Moon and Perron [43] represent the observation series $y_{it}$ as AR(1) processes with fixed effects and assume, such as Pesaran [45], the presence of common factors in the error terms. They consider the following dynamic panel model:

$$
y_{it} = \mu_i + x_{it},
$$

$$
x_{it} = \rho_i x_{it-1} + u_{it},
$$

$$
u_{it} = \lambda_i f_t + e_{it},
$$

where the observed $y_{it}$ variables ($i = 1, 2, \ldots, N; t = 1, 2, \ldots, T$) are generated by a deterministic component $\mu_i$ and an autoregressive process $x_{it}$, with $x_{i0} = 0$ for all $i$.

In order to model the correlation among the cross-sectional units, the error component $u_{it}$ is assumed to follow an approximate factor model, where $f_t$ is a ($K \times 1$) vector of unobservable random factors, $\lambda_i$ is the corresponding vector of non-random factor loading for cross-section $i$ and
$e_{it}$ is an idiosyncratic shock. The number of factors $K$ is possibly unknown.

As it is easy to note, in this framework panel data are assumed to be generated by idiosyncratic shocks and unobservable dynamic factors that are common to all the individual units but to which each individual reacts heterogeneously.

Model (2.1.2.1) can also be re-written as

$$\Delta y_{it} = (1 - \rho_i)_t u_i + \rho_i y_{it-1} + u_{it}. \tag{2.1.2.2}$$

Comparing (2.1.2.2) and (2.1.1.1), it is easy to note that Pesaran [45] and Moon and Perron [43] models are identical in the case, where a single common factor is present in the composite error term.

For the error term $u_{it}$ in (2.1.2.1) Moon and Perron make similar assumptions to those of Pesaran [45], namely

\begin{itemize}
  \item $e_{it} = \Gamma_t(L)\varepsilon_{it}$, where $\Gamma_t(L) = \sum_{j=0}^{\infty} \gamma_{ij}L^j$ and $\varepsilon_{it} \sim i.i.d.(0,1)$ across $i$ and $t$ have a finite eighth moment; furthermore, the possibility of cointegrating relationships among the integrated idiosyncratic shocks $E_{ij} = \sum_{s=1}^{t} e_{is}$ is excluded;
  \item $f_t = \Phi(L)\eta_{it}$, where $\Phi_t(L) = \sum_{j=0}^{\infty} \phi_{ij}L^j$ is a $K$-dimensional lag polynomial and $\eta_{it} \sim i.i.d.(0, I_K)$ so that $u_{it}$ is $i.i.d.$ across $i$ and $t$; furthermore, the covariance matrix of $f_t$ is (asymptotically) positive definite so that under the null hypothesis the nonstationary factors can be cointegrated;
  \item there exists at least one common factor in the data and their maximum number $K(l \leq K \leq K < \infty)$ is assumed to be known a priori. Also, the contribution of each factor to at least one of the $y_{it}$ is assumed to be significant by imposing $\frac{1}{N} \sum_{i=1}^{N} \lambda_i \lambda_i^T \rightarrow \sum_{\lambda_i} > 0$; however, this assumption does not imply that all cross-sections respond to all factors so that some of the factor loadings could be zero;
\end{itemize}
short-run variance $\sigma_{e_i}^2 (= \sum_{j=0}^{\infty} \gamma_{ij}^2)$, long-run variance $\omega_{e_i}^2 (= (\sum_{j=0}^{\infty} \gamma_{ij})^2)$ as well as the one-sided long-run covariance $\varphi_{e_i} (= \sum_{l=1}^{\infty} \sum_{j=0}^{\infty} \gamma_{ij} \gamma_{ij+l})$ are well defined for all idiosyncratic disturbances $e_{it}$ and have non-zero cross-sectional averages $\sigma_e^2 = \frac{1}{N} \sum_{i=1}^{N} \sigma_{e_i}^2$, $\omega_e^2 = \frac{1}{N} \sum_{i=1}^{N} \omega_{e_i}^2$ and $\varphi_e^2 = \frac{1}{N} \sum_{i=1}^{N} \varphi_{e_i}^2$.

In referring to the model (2.1.2.1), the null hypothesis of interest is

$$H_0 : \rho_i = 1 \text{ for all } i = 1, \ldots, N,$$

against the heterogeneous alternative

$$H_a : |\rho_i| < 1 \text{ for some } i.$$ 

It is straightforward to note from (2.1.2.1) that, under the null hypothesis, two components influence $y_{it}$: the integrated factors $\sum_{s=1}^{T} f_s$ and the integrated idiosyncratic errors $\sum_{s=1}^{T} e_{is}$. Moreover, while both integrated or cointegrated factors are allowed in the model, the possibility of cointegrating relations of the integrated idiosyncratic errors is ruled out.

When common factors are present in the panel, tests based on the assumption of cross-sectional independence among units suffer from size distortions. To overcome this difficulty, Moon and Perron [43] transform the model in order to eliminate the common components of the $y_{it}$ series and apply the unit root test on defactored series. The resulting test statistics have a normal asymptotic distributions as those of Im et al. [30] or Levin and Lin [37, 38]; moreover being computed from defactored data, they are also cross-sectional independent.

More specifically, to remove cross-sectional dependence in (2.1.2.1), Moon and Perron use a projection onto the space orthogonal to the factor loadings, (i.e., the space generated by the columns of the matrix of factor
loading $\Lambda = (\lambda_1, \ldots, \lambda_N)'$. Then, $\Lambda$ is estimated\(^{14}\) to construct the projection matrix $Q_\Lambda = I_N - \Lambda(\Lambda'\Lambda)^{-1}\Lambda'$.

Let $\hat{\Lambda}$ and $Q_{\Lambda_k}$ be the estimator of the matrix $\Lambda$ and of the corresponding projection matrix; further, denote the estimates of short-run and long-run variances, $\sigma^2_{e_t}$ and $\omega^2_t$, as $\hat{\sigma}_{e_t}$ and $\hat{\omega}_{t}$, respectively\(^{15}\), and let $\hat{\phi}_e$ and $\hat{\omega}_e^2$ be their cross-sectional averages.

The unit root test is implemented from the defactored data obtained as $YQ_\Lambda$ (being $Y$ the matrix of observed data). Specifically, the unbiased pooled estimator of $\rho$ suggested by Moon and Perron [43] is

$$\rho^*_\text{pool} = \frac{tr(Y_{-1}Q_{\Lambda_k}Y') - NT\hat{\phi}_e}{tr(Y_{-1}Q_{\Lambda_k}Y')}, \quad (2.1.2.4)$$

where $Y_{-1}$ is the matrix of lagged observed data and $tr(\cdot)$ the trace operator.

From estimator $\rho^*_\text{pool}$, two modified t-statistics based on pooled estimation of the first-order serial correlation coefficient of the data are suggested for the null hypothesis (2.1.2.2)

$$t^*_a = T\sqrt{N\left(\frac{\rho^*_\text{pool}}{2\hat{\phi}_e} - 1\right)} \frac{\sigma^2_{e_t}}{\hat{\sigma}_{e_t}^4}, \quad (2.1.2.5.a)$$

\(^{14}\) For this purpose, Moon and Perron suggest to employ the principal component method used in Stock and Watson [51] and Bai and Ng [3].

\(^{15}\) To this aim, Moon and Perron use kernel estimators (see Moon and Perron, [43]. Note that, once previous estimates have been obtained, the authors also discuss how to consistently estimate the number of factors $K$. 
\[ t_b^* = T \sqrt{N \left( \hat{\rho}_{\text{pool}}^+ - 1 \right)} \left( \frac{1}{NT^2} tr(Y_{-1}Q_{\Lambda_h}Y_{-1}) \right)^{1/2} \frac{\tilde{\sigma}_e^2}{\sigma^2}, \tag{2.1.2.5.b} \]

\( \hat{\rho}_{\text{pool}}^+ \) being the bias-corrected pooled autoregressive estimate of (2.1.2.4).

