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ACR-INDEX : A NEW MATHEMATICAL MEASURE CONSIDERING THE ARTICLE'S AGE, CITATIONS, AND RANK (ACR)

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Abstract

This paper introduces a new index, called ACR-index, to measure an individual's scientific research output based on their publications, by taking the age, number of citations and rank of each paper published into account. The paper presents several real author examples to illustrate the proposed ACR-index and compare it to several related indices such as h-index, g-index, and $i10$-index. We believe that the proposed index provides a realistic measure that reflects the researchers' impacts and their research accomplishments in the field especially to early- and mid-career researchers. It should be noted that the two researchers who have the same h-index will unlikely have the same ACR-index since it depends on the paper's-age, citations and rank of each publication.

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Based on a small (a couple of dozen individuals) sample size, we observe that in the science and engineering field, on average, ACR-index scores between 8-12 seem common for assistant professors, and that individuals who have ACR index of 30 after 10 years of research activity characterize outstanding researchers; and those with ACR index of 60 after 30 years characterize unique outstanding researchers.

1. Introduction

The h-index, also known as Hirsch index, was introduced by Hirsch [1] and is defined as the largest number h such that at least h articles in that each publication were cited at least h times based on a set of articles ranked in decreasing order of the number of citations that they received [1, 2]. In other words, a researcher with an index of h has published h papers each of which has been cited in other papers at least h times. For example, a researcher has an h-index of 10 has published 10 papers that have been cited at least 10 times. The h-index reflects both the number of published papers and the number of citations per publication. As expected the researcher's h-index increases over the years but it does not depend on the extra citation numbers of papers which already have h or more citations. Since Hirsch [1] introduced the h index in 2005, it has become the most widely used citation index among the researchers in scientific community as a base to measure a researcher's output as well as the academic performance and impact on their field.

Despite the fact that the h-index can be a good benchmark for capturing researcher's output and overall scientific achievements based on the total number of publications published (i.e., productivity) and the number of citations (i.e., quality of those publications) of an individual's research performance and as an easily accessible their h-index from Web of Science, Scopus, or Google Scholar databases, there are some drawbacks of using it alone, for example;

- The *h*-index is insensitive to the highly cited work because it doesn't take into account the number of highly cited papers beyond the *h* value or low-citation articles below the *h* value.

- If a researcher has a small number of papers but they are highly cited, the *h*-index will not fully reflect that researcher's accomplishments.

- The *h*-index does not give an accurate measure for early-career researchers who could have already made a good scientific contribution to the field but his/her publications simply have not been long enough time to accumulate citations yet.

- As expected the author's *h*-index and citations increase over the years so early-career researchers may have an *h*-index disadvantage when compared them to someone who has been publishing and in the profession for many years by their *h*-index. For example, it is easy to understand the facts that someone with a publication record of 5 years to have less number of citations and smaller *h*-index as a researcher who has been publishing for 20 years.

Many researchers [3-13] have been proposed other indices such as *g*-index, *e*-index, *r*-index, *m*-index, *k*-index, etc. in recent years to evaluate the research output and impact of a researcher addressing some of the drawbacks discussed above including the factors such as independent citations, self-citation, number of co-authors, article age [9, 10, 13]. However, these indices do not differ much from the *h*-index. Some of those include, for example;

- ***g-index.*** The *g*-index is defined as the unique largest number such that the top *g* articles received together at least g^2 citations [3]. For example, a *g*-index of 20 means that a researcher has published at least 20 articles that combined have received at least 400 citations. The *g*-index attempts to give more weight to highly-cited papers beyond the *h*-index value and is always higher than the *h*-index.

• ***e-index.*** The e-index aims for the highly cited papers based on the h-index which is useful for highly cited researchers and for comparing those with the same h-index but not for the low-citation papers below the h-value [4, 5].

• ***i10-index.*** The $i10$-index was introduced in 2011 by Google Scholar [6] and used in Google Scholar citation database, which counts the number of publications by an author that have at least 10 citations.

• ***r-index.*** The r-index was proposed as an alternative to the h-index with the aim of better capturing the impact and significance of a researcher's most highly cited papers [7]. The r-index is calculated by arranging a researcher's publications in descending order of the number of citations they have received, and then determining the highest number of consecutive papers that have at least that many citations.

