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A CHRYSIPPIAN MODAL θ-VALENT VIEW OF AN ATOM IN QUANTUM PHYSICS

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Abstract

In this note, we intend to propose a chrysippian modal θ-valent view of the hydrogen atom in quantum physics.

1. Introduction

Orbitals are specific regions of space where electrons may exist. The hydrogen atomic orbitals depend upon three quantum numbers n, l and m, the principal quantum number $n = 1, 2, \ldots$ specifies the energy of an electron

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and the size of the orbital; the secondary quantum number $l = 0, \ldots, n-1$ specifies the shape of an orbital with a particular principal quantum number; the magnetic quantum number $m = -1, \ldots, 0, \ldots, l$ specifies the orientation in space of an orbital of a given energy n and shape l.

For a hydrogen atom, with $n = 1$, the electron is in its ground state; if $n = 2$ the electron is an excited state. The total number of orbitals for a given value n is n^2.

According to what proceeds, it appears that quantum numbers are elements of the chains $(\mathbb{N}, \leq), (\mathbb{Z}, \leq), \ldots, (\mathbb{C}, \leq)$. From a given closed chain I of ordinal θ, one can define the canonical θ-valent LuKasiewicz algebra defined by I denoted $I_\theta = (I, \Phi_\alpha)$. In [1], F. Ayissi Eteme defines the θ-valent chrysippian ring as the modal θ-valent chrysippian completion of a θ-valent LuKasiewicz algebra. The θ-valent chrysippian ring is a algebraic representation of a non classical chrysippian multivalued logic named the modal θ-valent chrysippian logic [1].

From the modal θ-valent chrysippian logic F. Ayissi Eteme defines in [1] the notions of $m\theta$ sets, of $m\theta$ algebraic structures which allow to define in [3, 4, 5, 6, 7, 8, ...] the notions of $m\theta$ codes. In this note, we intend to look the quantum numbers as elements of the $m\theta$ sets $(\mathbb{N}_{p\mathbb{Z}}, F_\alpha)$, $(\mathbb{Z}_{p\mathbb{Z}}, F_\alpha)$, $(\mathbb{Q}_{p\mathbb{Z}}, F_\alpha)$, $(\mathbb{R}_{p\mathbb{Z}}, F_\alpha)$, $(\mathbb{C}_{p\mathbb{Z}}, F_\alpha)$ and then give a definition of a modal θ-valent chrysippian $(\text{chrm}\theta)$ quantum bit which would allow the implementation of the applications that come from $m\theta$ chrysippian logic as soon as $m\theta$ algebraic structures. In the Section 2, we present the modal θ-valent chrysippian completion of a Lukasiewicz algebra. In the Section 3, we present the intrinsic natural quantum logic of the hydrogen atom. In the Section 4, we define the intrinsic anatomy of the $m\theta$ set $(QSH_{p\mathbb{Z}}, F_\alpha)$. In the Section 5, we give the intrinsic $m\theta$ algebraic structure of the hydrogen atom H. Finally in Section 6, we give a conclusion of this paper.

2. The θ-Valent Chrysippian Completion of a LuKasiewicz θ-Valent Algebra

2.1. The θ-valent Lukasiewicz algebra and the θ-valent chrysippian ring (achθ)

Definition 2.1.1. Let J be a closed chain of ordinal θ and $J_\star = J \setminus \{0\}$.

A θ-valent Lukasiewicz algebra is a structure $(L, \vee, \wedge, 1, 0, (\Phi_\alpha)_{\alpha \in J_\star})$, where

(1) $(L, \vee, \wedge, 1, 0)$ is a closed distributive latice.

(2) $\forall \alpha \in J_\star$, Φ_α is an endomorphisme such that $\Phi_\alpha(0) = 0$ and $\Phi_\alpha(1) = 1$.

(3) $\forall \alpha, \beta \in J_\star$, $\Phi_\alpha \circ \Phi_\beta = \Phi_\beta$.

(4) $\forall \alpha, \beta \in J_\star$, $(\alpha \leq \beta \Rightarrow \Phi_\beta \leq \Phi_\alpha)$.

(5) $(\forall \alpha \in J_\star, \Phi_\alpha(x) = \Phi_\alpha(y)) \Rightarrow x = y$.

(6) $\forall \alpha \in J_\star$, Φ_α is an chrysippian operator (i.e., $\forall x \in L$, $\Phi_\alpha(x)$ is an chrysippian element, i.e., $\exists!\, y \in L$ such that $\Phi_\alpha(x) \wedge y = 0$ and $\Phi_\alpha(x) \vee y = 1$).