Moon and Perron show that as \( N \) and \( T \to \infty \), with \( N / T \to 0 \), the statistics (2.1.2.5.a) and (2.1.2.5.b) have a limiting standard normal distribution under the null hypothesis.

It is possible to note that, since the Moon and Perron’s model obtained by removing the cross unit dependence is similar to the LLC model with common autoregressive root, under the cross-unit independence hypothesis the convergence rate of the pooled estimator (corrected or not) of the autoregressive root is the same as the one obtained in the LLC model.

Finally, Moon and Perron simulations show that the tests are very powerful and have good size when no estimation of deterministic components is necessary, (i.e., only a deterministic constant is included in the model) for different specifications and different values of \( T \) and \( N \). When such estimation is necessary, the tests have no power beyond their size.

Note that the Moon and Perron [43] tests using defactored data allow for multiple common factors. Therefore, their use has to be recommended when cross-section dependence is expected to be due to several common factors.

### 2.1.3. Bai and Ng [3] test

Bai and Ng [3] propose a different procedure to test for panel unit root allowing for cross-section correlation as well as cointegration. It does not treat cross-section dependence as a disturbance as the previously presented tests did: the nature of the co-movements of economic variables are themselves an object of interest in the analysis.

Bai and Ng consider a balanced panel with \( N \) cross-section units with \( T \) time series observations and the following model representation:

\[ y_{it} = \alpha_i + \beta_i t + \lambda_i F_i + e_{it}, \tag{2.1.3.1.a} \]
\( F_t = F_{t-1} + f_t, \) 

(2.1.3.1.b)

\[ e_{it} = \rho_i e_{it-1} + \varepsilon_{it}, \] 

(2.1.3.1.c)

with

- \( f_t = \Phi(L)\eta_t, \) where \( \Phi(L) = \sum_{j=0}^{\infty} \phi_j L^j \) is a \( K \)-dimensional lag polynomial such that \( \text{rank}(\Phi(1)) = k_1 \) and \( \eta_t \sim \text{i.i.d.}(0, \Sigma_{\eta}) \) with finite fourth-order moment. Consequently, \( F_t \) are assumed to follow an AR(1) process containing \( k_1 \leq K \) independent stochastic trends and \( K - k_1 \) stationary components.

- the idiosyncratic terms \( e_{it} \) are also modelled as AR(1) processes and are allowed to be either \( I(0) \) or \( I(1) \); furthermore, \( e_{it} = \Gamma_i(L)e_{it} \) with \( \varepsilon \sim \text{i.i.d.}(0, \Sigma_{\varepsilon}^2) \).

Instead of directly testing the nonstationarity of \( y_{it} (i = 1, \ldots, N) \), this approach analyzes the common and individual components separately; this is why, it is referred as PANIC (Panel Analysis of

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16 This means that \( \Delta F_t \) has a short-run covariance matrix of full rank; at the same time, as \( \text{rank}(\Phi(1)) = k_1 \), the long-run covariance matrix has reduced rank and cointegration relations among the common factors are allowed. In such a context, similar to Moon and Perron [43], (asymptotically) redundant factors are excluded.

17 The cross-sectional independence of the idiosyncratic term is imposed only in order to validate pooled testing. In such a context \( \Sigma_{\eta} \) is not necessarily a diagonal matrix and this procedure seems to be more general than those by Moon and Perron [43], which assumes uncorrelation for the innovations of the common factors.

18 Note that a series defined as the sum of two components with different dynamics could have a very different dynamic from those of its constituents. As a consequence, if one component is \( I(1) \) and the other one is \( I(0) \), it could be difficult to establish if a unit root exists from the observation of \( y_{it} \) alone, especially, if this series contains a large stationary component. In fact, in this situation, tests for the null hypothesis of a unit root on \( y_{it} \) can be oversized while tests for the null hypothesis of stationarity will have no power (Schwert [50]).
Nonstationarity in the Idiosyncratic and Common Components). The aim of this procedure is to determine if nonstationarity comes from a pervasive ($F_t$) or an idiosyncratic source ($E_{it}$), and to construct valid pooled tests for panel data when the individuals are correlated. In terms of model (2.1.3.1), this means determining the number of non-stationary factors $k_1$ and testing whether $\rho_i = 1$ with $i = 1, \ldots, N$.

First of all, in order to analyze the factors $F_t$ and the idiosyncratic components $e_{it}$ that are both unobserved, Bai and Ng try to find consistent estimates of these series preserving their integration features.

They accomplish this goal by referring to an appropriate transformation of $y_{it}$; namely, they employ the first differences, i.e., $\Delta y_{it} = y_{it} - y_{it-1}$, if the only intercept is included in the model, (i.e., $y_{it} = c_i + \lambda_i F_t + e_{it}$) while they apply the detrended $y_{it}$, i.e., $y_{it} = \Delta y_{it} - \Delta \bar{y}$, where $\Delta \bar{y}_{it} = \frac{1}{T-1} \sum_{t=2}^{T} \Delta y_{it}$, in the presence of a linear trend ($y_{it} = c_i + \beta_i t + \lambda_i F_t + e_{it}$).

In other words, Bai and Ng [3] suggest to proceed estimating in a first step the common factors and idiosyncratic errors in $\Delta y_{it}$ or $y_{it}^d$ by a simple principal component method; and then, in a second step, re-cumulating these estimators, denoted as $\hat{F}_t$ and $\hat{e}_{it}$, respectively, in order to remove the effect of possible over-differencing

$$
\hat{F}_t = \sum_{s=2}^{T} \hat{f}_s, \quad \hat{E}_{it} = \sum_{s=2}^{T} \hat{e}_{is}.
$$

---

19 Note that the possibility that one or more common factors are integrated allows Bai and Ng test to consider the possible presence of cross-section cointegration relationships.

20 This means that the common variations must be obtained without referring to stationarity assumptions and/or cointegration restrictions.
At this time, the null hypothesis of a unit root is tested separately for the common factor $\hat{F}_t$ and for each idiosyncratic component $\hat{E}_{it}$.

**Common factors stationarity analysis**

In order to test the nonstationarity of the common factors, when only one factor is detected\(^{21}\), Bai and Ng [3] suggest to use an $ADF$ test; when several common factors are detected, they employ a modified version of Stock and Watson [51] common trend test.

In the former case, (i.e., $K = 1$), the univariate augmented autoregression which they refer to is

$$
\Delta \hat{F}_{it} = D_{it} + \theta_0 \hat{F}_{it-1} + \sum_{j=1}^{p} \theta_j \Delta \hat{F}_{it-j} + \xi_{it},
$$

where $\xi_{it}$ is the regression error and $D_{it}$ is a polynomial function containing either a constant $\alpha_i$ or a linear trend $\alpha_i + \beta_it$.

Now, let $ADF_{\hat{F}}^C$ and $ADF_{\hat{F}}^T$ denote the $t$-statistic for $\theta_0 = 0$ in the case, where $D_{it}$ contains only a constant and in the case, where it contains a linear trend, respectively. Bai and Ng [3] show that the limiting distributions of these statistics correspond to the distribution of the $DF$ test for the constant only or the linear trend case.

When several common factors are detected ($K > 1$), individually testing the factors for the presence of a unit root may overstate the number of common trends. This is why, in order to select $k_1$, (i.e., the number of common independent stochastic trends in the common factors) Bai and Ng implement an iterative procedure, similar to the Johansen [33] trace test for cointegration.

