IJAMML 18:1 (2024) 9-12 December 2024
ISSN: 2394-2258
Available at http://scientificadvances.co.in
DOI: http://dx.doi.org/10.18642/ijamml_7100122314

NEW RESULTS ON THE HIGHER GENERAL RANDIĆ INDICES OF *k*-REGULAR ELECTRICAL NETWORKS

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Abstract

In this paper, we firstly define the h-th (order) general Randić index of physico-chemical molecular graphs, denoted by $R_{\alpha}^{h}(G)$, which is defined as the sum of terms

$$[d(v_{i_1})d(v_{i_2})\cdots d(v_{i_{h+1}})]^{\alpha}$$

over all paths of length h contained in G, where α is any real number. Also, the higher general Randić indices of any k-regular electrical networks are completely computed.

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2020 Mathematics Subject Classification: 05C78.
Submitted by Jianqiang Gao.
Received September 12, 2024

Keywords: higher general Randić index, electrical networks.

1. Introduction

In physico-chemical graph theory, the vertices and the edges correspond to the atoms and the bonds, respectively. Let be $G = (V, E)$ a simple finite molecular or electrical network with the vertex set $V(G)$ and the edge set $E(G)$. Let $u, v \in V(G)$. The number of adjacent vertices of u is called degree of u, denoted by $d(u)$. The minimum and maximum degree of G is denoted by $\delta(G)$ and $\Delta(G)$, respectively. A molecular or electrical network with $\delta(G) = \Delta(G) = k$ is called k-regular. All other notation and terminology are referred to [1].

In 1998, Bollobás and Erdös [2] generalized the Randić index by replacing $-\frac{1}{2}$ with any real number α, which is called the general Randić molecular index of a molecular structure electrical network $G = (V, E)$ as follows:

$$R_\alpha(G) = \sum_{uv \in E(G)} (d_u d_v)^\alpha.$$

The h-th (order) general Randić index of graph G, denoted by $R_\alpha^h(G)$ is defined as

$$R_\alpha^h(G) = \sum_{P^h = v_{i_1} v_{i_2} \cdots v_{i_{h+1}} \subseteq G} [d(v_{i_1}) d(v_{i_2}) \cdots d(v_{i_{h+1}})]^\alpha,$$

where summation is over all paths P^h of length of h contained in G and α is any real number. Obviously, $R_\alpha^1(G) = R_\alpha(G)$ and some results about the general Randić index and the higher general Randić index have been obtained in [3].

2. Main Results

Theorem 2.1. *Let G be a k-regular electrical network with vertices number n and α be any real number. Then its general Randić index is*

$$R_\alpha(G) = \frac{n \cdot k^{2\alpha+1}}{2}. \tag{2.1}$$

Proof. Since G be a k-regular electrical network with vertices number n, $kn = 2|E(G)|$. Then $|E(G)| = \frac{kn}{2}$. By the definition of the general Randić index, we have

$$R_\alpha(G) = \sum_{uv \in E(G)} (d_u d_v)^\alpha = \frac{kn}{2} \cdot (k \cdot k)^\alpha = \frac{n \cdot k^{2\alpha+1}}{2}. \qquad \square$$

Corollary 2.1. *Specially, denote C_n and K_n be a cycle electrical network and a complete electrical network with n vertices, respectively. By formula* (2.1), *then we have*

$$R_\alpha(C_n) = n \cdot 4^\alpha, \tag{2.2}$$

$$R_\alpha(K_n) = \frac{n \cdot (n-1)^{2\alpha+1}}{2}, \tag{2.3}$$

where α is any real number. □

Acknowledgement

This research is supported by 2024 Special Projects for Graduate Education and Teaching Reform from China University of Geosciences, Beijing (Grant No. JG2024021 and No. JG2024013), 2023 Experimental Technology Research and Applied Teaching Reform Project of China University of Geosciences, Beijing (Grant No. SYJS202305), and 2024 Subject Development Research Fund Project of China University of Geosciences, Beijing (Grant No.2024XK208).

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DOI: https://doi.org/10.1007/s10910-009-9639-9

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