• ***m-index.*** The m-index takes into account the number of years since first publication and is more relevant to an earlier career researcher than the h-index. It is defined as h/n, where h is the h-index and n is the number of years since the first published paper of the researcher [8].

• ***k-index.*** The k-index measures the productivity of an individual based on number and order of co-author, the total number of citations and the aging factor of each article [9, 10].

By addressing the drawbacks and disadvantages of various indices above including the h-index, this paper introduces a new metric, called ACR-index, by taking into account the age, the number of citations and the rank of each publication that can reflect an individual's scientific research output especially for early-career researchers based on their publications. The paper also analyzes several real author examples to illustrate the proposed ACR-index along with the comparisons of several common indices such as h-index, g-index, and $i10$-index.

2. ACR-Index

This section discusses the mathematical derivation and results for calculating the proposed ACR-index. A list of notation is given below.

Notation

The citations of all n published papers of a researcher are ranked in decreasing order

r_i ranking of paper i with citations in descending order, for $i = 1, 2, \ldots, n$;

n the number of academic publications he or she has authored or co-authored;

c_i the number of citations for paper i;

P the present time, unit time is year (Note: It is difficult to collect all published papers data in term on months, although it is possible in some articles, so the unit time for P value is year when using the Lemma below in calculating ACR index);

y_i the year of the paper i-th published;

k^* ACR-index value, a positive integer value.

Assume that the citations of all the publications of a researcher are ranked in decreasing order. When two papers have the same number of citations, they will receive the same rank but the latest published paper will be listed first. We now present the results that shows how to obtain the ACR-index, i.e., optimal value k^*.

Lemma. *Assume that the citations are ranked in descending order. For given values of* P, r_i, y_i, c_i *and* n, *there exists a unique ACR-index value,* k^*, *such that*

$$k^* = \begin{cases} \sup\limits_{1 \le k \le n} \{k \le n : A_k \ge S_k\}, \\ 1 \qquad\qquad if\ A_1 < S_1, \end{cases} \tag{1}$$

where

$$A_k = \sum_{i=1}^{k}\left(\sqrt{\frac{c_i}{r_i + 2}}\right) \; and \; S_k = \sum_{i=1}^{k}\left(\sqrt{(P - y_i)}\right). \tag{2}$$

Proof. Let $f(i) = \dfrac{c_i}{r_i + 2}$. Then we obtain

$$f(i) - f(i+1) = \frac{(r_i + 2)(c_i - c_{i+1}) + c_i}{(r_i + 2)(r_i + 3)} \geq 0.$$

Thus, the function $f(i)$ is decreasing in i for all $i = 1, 2, \ldots, n$. Similarly, since $f > 0$ is a decreasing function, it can be shown that the function A_k is increasing in k and concave. The function S_k is also increasing in k. Therefore, there exists a unique ACR-index value, k^* such that

$$k^* = \begin{cases} \sup\limits_{1 \leq k \leq n} \{k \leq n : A_k \geq S_k\}, \\ 1 \qquad\qquad \text{if } A_1 < S_1. \end{cases}$$

Q.E.D.

In other words, the ACR-index is the largest number k of publications such that $A_k \geq S_k$, where A_k and S_k are given in Equation (2).

It should be noted that the ACR-index is not necessary higher than the h-index as shown in the examples that we will now discuss in Section 3.

3. Application Analysis

In this section we use four real author (namely, Author #1, #2, #3, and #4) citations data obtained from Google Scholar (GS) database [6] as of April 2023 to illustrate the proposed ACR-index and compare it to several common indices such as h-index, g-index, and $i10$-index. So the present time value here is $P = 2023$.

Table 1 shows Author #1 citation data obtained from GS database. The total number of citations received by Author #1 is 15,860. *Citations* (see Table 1) denotes the number of citations to paper i on rank i. For example, the paper 1, with rank 1, has received 1275 citations that was published in 1996 (i.e., $y_1 = 1996$). So the age of this publication is 27. Using Equation (2), we obtain the values A_k and S_k as shown in Table 1. The bold numbers in Table 1 show how the ACR-index can be obtained. So, Author #1 has a ACR-index of 42 (i.e., $k^* = 42$, see Figure 1) and h-index of 59 which is much higher than the ACR-index in this case.