---

\(^{21}\) The number of factors is estimated following the Bai and Ng’s [2] procedure. It is straightforward to verify that $k_1 = 0$ corresponds to the case, where there are $N$ cointegrating vectors for $N$ common factors, i.e., all factors are $I(0)$. 
They consider demeaned or detrended factor estimates, depending on whether the model (2.1.3.1) contains the intercept only, or also a time trend. Specifically, they refer to $\tilde{F}_t$ which is alternatively defined as $\tilde{F}_t = F_t - \bar{F}_t$, with $\bar{F}_t = (T-1)^{-1} \sum_{t=2}^{T} F_t$, in the intercept only case and as the residuals from a regression of $\hat{F}_t$ on a constant and a time trend, in the linear trend case.

Referring to such defined $\tilde{F}_t$, the proposed test procedure can be described as follow.

Firstly, let $m = K$.

1. If $\hat{\beta}_\perp$ are the $m$ eigenvectors associated to the $m$ largest eigenvalues of $T^{-2} \sum_{t=2}^{T} \tilde{F}_t \tilde{F}_t'$ and $\hat{X}_t$ is the matrix defined as $\hat{X}_t = \hat{\beta}_\perp \hat{F}_t$, it is possible to consider two different statistics:

   (a) Let $K(j) = 1 - j / (J + 1)$, $j = 1, ..., J$; in this case, the considered statistic $MQ^c(m)$, in the constant only case, or $MQ^t(m)$, in the linear trend case, is defined as:

   $$T[\hat{\nu}_c(m) - 1],$$

   where $\hat{\nu}_c(m)$ is the smallest eigenvalue of

   $$\hat{\Phi}_c(m) = \frac{1}{2} \left[ \sum_{t=2}^{T} (\hat{X}_t \hat{X}_{t-1}' + \hat{X}_{t-1} \hat{X}_t') - T(\hat{\Sigma}_1 + \hat{\Sigma}_1') \right] \left( \sum_{t=2}^{T} \hat{X}_{t-1} \hat{X}_{t-1}' \right)^{-1},$$

   with $\hat{\Sigma}_1 = \sum_{j=1}^{J} K(j) \left( \frac{1}{T} \sum_{t=2}^{T} \xi_{t-1-j} \xi_{t}^\prime \right) \xi_{t}$ being the residuals from estimating a first order VAR in $\hat{X}_t$. 


(b) For $p$ (the augmentation for the autoregression in (2.1.3.3)) fixed that does not depend on $N$ or $T$, the considered statistic $MQ_f(m)$, in the constant only case, or $MQ_f^*(m)$, in the linear trend case, is defined as

$$T[\hat{\nu}_f(m) - 1],$$

where $\hat{\nu}_f(m)$ is the smallest eigenvalue of

$$\hat{\Phi}_f(m) = \frac{1}{2} \left[ \sum_{t=2}^{T} (\hat{x}_t \hat{x}_{t-1}^\prime + \hat{x}_{t-1} \hat{x}_t^\prime) \right] \left( \sum_{t=2}^{T} \hat{x}_{t-1} \hat{x}_{t-1}^\prime \right)^{-1},$$

where $\hat{x}_t = \hat{I}(L) \hat{X}_t$ is obtained by filtering $\hat{X}_t$ by $\hat{I}_L$, that is, the polynomial coefficients of an estimate $\text{Var}(p)$ in $\Delta \hat{X}_t$, i.e., $\hat{I}(L) = I_m - \hat{I}_1 L - ... - \hat{I}_p L^p$.

2. If the null hypothesis $H_0 : k_1 = m$ is rejected, it is necessary to set $m = m - 1$ and return to Step 1. If the null hypothesis is not rejected, we set $k_1 = m$ and we can stop.

Then, if there are $K > 1$ common factors, Bai and Ng consider two tests: the first corrects for serial correlation of arbitrary form by non-parametrically estimating the relevant nuisance parameters. The second filters the factors under the assumption that they have finite order VAR representations. This is why they have been called $MQ_0$ and $MQ_f$, respectively. It is obvious that the $MQ_f$ test is valid only when the common trends can be represented as finite order AR($p$) processes whereas $MQ_0$ is more general.

The limiting distributions of these tests are non-standard; Bai and Ng provide 1%, 5%, and 10% critical values for all four statistics and various $m$.

\footnote{Note that $MQ_f^*(m)$ and $MQ_f^+(m)$ statistics are modified version of the $Q_0$ and $Q_f$ tests developed in Stock and Watson [51].}
- **Idiosyncratic components stationarity analysis**

  In order to test the non-stationarity of the idiosyncratic components, a method based on meta-analysis is used. Specifically, Bai and Ng implement a methodology that consists in pooling individual $ADF$ $t$-statistics computed for each defactored $\hat{E}_{it}$ in a model with no deterministic term such as

  \[
  \Delta \hat{E}_{it} = d_{i0} \hat{E}_{it-1} + \sum_{i=1}^{p} d_{ij} \Delta \hat{E}_{it-j} + v_{it}, \tag{2.1.3.4}
  \]

  where $v_{it}$ denotes a regression error.

  Let $ADF^c_E(i)$ (if a constant is included in the DGP) and $ADF^r_E(i)$ (if a constant and a linear trend are included in the DGP) be the individual $t$-statistics to test the hypothesis $H_0$. The limiting distribution of $ADF^c_E(i)$ coincides with the usual $DF$ distribution for the case of no constant and the 5% critical value is -1.95. Instead, the asymptotic distribution of $ADF^r_E(i)$ is proportional to the reciprocal of a Brownian bridge. Unfortunately, since the critical values for this distribution are not tabulated, simulations are required.

  Thus, contrary to the other panel unit root tests previously described, these statistics do not take the advantages of a limiting standard normal distribution. This happens because the panel information has been used to consistently estimate $e_{it}$, but not to analyze its dynamic properties.

  As Bai and Ng pointed out, PANIC procedure is characterized by some significant features: first, the tests on the factors do not depend on whether $e_{it}$ is $I(1)$ or $I(0)$, as well as the tests on the idiosyncratic errors do not depend on whether $F_t$ is $I(1)$ or $I(0)$; second, the unit root tests for $e_{it}$ is valid whether $e_{jt}$, $j \neq i$, is $I(1)$ or $I(0)$, and in any event, such knowledge is not necessary.

23 This procedure was originally introduced in Maddala and Wu [42] as well as in Choi [14].
The independence of the limiting distribution of $ADF^c_E(i)$ and $ADF^\tau_E(i)$ on the common factors makes possible for Bai and Ng [3] to propose a pooled Fisher-type test\(^{24}\) as suggested in Maddala and Wu [42] or Choi [14].

The test statistic is given by

\[
P^o_E = \frac{-2 \sum_{t=1}^N \log P^o_E(i) - 2N}{\sqrt{4N}} \rightarrow \mathcal{N}(0, 1),
\]

(2.1.3.5)

where $P^o_E$ denotes $P^c_E$ or $P^\tau_E$, depending on the deterministic specification, and $P^o_E(i)$ is the associated $p$-value of the $ADF$ test on the estimated residual $\hat{\epsilon}_{it}$.

For $N$ and $T \to \infty$, this statistic converges in distribution to a standard normal distribution, but only if independence among the error terms is assumed: in this case, pooled testing is valid and it is possible to derive the statistic distribution.\(^{25}\) Simulations show that Bai and Ng test

\(^{24}\) In principle, also an IPS-type test using a standardized average of the above described $t$-statistics should be possible.