Definition 2.1.2. One calls a θ-valent chrysippian ring a tuple $(A, (\Omega_\alpha)_{\alpha \in I_*})$ denoted (A, Ω_α) for short, where

(1) A is a boolean ring.

(2) I is a closed chain 0, 1 of ordinal θ and $I_* = I \setminus \{0\}$.

(3) $\forall \alpha \in I_*$, Ω_α is a boolean endomorphism of A such that $\forall \alpha, \beta \in I_*$, $(\alpha \neq \beta \Rightarrow \Omega_\alpha \neq \Omega_\beta)$.

(4) $\forall\alpha, \beta \in I_*, \Omega_\beta \circ \Omega_\alpha = \Omega_\alpha$.

(5) $(\forall\alpha \in I_*, \Omega_\alpha(x) = \Omega_\alpha(y)) \Rightarrow x = y$.

Definition 2.1.3. Let (A, Ω_α) be a θ-valent chrysippian ring.

(1) An element $x \in A$ is said θ-invariant if $\forall\alpha \in I_*, \Omega_\alpha(x) = x$.

(2) Let B be a sub set of A, B is said θ-invariant if $\forall x \in B$, x is θ-invariant.

Proposition 2.1.1. *Let (A, Ω_α) be a θ-valent chrysippian ring. An element $x \in A$ is θ invariant if $\exists\mu \in I_*, \Omega_\mu(x) = x$.*

Proof. Let $x \in A$, if $\exists\mu \in I_*$ such that $\Omega_\mu(x) = x$. Let $\alpha \in I_*$, $\Omega_\alpha(x) = \Omega_\alpha(\Omega_\mu(x)) = \Omega_\mu(x) = x$.

Theorem 2.1.1. *Let (A, Ω_α) be a θ-valent chrysippian ring.*

Let $LA = \{x \in A / \forall\alpha, \beta \in I_*, (\alpha \leq \beta \Rightarrow \Omega_\beta(x) \leq \Omega_\alpha(x))\}$.

If $\forall\alpha, \beta \in I_*, (\alpha \neq \beta \Rightarrow \Omega_\alpha \neq \Omega_\beta)$, then $(LA, {}^{\Omega_\alpha}|_{LA})$ is a θ-valent LuKasiewicz algebra.

Proof. $0_A, 1_A, \Omega_\alpha(x) \in LA$ for every $\alpha \in I_*, x \in A$; thus $LA \neq \phi$, LA is a sub distributive latice of A

$$\forall\alpha, \beta \in I_*, \Omega_\alpha \neq \Omega_\beta \Rightarrow {}^{\Omega_\alpha}|_{LA} \neq {}^{\Omega_\beta}|_{LA}.$$

2.2. The $m\theta$ chrysippian completion of a θ-valent LuKasiewicz algebra

Let (L, Ω_α) be a θ-valent LuKasiewicz algebra. Let $B = C(L)$ the boolean ring of chrysippian elements of L.

For $x \in L$, let set $x_\Phi = (\Phi_\alpha(x))_{\alpha \in I_*}$ a family of elements of $B : x_\Phi \in B^{I_*}$. The map $x \mapsto x_\Phi$ injects L in B^{I_*}.

B^{I_*} is a boolean ring for its product laws:

– $(\mathscr{U}_\alpha)_\alpha \wedge (\mathscr{V}_\alpha)_\alpha = (\mathscr{U}_\alpha \wedge \mathscr{V}_\alpha)_\alpha$

– $(\mathscr{U}_\alpha)_\alpha \vee (\mathscr{V}_\alpha)_\alpha = (\mathscr{U}_\alpha \vee \mathscr{V}_\alpha)_\alpha$

– $\urcorner(\mathscr{U})_\alpha = (\urcorner \mathscr{U}_\alpha)_\alpha$.

Thus $1_\Phi = (1_\alpha)$ and $0_\Phi = (0_\alpha)$.

For $\alpha \in I_*$, $1_\alpha = 1$ and $0_\alpha = 0$.

We then write $1 = 1_\Phi$ and $0 = 0_\Phi : B \subseteq B^{I_*}$ (B^{I_*}, Ω_α) is a θ-valent chrysippian ring $\forall \alpha \in I_*$, $\forall \mathscr{U} = (\mathscr{U}_\alpha)_\alpha \in B^{I_*}$, $\Omega_\alpha(\mathscr{U}) = \mathscr{U}_\alpha$.