Table 1. Author #1 data (given $P = 2023$, i.e., year 2023)

			Publication			
Rank	**Title**	**Citations**	**Year**	A_k	S_k	
1	Paper 1	1275	1996	20.61552813	5.196152423	
2	Paper 2	762	2000	34.41770187	9.991983946	
3		722	2007	46.43435698	13.99198395	
4		590	2003	56.3506735	18.4641199	
5		472	2006	64.56216371	22.58722553	
…						
40		88	2004	172.8200734	165.1937801	
41		85	2000	174.2260408	169.9896116	
42		**82**	**2007**	**175.5911915**	**173.9896116**	**ACR-index (k^* value) = 42**
43		77	2014	176.8992859	176.9896116	
…						
59		59	2005	194.8795216	243.4588893	h-index = 59

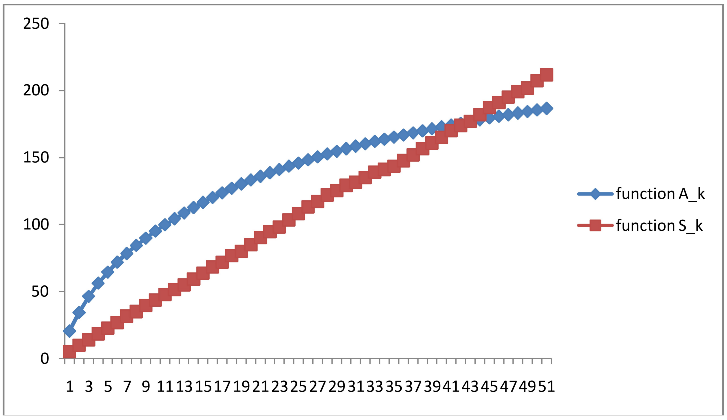

Figure 1. Author #1 data - ACR-index = 42.

Similarly, Tables 2-4 show Author #2-4 citation data sets obtained from GS database, respectively. The total number of citations received by Author #2, 3, and 4 are 4920, 912, and 48352, respectively. The bold numbers in Tables 2-4 show how to obtain the ACR-index values as also shown in Figures 2-4. Table 5 presents the summary results of the ACR-index and other indices for all four authors.

Table 2. Author #2 data (given $P = 2023$, i.e., year 2023)

Rank	Title	Citations	Publication Year	A_k	S_k	
1	Paper 1	2100	2002	26.45751311	4.582575695	
2	Paper 2	1276	1996	44.31808421	9.778728118	
3		473	2006	54.04433734	13.90183374	
4		173	1996	59.41400524	19.09798617	
5		152	2000	64.07386422	23.89381769	
…						
15		27	2006	86.64681497	71.01214255	
16		24	1997	87.80151551	76.11116206	h-index = 16
17		15	1992	88.69003883	81.67892642	
18		**13**	**1997**	**89.4962646**	**86.77794594**	**ACR-index = 18**
19		11	2006	90.22001147	90.90105156	
20	paper 20	9	1992	90.85961361	96.46881593	

Table 3. Author #3 data (given P = 2023, i.e., year 2023)

Rank	Title	Citations	Publication Year	A_k	S_k	
1	Paper 1	244	2018	9.018499506	2.236067977	
2	Paper 2	170	2018	15.53770191	4.472135955	
3		162	2017	21.2298017	6.921625698	
4		72	2020	24.69390331	8.653676505	
5		67	2019	27.78767586	10.65367651	
…						
10		15	2013	35.46809269	20.23016773	
11		14	2018	36.50584174	22.46623571	h-index = 11
12		11	2018	37.392247	24.70230368	
…						
16		7	2022	40.26834748	31.34805499	
…						
23		**1**	**2020**	**42.72591015**	**42.07081059**	**ACR-index = 23**

Table 4. Author #4 data (given P = 2023, i.e., year 2023)

Rank	Title	Citations	Publication Year	A_k	S_k	
1	Paper 1	6049	2019	44.90360045	2	
2	Paper 2	4764	2009	79.4144683	5.741657387	
3		3678	2016	106.5364151	8.387408698	
4		2183	2015	125.6108308	11.21583582	
5		1701	2009	141.1992881	14.95749321	
…						
85		95	1995	344.9186491	335.145908	
86		92	2014	345.9411238	338.145908	
87		91	1997	346.9522974	343.2449275	
88		**89**	**2014**	**347.9467263**	**346.2449275**	**ACR-index = 88** h-index = 88
89		86	2008	348.9188657	350.1179108	