\(^{25}\) This seems a contradiction: the aim of Bai and Ng [3] test was specifically to take into account this individual dependence. Nevertheless, note that Bai and Ng do not assume the cross-sectional independence hypothesis on the whole series $\gamma_{it}$ as Im et al. [30] or Maddala and Wu [42] do, but they only hypothesize the asymptotic independence among the individual components $\epsilon_{it}$. Under this hypothesis, the test statistics based on the estimate components $\hat{\epsilon}_{it}$ are asymptotically independent and the $p$-values $P_{\hat{\epsilon}_{i}}$ are also independently distributed according to an uniform law on $[0, 1]$. In this way, the hypothesis that all individual components $\epsilon_{it}$ for $i = 1, \ldots, N$ are $I(1)$ is sufficient to assure that the test statistic $P^c_E$ or $P^\tau_E$ is standard normally distributed, for all panel sizes $N$ (for the large size panel case, see Choi [14]).
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has good finite sample properties in term of size and a power even for small panel ($N = 40$).

In conclusion, the PANIC approach has the important advantage of allowing to obtain consistent estimators for the common factors and idiosyncratic components, whether they are stationarity or non-stationarity. Furthermore, it solves the problem of the size distortion and takes advantage of the cross-section relationship information (Banerjee and Zanghieri [7]) in order to obtain pooled tests that are more powerful than the univariate ones.

2.2. Other approaches

Among other suitable solutions for the problem of cross-section correlation, the procedures proposed by Breitung and Das [10], Chang [12] and Choi and Chue [16] seem to be particular interesting and are illustrated in the next few pages.

2.2.1. Breitung and Das [10] test

Breitung and Das [10] propose a test procedure based on OLS $t$-statistics with panel corrected standard error (PCSE) (Joansson [32]), that does not require any Monte Carlo simulations to be computed, and can be used differently from the GLS test procedure even in cases, where $T$ is less than $N$. Specifically, under a weak error dependence assumption, they consider the autoregressive model

$$
\Delta y_t = \phi y_{t-1} + \Gamma_1 \Delta y_{t-1} + \Gamma_2 \Delta y_{t-2} + \ldots + \Gamma_p \Delta y_{t-p} + \epsilon_t,
$$

(2.2.1.1)

Note that the re-accumulation of the unobserved component estimates removes the effect of possible overdifferencing when the factors or the errors are stationary.

See note 19.

Specifically, they assume that all the eigenvalues of the error covariance matrix are bounded when $N \to \infty$. 

26 Note that the re-accumulation of the unobserved component estimates removes the effect of possible overdifferencing when the factors or the errors are stationary.

27 See note 19.

28 See, for example, Driscoll and Kraay [20], Conley [17], O’Connell [44], Maddala and Wu [42], Taylor and Sarno [53], Chang [13], Harvey and Bates [27].

29 Specifically, they assume that all the eigenvalues of the error covariance matrix are bounded when $N \to \infty$. 

28
where $\Delta y_t = [\Delta y_{1t}, \Delta y_{2t}, \ldots, \Delta y_{Nt}]$, $y_{t-1} = [y_{1,t-1}, y_{2,t-1}, \ldots, y_{N,t-1}]$, $\Gamma_j = \text{diag}(\gamma_{j1}, \ldots, \gamma_{jN})$ for $j = 1, \ldots, p$, and the error vector $u_t = [u_{1t}, u_{2t}, \ldots, u_{Nt}]$ is i.i.d. with zero mean, positive definite variance-covariance matrix $\Omega$ and finite fourth-order moment\(^{30}\).

In order to test the null hypothesis of interest $H_0 : \phi = 0$, against the alternative $H_0 : \phi < 0$, they suggest using the robust $t$-statistic

$$t_{rob} = \frac{\sum_{t=1}^{T} y_{t-1}^* \Delta y_t^*}{\sqrt{\sum_{t=1}^{T} y_{t-1}^* \hat{\Omega} y_{t-1}^*}}$$

(2.2.1.2)

where $y_t^* = y_t - \Gamma_1 y_{t-1} - \Gamma_2 y_{t-2} - \ldots - \Gamma_p y_{t-p}$\(^ {31}\) and $\hat{\Omega}$ is a consistent estimator of $\Omega$. Note that under the null hypothesis, the “pre-whitened” series $y_t^*$ is a vector random walk and $\Delta y_t^*$ is white noise with $E(y_t^* y_t^{*\dagger}) = \Omega$. Being the GLS estimator more efficient than the OLS one, they also consider a more powerful test based on the GLS estimator of $\phi$ and computed with the pre-whitened series

$$t_{gls} = \frac{\sum_{t=1}^{T} y_{t-1}^* \hat{\Omega}^{-1} \Delta y_t^*}{\sqrt{\sum_{t=1}^{T} y_{t-1}^* \hat{\Omega}^{-1} y_{t-1}^*}}$$

(2.2.1.3)

that obviously could only be computed if $\hat{\Omega}$ is not singular, and so when $T > N$.

\(^{30}\) In their study, Breitung and Das\([10]\) also consider simple adjustment procedure in order to include individual specific intercepts and time trends in the model.

\(^{31}\) Breitung and Das show that their results are still valid even if the matrices $\Gamma_j$ are replaced by consistent estimates.
Breitung and Das [10] show that the two statistic are asymptotically standard normally distributed as \((T, N \to \infty)_{seq}\). The authors also show by means of Monte Carlo simulations that the robust OLS t-statistic performs well with respect to size and power, whereas the GLS ones may suffer from severe distortions in small and moderate sample sizes.

Breitung and Das [11] extend their previous study and analyze the behaviour of a series of panel unit root tests assuming a factor structure approach. Specifically, they consider their previous tests computed on non pre-whitened \(y_t\) series in three different situations and show that:

- when both common and idiosyncratic components are non-stationary, the robust OLS t-statistic has a non standard limiting distribution if \(\Omega\) does not have bounded eigenvalues but follows a factor structure; whereas, the GLS test, that apply a transformation in order to remove the common factors, has a standard normal distribution;

- when the idiosyncratic components are stationary while the common components are \(I(1)\) (cross-unit cointegration case), both statistics diverge as \(T, N \to \infty;\)

---

32 Note that when the autoregressive representation (2.2.1.1) is not the same for all units, an individual specific autoregression must be fitted for each unit and the series are prewhitened by using \(\tilde{y}_{it} = y_{it} - \tilde{\gamma}_1y_{i,t-1} - \cdots - \tilde{\gamma}_{p_i}y_{i,t-P_i}\). In these cases, Breitung and Das [10] suggest to estimate the lag order \(p_i\) by using a consistent information criterion as the Schwarz one (Lütkepohl and Krätzig [41]) applied to the individual series \(y_{it}, \ldots, y_{it} (i = 1, \ldots, N)\).

33 In that case, the common factor structure can be incorporate in the GLS statistic imposing some structure on the covariance matrix \(\Omega\), i.e., \(\Omega = \Lambda\Lambda' + \Sigma\) with \(\Lambda\) matrix of factor loadings and \(\Sigma\) covariance matrix of the idiosyncratic innovations. In order to estimate \(\Omega\) consistent estimators of \(\Lambda\) and \(\Sigma\) are required and the principal component procedure suggested by Bai and Ng [3] or Moon and Perron [43] can be adopted.

34 Breitung and Das [11] observe that in such a context also the prewhitening approach may fail, if the short-run dynamics are individual specific.
- when the common components are stationary while the idiosyncratic components are $I(1)$ the OLS-based statistic is not applicable since it tends to infinity and so tends to indicate stationary time series as $T, N \to \infty$. On the other hand GLS test, removing the cross-section dependence, is asymptotically valid.

### 2.2.2. Chang [12] test

Chang [12] proposes an alternative non-linear instrumental variable (IV) approach in order to solve the nuisance parameter problem afflicting the distribution of the unit root tests in the presence of cross-sectional correlation. To do this, Chang [12] tries to make the panel statistics asymptotically invariant to cross-sectional dependence: for each cross-section unit, the author estimates the $AR$ coefficient from an usual $ADF$ regression using the instruments generated by an integrable transformation of the lagged values of the endogenous variable. Then, for testing the unit root based on these $N$ nonlinear IV estimators, Chang constructs $N$ individual $t$-statistics that have limiting standard normal distribution under the null hypothesis. Finally, a cross-sectional average of the individual IV $t$-ratio statistics is considered, as in the IPS approach.