Definition 2.2.1. (B^{I_*}, Ω_α) is called the $m\theta$ chrysippian completion of the θ-valent LuKasiewicz algebra (L, Ω_α) denoted $B^\theta(L)$.

$B^\theta(L) = (B^{I_*}, \Omega_\alpha)$.

Example 2.2.1. Let I be closed chain $0, 1$ of ordinal θ. Let $x \in I$, $\alpha \in I_*$ and $\Phi_\alpha(x) = \begin{cases} 1 & \text{if } \alpha \leq x \\ \\ 0 & \text{if not} \end{cases}$ $I_\theta = (I, \Phi_\alpha)$ is the canonical θ-valent LuKasiewicz algebra defined by I.

$C(I) = \{0, 1\}$.

Notation. The $m\theta$ chrysippian completion of $I_\theta = (I, \Phi_\alpha)$, $B^\theta(I_\theta)$ will be denoted in what follows 2^θ or 2^{θ_n} if I is an n-valent chain with $n > 2$.

3. The Intrinsic Natural Quantum Logic of the Hydrogen Atom

In what follows, H represents the hydrogen atom, one of those atoms known as the simplest; QSH the set of its quantum states; $\mathbb{N}QSH$ that of its natural energy levels: its principal quantum numbers. It is assumed that $\mathbb{N}QSH \cong \mathbb{N}$. Let $p \in \mathbb{N}^*$, $\theta = \theta_p$ its ordinal, p taken say, as the limit visible energy level of the spectrum of H.

Psychologically note SH_0 or SH_Z the set that I call the Z-quantum states of $H : SH_Z$, any state when H does not longer exist from a quantum point of view : the ∞-excitation states of H; SH_1 is the set of ground states of H, say the least excited states of H; SH_2 that of the first excited states after $SH_1, \ldots, SH_{p-1}$ the last excited states of H before SH_Z.

Let $I_p = \{0, 1, \ldots, p-1\}$

$I_{H_p} = \{SH_0, SH_1, \ldots, SH_{p-1}\}$.

Proposition 3.1. *Let* $h : \begin{matrix} I_p \to I_{H_p} \\ j \mapsto SH_j \end{matrix}$ *be a map. Let* $(i, j) \in \{1; \ldots; p-1\} \times \{0; 1; \ldots; p-1\}$ *and* $\varphi_i(j) = \begin{cases} p-1 & \text{if } i \leq j, \\ 0 & \text{if not.} \end{cases}$

(1) $(I_p; \varphi_i)$ *is a* θ*-valent LuKasiewicz algebra that h-induces a* θ*-valent same structure on* I_{H_p} *as follows*:

$$\forall(\alpha, \beta) \in \{1; \ldots; p-1\} \times \{0; 1; \ldots; p-1\},\ \varphi_\alpha(SH_\beta) = \begin{cases} SH_{p-1} & \text{if } \alpha \leq \beta, \\ SH_0 & \text{if not.} \end{cases}$$

(2) $(I_{H_p}; \varphi_\alpha)$ *is a* θ*-valent LuKasiewicz chain.*

Proof. h bijects the natural order of I_p over I_{H_p}, and then the proof results.

Definition 3.1. $(I_{H_p}; \varphi_\alpha)$ is called the Moisil θ-valent chain of energy levels of H.

Notation. Let denote $M_{I_{H_p}} = (I_{H_p}; \varphi_\alpha)_{\alpha=1; \ldots; p-1}$.

Proposition 3.2. *Let* $chrm\theta I_{H_p} = (B^\theta(I_{H_p}), \omega_\alpha)$ *the* $m\theta$ *chrysippian completion of* $M_{I_{H_p}}$ $chrm\theta I_{H_p} \overset{\theta}{\cong} 2^\theta$.

Proof. It comes by definition.

Definition 3.2. $\text{chrm}\theta I_{H_p}$ is called the $\text{chrm}\theta$ completion of natural $\text{chrm}\theta$ energy levels of H: The intrinsic natural logic of energy levels of H.

Proposition 3.3. (1) $x \in M_{I_{H_p}} \Rightarrow (\alpha \leq \beta \Rightarrow \varphi_\beta(x) \leq \varphi_\alpha(x))$.