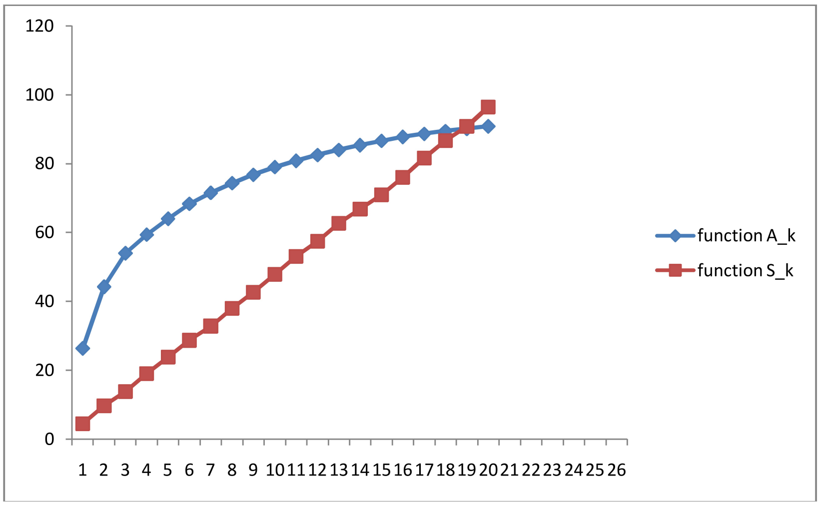

Figure 2. Author #2 data - ACR-index = 18.

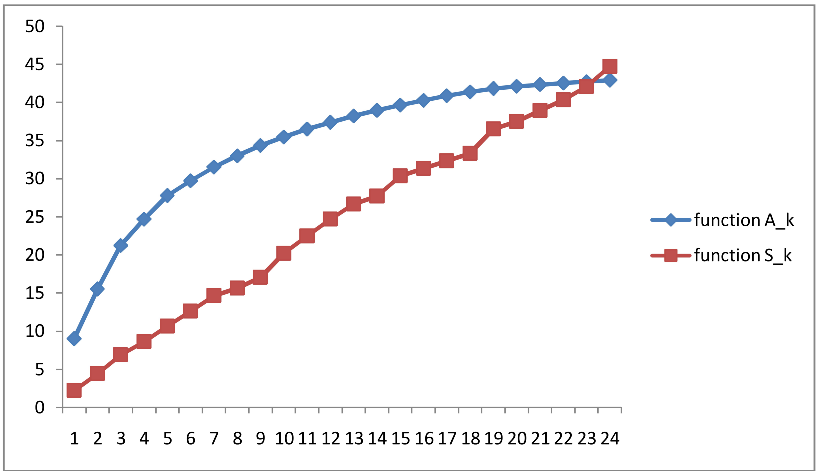

Figure 3. Author #3 data - ACR-index = 23.

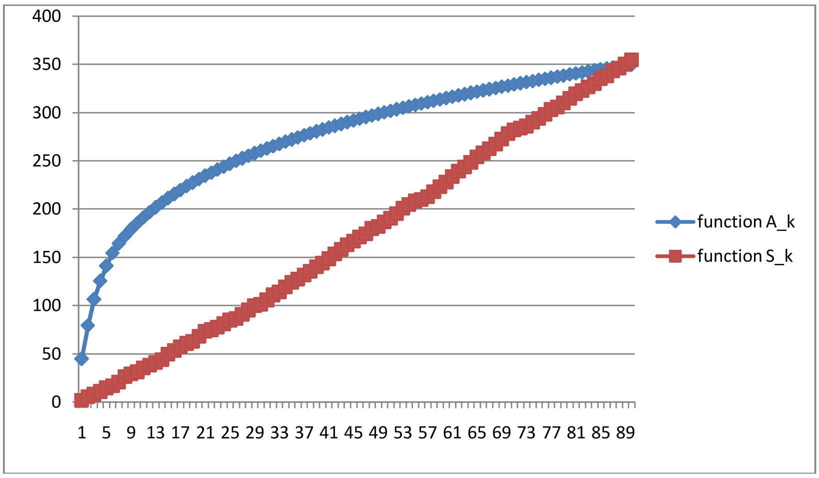

Figure 4. Author #4 data - ACR-index = 88.

Note that the ACR-index is not necessary higher than the *h*-index as shown in Figure 5 where the *g*-index is always higher than the *h*-index as given in Table 5.