Specifically, Chang considers a panel model generated by a first-order autoregressive regression\(^{35}\) such as

$$y_{it} = \rho_i y_{i,t-1} + u_{it},$$

where, as usual, $i = 1, \ldots, N$ denotes individual cross-sectional units and $t = 1, \ldots, T_i$ denotes time-series observations. Note that the total number $T$ for each individual $i$ may differ across units, i.e., unbalanced panels are allowed.

The initial values $(y_{i1}, \ldots, y_{iN})$ are set at zero for simplicity. The error term $u_{it}$ is given by an $AR(p)$ invertible process

$$\lambda'(L)u_{it} = \epsilon_{it},$$

---

\(^{35}\) The model with deterministic components can be analyzed similarly using demeaned or detrended data.
where $\lambda'(L) = 1 - \sum_{j=1}^{p_i} \beta_{ij}L^j$, with $L$ denoting the usual lag operator.

Cross-sectional dependence of the innovations $\varepsilon_{it} \sim i.i.d. (0, \sigma_{\varepsilon_i}^2)$ that generate the errors $u_{it}$'s is allowed.

The null hypothesis of interest is

$$H_0 : \rho_i = 1 \text{ for all } y_{it},$$

against the alternative hypothesis

$$H_a : |\rho_i| < 1 \text{ for some } y_{it}.$$

Thus, the null hypothesis implies that all $y_{it}$'s have unit roots, and it is rejected, if one at least of the $y_{it}$'s is stationary with $|\rho_i| < 1$. In this way, rejection of the null hypothesis does not imply that the whole panel is stationary.

Giving the (2.2.2.1) and (2.2.2.2), it is possible to rewrite the model as

$$y_{it} = \rho_i y_{it-1} + \sum_{j=1}^{p_i} \beta_{ij} u_{it-j} + \varepsilon_{it},$$

and, since $\Delta y_{it} = u_{it}$ under the null hypothesis of unit root, the above regression becomes

$$y_{it} = \rho_i y_{it-1} + \sum_{j=1}^{p_i} \beta_{ij} \Delta y_{it-j} + \varepsilon_{it}. \quad (2.2.2.3)$$

In order to deal with the cross-sectional dependence, Chang uses the instrument generated by a non-linear function $F(\cdot)$ of lagged values $y_{it-1}$, i.e., $F(y_{it-1})$, which is called the Instrument Generating Function (IGF).

$F(\cdot)$ is a regularly integrable function which satisfies $\int_{-\infty}^{+\infty} xF(x) \, dx \neq 0$, i.e., the nonlinear instrument $F(\cdot)$ is correlated with the regressor $y_{it-1}$. 
For the lagged demeaned differences \( x_{it} = (\Delta y_{it-1}, \ldots, \Delta y_{it-p_i}) \), the variables themselves are used as instruments.

Let \( X_i = (x_{i,0}, \ldots, x_{iT}) \) be the \((T, P_i)\) matrix of the lagged differences, \( y_{it} = (y_{i,0}, \ldots, y_{iT-1}) \) the vector of lagged values and \( \varepsilon_i = (\varepsilon_{i,0}, \ldots, \varepsilon_{iT}) \) the vector of residuals.

The augmented regression (2.2.2.3) can be written in matrix form as

\[
y_i = y_{ii} \rho_i + X_i \beta_i + \varepsilon_{ii},
\]

(2.2.2.4)

where \( \beta_i = (\beta_{i,0}, \ldots, \beta_{i,P_i}) \). Under the null hypothesis, the nonlinear IV estimator of the parameter \( \rho_i \) denoted as \( \hat{\rho}_i \), is given by

\[
\hat{\rho}_i = \left[ F(y_{ii})' y_{ii} - F(y_{ii})' X_i (X_i' X_i)^{-1} X_i' y_{ii} \right]^{-1} \left[ F(y_{ii})' \varepsilon_{ii} - F(y_{ii})' X_i (X_i' X_i)^{-1} X_i' \varepsilon_{ii} \right]
\]

and its variance by

\[
\hat{\sigma}^2_{\rho_i} = \hat{\sigma}^2_{\varepsilon_i} \left[ F(y_{ii})' y_{ii} - F(y_{ii})' X_i (X_i' X_i)^{-1} X_i' y_{ii} \right]^2 \left[ F(y_{ii})' F(y_{ii}) - F(y_{ii})' X_i (X_i' X_i)^{-1} X_i' F(y_{ii}) \right],
\]

where \( \hat{\sigma}^2_{\varepsilon_i} = \frac{1}{T} \sum_{t=1}^{T_i} \hat{\varepsilon}_{it}^2 \) and \( \hat{\varepsilon}_{it} \) is the fitted residual from augmented regression (2.2.2.3).

For testing the unit root hypothesis \( H_0 : \rho_i = 1 \) for all \( y_{it} \), Chang constructs a \( t \)-ratio statistic from the nonlinear IV estimator \( \hat{\rho}_i \), denoted by \( Z_i \), in this way:

\[
Z_i = \frac{\hat{\rho}_i - 1}{\hat{\sigma}_{\hat{\rho}_i}} \xrightarrow{d} N(0, 1) \text{ for all } i = 1, \ldots, N,
\]

where the asymptotical convergence to a standard normal distribution is assured, if a regularly integrable function is used, as an IGF.

This asymptotic Gaussian result is fundamentally different from the usual unit root limit theories and it is essentially due to the nonlinearity
of the IV. More importantly, the limit distributions of individual $Z_i$ statistics are cross-sectionally independent. Hence, these asymptotic orthogonalities lead to propose a panel unit root test based on the cross-sectional average of these individual independent statistics. Chang proposes an average IV $t$-ratio statistic, defined as

$$S_N = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} Z_i,$$

where $N^{-1/2}$ is just as a normalization factor. It is straightforward to show that $S_N$ has a limiting standard normal distribution\(^{36}\) and, then it is possible to do simple inference even for unbalanced panels with general cross-sectional dependence. Moreover, Chang’s limit theory does not require a large spatial dimension; consequently $N$ may take any value, large or small.

Chang’s [12] approach is very general and has good finite sample properties. The simulation results show that the finite sample sizes of $S_N$, calculated using the standard normal critical values, are close to the nominal ones. Moreover, the $S_N$ test seems to be more powerful than the $IPS$ one and improves significantly upon the $t$-bar test under cross-sectional dependency, especially, for panels with smaller $T$ and $N$.

However, Im and Pesaran [31] showed that Chang’s test is valid only, if $N$ is fixed as $T \to \infty$. Their Monte Carlo simulations show that Chang’s test is considerably over-sized for moderate degrees of cross-section dependence, even for relatively small values of $N$. Furthermore, the nonlinear transformation involved in the Chang’s procedure may compromise the power of the test and it is not clear how to choose the transformation in an optimal way.

2.2.3. Choi and Chue [16] test

Choi and Chue [16] propose subsampling techniques to deal with cross-sectional dependence. Their procedure is very general and can be

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\(^{36}\) It should be noted that the usual sequential asymptotic is not used here. The limit theory is derived for $T \to \infty$, which in not followed by $N \to \infty$. 
applied to possibly nonstationary panel data. Moreover, it allows the regressors to be stationary, nonstationary or a mixture of both types and it does not require the estimation of the cross-sectional correlation, so far reducing the risk of size distortion due to misspecification with respect to a factor structure approach. It also allows for cross-sectional cointegration without requiring knowledge of the cointegration coefficients and ranks. Choi and Chue’s [16] tests have finite sample distributions that do not depend on nuisance parameters. Specifically, in the finite N, infinite T case, they consider a panel unit root test \( z_{Nn} \) computed on the whole disposable sample and observe that its limiting distribution is likely to depend on nuisance parameters for cross-sectionally correlated or cointegrated panels. Nevertheless, it is possible to approximate this limiting distribution by means of the subsampling method. Given a panel unit root test \( z_{Nbs} \) computed with smaller blocks of consecutively observed time series \( \{y_{is}, \ldots, y_{i,s+b-1}\}_{i=1}^{N} \) starting from \( s(1 \leq s \leq T - b - 1) \) and with block size \( b \), the empirical distribution function based on the computed values of the tests can be expressed as

\[
L_{NTb}^z(x) = \frac{1}{T-b+1} \sum_{s=1}^{T-b+1} 1\{z_{Nbs} \leq x\},
\]

where \( 1\{z_{Nbs} \leq x\} = 1 \), if \( z_{Nbs} \leq x \) and \( 1\{z_{Nbs} \leq x\} = 0 \), if \( z_{Nbs} > x \). Assuming \( b \to \infty \) and \( b/T \to 0 \) as \( T \to \infty \), \( L_{NTb}^z(x) \) approximates the limiting distribution uniformly in \( x \) and can be applied in a panel unit root context. Specifically, Choi and Chue subsample the LLC, IPS and Fisher-type tests for panel unit root and show that they are consistent with reasonably finite-sample properties, mostly in the noncointegrated panel case. Nevertheless, as Choi and Chue [16] point out subsampling is not always the best method to use, since it depends on the nature of the problem. Sometimes other methods, (i.e., Chang [13]) may work better in finite samples.

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37 Choi and Chue [16] procedure includes panel unit root and cointegration tests as special cases.
2.3. Comparison among the previous presented tests

From the previous discussions, it is straightforward to note that Bai and Ng [3], Pesaran [45] and Moon and Perron [43] tests assume the same dynamic structure of the data and are computationally simple to implement (they only require some tabulated critical values and the selection of the number of common factor $K$). Specifically, as Gengenbach et al. [22] point out, the use of a factor model allows to represent correlation or cointegration between panel units in a suitable way and the assumption of independence between common factors and error terms (required for pooled testing) seems to be more unrestrained than the assumption of independence between cross-sections (made in the first generation of unit root tests).

Nevertheless, Bai and Ng [3] approach is more general than the ones of Pesaran [45] and Moon and Perron [43]. In particular, one can observe that

- in Bai and Ng [3] the non-stationarity of the series can be due to common or idiosyncratic sources (so that the integration order of the two factors can differ), while Pesaran [45] and Moon and Perron [43] assume common and idiosyncratic stochastic trends under the null hypothesis.\[^{38}\]

- Pesaran [45] and Moon and Perron [43] do not allow for cointegration among the $y_{it}$ as well as between the observed data and the common factors but Bai and Ng [3] models include both possibilities.

- Pesaran’s [45] and Bai and Ng’s [3] models may include either a constant or a linear trend; whereas Moon and Perron [43] test is proposed for the case in which only a constant is present.

Due to these features, while the Bai and Ng [3] test is able to distinguish cases, where the observed non-stationarity depends only on a non-stationary common factor; in such circumstances the other two tests tend to reject the non-stationarity of the series.

\[^{38}\text{Note that on the basis of these observations, the null hypothesis assumed by Pesaran [45] and Moon and Perron [43] is a special case of the Bai and Ng's one [3].}\]
Gengenbach et al. [22] analyze the small sample behaviour of the proposed tests and show that:

- Moon and Perron [43] test is more powerful than the Pesaran [45] one, but the latter is simpler to compute;

- in order to detect the non-stationarity of the idiosyncratic components, the $P^c_E$ test is more powerful than the $ADF^c_E$;

- in order to detect the non-stationarity of the common factor the $ADF^c_F$ test shows good small sample properties when $N$ and $T \geq 50$ and the serial correlation in the common factor is not too persistent;

- in a multi-factor setting, the $MQ^c_E$ seems to overcome the $MQ^c_f$ test but both tests fail to distinguish high stationary serial correlation from non-stationarity in the common factors\(^{39}\).

Due to this observations, Gengenbach et al. [22] provide a procedure for testing the presence of unit root in panels with dynamic factors.

First, on the basis of this approach, if the cross-sectional dependence is suspected to be generated by a single common factor, the Pesaran’s [45] CIPS test is used for testing for the non-stationarity in the data; on the contrary, if cross-section dependence is suspected to be generated by multi common factors, a Moon and Perron [43] test is preferred.

Also, the $P^c_E$ and the $ADF^c_E$ tests are used in order to test for the non-stationarity of the idiosyncratic components and the common factors, respectively.

Now, the rejection by the Bai and Ng [3] tests of the null hypothesis of unit roots for the idiosyncratic components but not for the common factor, and the rejection by the Pesaran [45] and Moon and Perron [43]

\(^{39}\) This topic is well analysed also by Breitung and Das [11].
tests of their nonstationarity hypothesis, can be considered as a signal of the presence of cross-section cointegration.

Gutierrez [23] analyzes by Monte Carlo simulations the finite sample properties of the Choi [15], Bai and Ng [3], Moon and Perron [43] and Phillips and Sul [47] tests in the presence of cross-section correlation between the panel units. He concludes that in term of power for different values of \( N \) and \( T \), the performance of all tests grow as \( N \) and \( T \) increase\(^{40}\), but the test showing the best performance is the Moon and Perron [43] one.

Regarding the Bai and Ng [3] test, it shows good size and power in order to test the presence of a unit root in the idiosyncratic component, but low power for testing the non-stationarity of the common factors. Recently, Kapetanios [34] has tried to extend the work of Bai and Ng considering feasible alternative factor extraction and estimation approaches to the principal component in order to remove factors. By means of Monte Carlo simulations, he compares the resulting tests, and shows that the performance of the factor extraction methods depends on the dynamic nature and the number of factors. This poses significant problems for the available factor-based panel unit root tests whose related rejection probabilities exceed the nominal significance level in many cases, and sometimes, it could be even better not to introduce any correction for cross-sectional dependence. Kapetanios [34] concludes that, although in such a context, the use of Pesaran’s [45] test seems to be a promising alternative, more work is needed in order to guarantee that the second generation of tests dominates in all cases the performance of the first generation of tests.

3. The Second Generation of Panel Stationarity Tests

All previous test procedures evaluate the null hypothesis of unit root but, as Kwiatkowski et al. [35] have noted, the way in which classical hypothesis testing is carried out leads to accept the null hypothesis unless

\(^{40}\) Note that the inclusion of a deterministic trend in the model reduces the power of all tests.
there is strong evidence to the contrary. In time-series literature, this is confirmed by the little power showed by standard unit root tests against relevant alternatives and by the failure of this tests in rejecting, the null hypothesis for a wide range of economic series. In order to overcome, such impasse and decide by classical methods about the stationarity of economic data, DeJong and Whiteman’s [18] studies suggest to perform tests of the null hypothesis of stationarity as well as tests of the null hypothesis of a unit root. In the context of the first generation of panel tests, this effort has been accomplished by Hadri [24] first and by Choi [14] through a simple extension of their previous panel unit root tests.

Under the assumption of cross-section independence, testing for stationarity in panel data instead of single time-series leads to the same advantage invoked for panel unit root tests: as \( N \) grows, the power of the test increases and the distributions of the test statistics get asymptotically normal. On the contrary, when the independence assumption is violated, pooled tests tend to over-reject the null hypothesis, whether the null hypothesis is a unit root or stationarity (O’Connell [44]). Moreover, as in the time-series case, panel tests for the null hypothesis of stationarity tend to show serious size distortions when the null hypothesis is close to the alternative of a unit root.