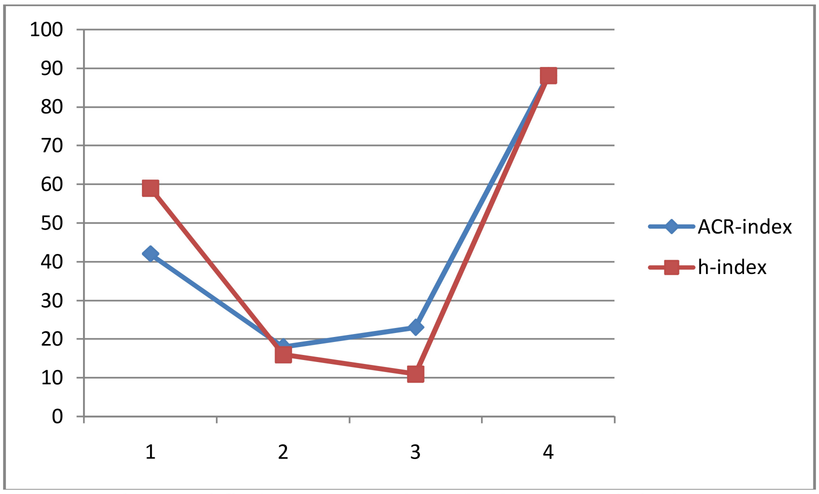

Figure 5. ACR-index versus *h*-index for four authors' citations.

Table 5. A comparison of the ACR-index and several common indices h-index, g-index, and i10-index

	ACR-index	h-index	g-index	i10-index	Total citations
Author #1	42	59	118	183	15860
Author #2	18	16	39	20	4920
Author #3	23	11	26	13	912
Author #4	88	88	205	429	48352

What is interesting here is to compare the ACR-index and the total number of citations of Author #2 and Author #3, for example. Author #2 has a significant higher number of citations (4,920 citations) who has been publishing for at least 25 years (see Table 2) compared to Author #3 who has only 912 citations with a publication record of about 10 years and is in early-career (see Table 3). However, Author #2 has the ACR-index of 18 where Author #3 has the ACR-index of 23 which is significantly higher than Author #2. It is worth to note that Author #2 who has been in the industry so the publications may not necessary a high priority compared to someone who is in the academic institution. From Table 3, Author #3 is still an early-career researcher and can be considered as a rising star in the field who has shown a significant research impact with the ACR-index of 23.

Table 6 shows the ACR-index and h-index results of 7 academic researchers and 1 industry scientist (#3 in Table 6) along with their total number of citations based on Google scholar data where we define the category as follows: Early careers (E): refers to research scholars who are early on in their research within 10 years of receiving their PhD; Middle careers (M): to those who are within 20 years of receiving their PhD; and Senior level (S): to those after 30 years of receiving their PhD.

Table 6. ACR-index of several researchers with career levels

Researcher	Category (early-, mid-senior career)	ACR-index	*h*-index	Total citations
#1	S	42	59	15860
#2	S	88	88	48352
#3	S	18	16	4920
#4	M	31	21	3210
#5	E	21	16	1055
#6	E	23	11	912
#7	E	11	10	415
#8	E	8	10	370

Based on a small (a couple of dozen individuals) sample size, we observe that in the science and engineering field, on average, ACR-index scores between 8-12 seem common for assistant professors, between 15-22 seem common for associate professors, and between 25-30 for full professors. We also found that individuals who have ACR index of 30 after 10 years of research activity characterize outstanding researchers; ACR index of 45 after 20 years of research activity characterize outstanding researchers; and index of 60 after 30 years characterize unique outstanding individuals.

4. Conclusion

The paper presents a new metric, ACR-index, taking into account the age, the citations and the rank of each publication to measure an individual’s scientific research output. We believe that the proposed ACR-index provides a realistic citation measure that reflects the researchers' impacts and their research accomplishments especially to early-career researchers as discussed in Section 3. The ACR-index also can help to recognize the rising starts in the field for early-career and mid-career researchers based on their publication records. The ACR-index is simple and easy to calculate using the mathematical results in Section 2 and following numerical calculations given in Section 3.

It's worth noting that while the ACR-index can be useful in evaluating a researcher's impact and productivity, it should not be relied on solely to make important decisions such as promotion, hiring and academic tenure.

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