As a consequence, caution should be exercised in interpreting the results of this kind of tests.

Nevertheless, performing panel stationarity tests in conjunction with panel unit root tests would be useful. The advantage is the possibility to distinguish among series that appear to be stationary, series that appear to have a unit root, and series for which it is not possible to establish, if they are stationary or integrated.

In the context of panel tests dealing with the presence of cross-sectional dependence, there are only few hand-outs in order to investigate the null hypothesis of stationarity, even if the literature in this topic is quickly growing. The main contributions are those by Bai and Ng [4] and
Harris et al. [26], both of which extend the principal component analysis for panel unit root testing proposed by Bai and Ng [3].

Referring to model (2.1.3.1), Bai and Ng [4] note that an observed series $y_{it}$ is stationary, if both $F_i$ and $e_{it}$ are stationary and, on the other hand, $y_{it}$ is nonstationary, if any one of the $K$ common factors or $e_{it}$ contains a unit root. Once again, the Bai and Ng’s procedure does not analyze directly the observed data, but it tests the common and idiosyncratic components separately. In order to obtain consistent estimates of that components without a priori information about their stationarity features, the authors retrace the steps illustrated in Section 2.1.3 and estimate $\lambda_i$ and $F_i$ by applying the principal components methodology to the first differenced data. These estimates are then re-cumulated in order to obtain estimates in level form, and tests verifying the null hypothesis of stationarity are applied to them. Specifically, Bai and Ng [4] applied the KPSS test (Kwiatkowski et al. [35]) in order to test the null hypothesis of stationarity for idiosyncratic errors and estimated factors.42

Bai and Ng [4] simulations show that testing on the component is more accurate than testing the summed series. However, as in the general case of stationary tests, also Bai and Ng [4] test suffers from size distortions and tends to over-reject the null hypothesis of stationarity. Various attempts have been accomplished in order to improve the power of these tests43, but they are not resolutive in the size problem.

Harris et al. [26] propose a test based on a vector version of their stationarity test (Harris et al. [25]). In empirical applications, they refer to the following model

41 Note that also the Choi and Chue’s [16] subsampling method can be used in order to test the null hypothesis of stationarity under cross-section correlation. In this case, the $\xi_{Nn}$ test will be a panel test for the null hypothesis of stationarity as the Hadri’s [24] one.

42 In their work, Bai and Ng [4] also consider alternative suitable stationary tests, i.e., the Leybourne and McCabe’s [40] test or the Joansson [32] one.

43 See, for example, Joansson [32] and Lanne and Saikkonen [36].
where \( x_{it} \) is a vector of deterministic terms. In order to test the null hypothesis of joint stationarity,

\[ H_0 : |\phi_i| < 1 \text{ for all } i, \]

against the unit root alternative

\[ H_1 : |\phi_i| = 1 \text{ for at least one } i, \]

they use a statistic, that is, a sum of the studentized lag-\( k \) sample autocovariances across the panel, formally

\[ \tilde{S}_k = \tilde{C}_k + \tilde{c}, \]

where \( \tilde{C}_k = T^{-1/2} \sum_{t=k+1}^{T} \tilde{a}_{kt}, \tilde{a}_{kt} = \sum_{i=1}^{N} \tilde{z}_{it} \tilde{z}_{it-k}, \tilde{z}_{it} \) are the standardized OLS residuals, \( \tilde{c} \) is a finite-sample correction term introduced in the aggregate numerator in order to reduce the effects of the estimation errors on the statistic distribution under the null (see Harris et al. [26] for details), \( \hat{o}(\tilde{a}_{kt}) \) is a long-run variance estimator and \( k \) is chosen so that \( k \to \infty \) and \( k / T \to 0 \) as \( T \to \infty \) \(^{44}\).

When \( T \to \infty \), fixed \( N \)\(^{45}\), \( \tilde{S}_k \) has a limiting standard normal distribution under the null hypothesis, while it diverges to \( +\infty \) under the alternative.

\(^{44}\) Such a choice of \( k \) removes any need to explicitly model the time-series dynamic of each series in the panel. By this way, they can be heterogeneous. It is also straightforward to note that being \( \tilde{S}_k \) the studentized version of \( \tilde{C}_k \), it is robust to the presence of cross-sectional dependence.

\(^{45}\) As a matter of fact, this is a reasonable assumption for many macroeconomic applications.
This test shows satisfactory performance in term of size and power and Harris et al. [26] observe that it can also be applied to estimated factor models with significantly more power than when it is applied to the raw data, even if the model is misspecified.

4. Conclusions

This work gives a review of the main results presented in the panel unit root test literature. Much research has been carried out recently on the topic of econometric nonstationary panel data, especially, because of the availability of new data sets, (e.g., the Penn World Tables by Summer and Heston [52]) in which the time-series dimension and the cross-section dimension are of the same order.

Giving a larger quantity of information, new data sets ask for new tools of analysis. In order to work with this new kind of data, it is necessary that new tools - including advantages and limits - are well-known to the researchers interested in this kind of investigations.

The aim of this paper is to provide a survey of the topic, making it easy to see the directions in which the research has developed, sorting out what appears worthwhile from the dead ends, and determining future areas in which it would be productive to undertake further analysis.

In particular, within the panel unit root test framework, two directions have been developed since the seminal work by Levin and Lin [37], leading to two generations of panel unit root tests. The first one concerns heterogeneous models with contributions by Im et al. [30], Maddala and Wu [42], Choi [14] and Hadri [24]. The second-and more recent one-aims at taking cross-sectional dependence into account. This latter category of tests is still developing, given the diversity of the potential cross-sectional correlations.

Nevertheless, researchers should bear in mind that performing panel stationarity tests in conjunction with panel unit root tests would be useful. The advantage is the possibility to distinguish among series that appear to be stationary, series that appear to have a unit root, and series for which it is not possible to establish, if they are stationary or integrated.
References


## Appendix

### Table A.1. The first generation of panel unit root tests

<table>
<thead>
<tr>
<th>Test</th>
<th>Hypothesis test</th>
<th>Model specification</th>
<th>Advantages (+)/disadvantages (-)</th>
<th>Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>LLC</strong> Levin et al.</td>
<td>• nonstationarity for all individual</td>
<td>• individual effects</td>
<td>+ unbalanced panels are allowed, but further simulations are required&lt;br&gt;- it requires an infinite number of groups&lt;br&gt;- all the groups are assumed to have the same type of nonstochastic components&lt;br&gt;- the critical values are sensitive to the choice of lag lengths in the individual ADF regressions&lt;br&gt;- does not allow that some groups have a unit root and others do not</td>
<td>• it is a pooled test&lt;br&gt;- more relevant for panel of moderate size (10 &lt; N &lt; 250 and 25 &lt; T &lt; 250)&lt;br&gt;- superconsistency of the estimators&lt;br&gt;- there is a loss of power when time trends are included</td>
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<tr>
<td>[39]</td>
<td>• homogeneous alternative</td>
<td>• time trends</td>
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<td></td>
<td></td>
<td>• heterogeneous serial correlation structure of the errors</td>
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<td></td>
<td></td>
<td>• individual effects</td>
<td>+ unbalanced panels are allowed, but further simulations are required&lt;br&gt;- it requires an infinite number of groups&lt;br&gt;- all the groups are assumed to have the same type of nonstochastic components&lt;br&gt;- the critical values are sensitive to the choice of lag lengths in the individual ADF regressions&lt;br&gt;- does not allow that some groups have a unit root and others do not</td>
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<td>• time trends</td>
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<td></td>
<td></td>
<td>• heterogeneous serial correlation structure of the errors</td>
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<tr>
<td><strong>IPS</strong> Im et al. [30]</td>
<td>• nonstationarity for all individual</td>
<td>• individual linear trend</td>
<td>+ unbalanced panels are allowed, but further simulations are required&lt;br&gt;- it requires an infinite number of groups&lt;br&gt;- all the groups are assumed to have the same type of nonstochastic components&lt;br&gt;- the critical values are sensitive to the choice of lag lengths in the individual ADF regressions&lt;br&gt;- does not allow that some groups have a unit root and others do not</td>
<td>• it is an averaged t-test&lt;br&gt;- there is a loss of power when time trends are included&lt;br&gt;- generally, it is more powerful than LLC and Fisher-type tests</td>
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<td></td>
<td>• heterogeneous alternative</td>
<td>• heterogeneous serial correlation structure of the errors</td>
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<td></td>
<td></td>
<td>• individual linear trend</td>
<td>+ unbalanced panels are allowed, but further simulations are required&lt;br&gt;- it requires an infinite number of groups&lt;br&gt;- all the groups are assumed to have the same type of nonstochastic components&lt;br&gt;- the critical values are sensitive to the choice of lag lengths in the individual ADF regressions&lt;br&gt;- does not allow that some groups have a unit root and others do not</td>
<td>• it is an averaged t-test&lt;br&gt;- there is a loss of power when time trends are included&lt;br&gt;- generally, it is more powerful than LLC and Fisher-type tests</td>
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<td></td>
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<td>• heterogeneous serial correlation structure of the errors</td>
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<td><strong>Fisher-type test</strong></td>
<td>• nonstationarity for all individual</td>
<td>• individual fixed effects and time trend</td>
<td>+ unbalanced panels are allowed&lt;br&gt;- it can be carried out for any unit root test derived&lt;br&gt;- it is possible to use different lag lengths in the individual ADF regressions&lt;br&gt;- the p-value have to be derived by Monte Carlo simulations&lt;br&gt;- problems of size distortion with serial correlated errors</td>
<td>• it is a combination test&lt;br&gt;- there is a loss of power when time trends are included&lt;br&gt;- with cross-sectional correlated errors it is more powerful than LLC</td>
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<tr>
<td>Maddala and Wu [42]</td>
<td>• heterogeneous alternative</td>
<td>• heterogeneous serial correlation structure of the errors</td>
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<tr>
<td>and Choi [14]</td>
<td></td>
<td>• individual fixed effects and time trend</td>
<td>+ unbalanced panels are allowed&lt;br&gt;- it can be carried out for any unit root test derived&lt;br&gt;- it is possible to use different lag lengths in the individual ADF regressions&lt;br&gt;- the p-value have to be derived by Monte Carlo simulations&lt;br&gt;- problems of size distortion with serial correlated errors</td>
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<td></td>
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<td>• individual fixed effects and time trend</td>
<td>+ unbalanced panels are allowed&lt;br&gt;- it can be carried out for any unit root test derived&lt;br&gt;- it is possible to use different lag lengths in the individual ADF regressions&lt;br&gt;- the p-value have to be derived by Monte Carlo simulations&lt;br&gt;- problems of size distortion with serial correlated errors</td>
<td>• it is a combination test&lt;br&gt;- there is a loss of power when time trends are included&lt;br&gt;- with cross-sectional correlated errors it is more powerful than LLC</td>
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<td>• heterogeneous serial correlation structure of the errors</td>
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<td>• individual fixed effects and time trend</td>
<td>+ unbalanced panels are allowed&lt;br&gt;- it can be carried out for any unit root test derived&lt;br&gt;- it is possible to use different lag lengths in the individual ADF regressions&lt;br&gt;- the p-value have to be derived by Monte Carlo simulations&lt;br&gt;- problems of size distortion with serial correlated errors</td>
<td>• it is a combination test&lt;br&gt;- there is a loss of power when time trends are included&lt;br&gt;- with cross-sectional correlated errors it is more powerful than LLC</td>
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<td>• heterogeneous serial correlation structure of the errors</td>
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<td><strong>Hadri [24]</strong></td>
<td>• stationarity for all individual</td>
<td>• individual specific variances and correlation patterns</td>
<td>+ it avoids oversized tests due to treating not only N but also T asymptotic&lt;br&gt;- the moments of the asymptotic distribution of the test are exactly derived</td>
<td>• it is a residual based LM test</td>
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<tr>
<td></td>
<td>• homogeneous alternative</td>
<td>• individual specific variances and correlation patterns</td>
<td>+ it avoids oversized tests due to treating not only N but also T asymptotic&lt;br&gt;- the moments of the asymptotic distribution of the test are exactly derived</td>
<td>• it is a residual based LM test</td>
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<td>• individual specific variances and correlation patterns</td>
<td>+ it avoids oversized tests due to treating not only N but also T asymptotic&lt;br&gt;- the moments of the asymptotic distribution of the test are exactly derived</td>
<td>• it is a residual based LM test</td>
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Table A.2. The second generation of panel unit root tests: the problem of cross-sectional dependence

<table>
<thead>
<tr>
<th>Test</th>
<th>Hypothesis test</th>
<th>Model specification</th>
<th>Advantages (+)/disadvantages(-)</th>
<th>Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pesaran [45]</td>
<td>• nonstationarity for all individual</td>
<td>• fixed effects</td>
<td>+ unbalanced panels are allowed</td>
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<td></td>
<td>• heterogeneous alternative</td>
<td>• time trends</td>
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<td>• cross-section dependence and/or serial correlation</td>
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<tr>
<td>Moon and Perron [43]</td>
<td>• nonstationarity for all individual</td>
<td>• one-way error component model</td>
<td>+ unbalanced panels are allowed</td>
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<tr>
<td></td>
<td>• heterogeneous alternative</td>
<td>• identical panel composition</td>
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<td>• heterogeneous restrictions</td>
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<td>• the test suffers from size distortion when common factors are present on cross-sectional independence</td>
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<td>• It is very powerful and has good size when only a deterministic constant is included in the model</td>
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<td>• recommended when cross-section dependence is expected to be due to several common factors</td>
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<td>Bai and Ng [3]</td>
<td>• nonstationarity for all individual</td>
<td>• fixed individual effects</td>
<td>+ unbalanced panels are allowed</td>
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<td>• heterogeneous alternative</td>
<td>• time trends</td>
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<td>• it is possible to determine, if nonstationarity comes from a pervasive or an idiosyncratic source</td>
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<td>• this procedure solves the problem of the size distortion</td>
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<td>• it is a pooled test, then it is more powerful than univariate unit root test</td>
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<td></td>
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<td></td>
<td></td>
<td>• good finite sample properties (even for small $N$)</td>
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<tr>
<td>Chang [12]</td>
<td>• nonstationarity for all individual</td>
<td>• fixed effects</td>
<td>+ unbalanced panels are allowed</td>
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<td></td>
<td>• heterogeneous alternative</td>
<td>• time trends</td>
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<td>• it is a nonlinear instrumental variable approach</td>
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<td>• good finite sample properties</td>
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<td>• more powerful than IPS, especially, for little panels</td>
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<tr>
<td>Harris et al. [26]</td>
<td>• stationarity for all individual</td>
<td>• fixed individual effects</td>
<td>+ unbalanced panels are allowed</td>
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<td></td>
<td>• heterogeneous alternative</td>
<td>• time trends</td>
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<td>• the test suffers from size distortions and tends to over-reject the null hypothesis of stationarity</td>
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<tr>
<td>Bai and Ng [4]</td>
<td>• stationarity for all individual</td>
<td>• general deterministic models</td>
<td>+ simple to compute</td>
<td></td>
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<tr>
<td></td>
<td>• heterogeneous alternative</td>
<td>• time trends</td>
<td>• the fitted deterministic model can differ for each series</td>
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<td></td>
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<td>- it is valid only for $N$ fixed, $T \to \infty$</td>
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<td>• no need to compute critical values</td>
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