(2) $x \in \text{chrm}\theta I_{H_p} \setminus M_{I_{H_p}} \Rightarrow \exists \mu \lneqq \mu'$ in $\{1, \ldots, p-1\}$, $\omega_\mu(x) \lneqq \omega_{\mu'}(x)$.

Proof. Results from the definition of $M_{I_{H_p}}$ and $\text{chrm}\theta I_{H_p}$.

Remark. With QSH the set of all quantum states of H; the $\text{chrm}\theta$ logic of energy levels of H:

$\text{chrm}\theta I_{H_p} = B^\theta((I_{H_p}), \omega_\alpha) \overset{\theta}{\cong} 2^\theta$ associates an intrinsic natural $\text{chrm}\theta$ set structure $QSH_{p\mathbb{Z}}$, the $m\theta$ set of $m\theta$ quantum states of H, better say of $H_{pZ} : (QSH_{p\mathbb{Z}}, F_\alpha)$ about what structure very many observations should worth making.

4. The Intrinsic Anatomy of $(QSH_{p\mathbb{Z}},\ F_{\alpha})$

If it is admitted that an energy state x_H in QSH of H is known if its electronic layer repaired by $n \in \mathbb{N}$, its sub layer l of the layer n, $0 \leq l \leq n-1$, and its state m in the sub layer

$l : m = -l, -l+1, \ldots, -1, 0, 1, \ldots, l$ are all known , one defines:

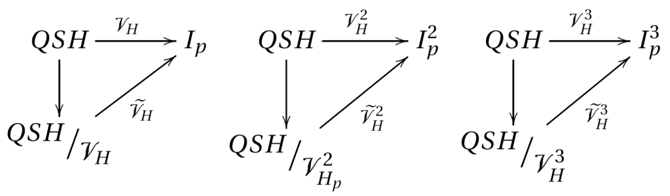
$\mathscr{V}_H : \begin{array}{ccc} QSH & \to & I_p \\ x_H & \mapsto & \mathscr{V}_H(x_H) = n \end{array}$, where n is the principal quantum number of x_H,

$$\mathscr{V}_H^2 : \begin{array}{ccc} QSH & \to & I_p^2 \\ x_H & \mapsto & (\mathscr{V}_H(x),\ l) = \mathscr{V}_H^2(x_H) \end{array} ;\ 0 \leq l \leq n-1.$$

$$\mathscr{V}_H^3 : \begin{array}{ccc} QSH & \to & I_p^3 \\ x_H & \mapsto & \mathscr{V}_H^3(x_H) = (\mathscr{V}_H^2(x),\ m) = (n,\ l,\ m) \end{array};$$

$$m = -l, -l+1, \ldots, -1, 0, 1, \ldots, l.$$

The following diagrams are commutative:



By definition $\mathscr{V}_H(\mathscr{V}_H^2,\ \mathscr{V}_H^3)$ is surjective and $\widetilde{\mathscr{V}}_H\ (\widetilde{\mathscr{V}}_H^2,\ \widetilde{\mathscr{V}}_H^3)$ injective by construction. Thus $\widetilde{\mathscr{V}}_H\ (\widetilde{\mathscr{V}}_H^2,\ \widetilde{\mathscr{V}}_H^3)$ bijects $QSH/\mathscr{V}_H\left(QSH/\mathscr{V}_H^2,\ QSH/\mathscr{V}_H^3\right)$ over $I_p(I_p^2,\ I_p^3)$.

Therefore since the chrmθ extension of I_p is chrmθ$I_p \overset{\theta}{\cong} 2^{\theta}$, then chrmθ$^{QSH}/_{\mathscr{V}_H}\left({}^{QSH}/_{\mathscr{V}_H^2},\ {}^{QSH}/_{\mathscr{V}_H^3}\right)$ thus is well defined and then, say: chrmθ $^{QSH}/_{\mathscr{V}_H} \overset{\theta}{\cong} 2^{\theta}\ (\widetilde{\mathscr{V}}_{H_p})$; chrmθ $^{QSH}/_{\mathscr{V}_H^2} \overset{\theta}{\cong} 2^{\theta^2}\ (\widetilde{\mathscr{V}}^2_{H_p})$; chrmθ $^{QSH}/_{\mathscr{V}_H^3} \overset{\theta}{\cong} 2^{\theta^3}\ (\widetilde{\mathscr{V}}^3_{H_p})$.

$\widetilde{\mathscr{V}}_{H_p}\ (\widetilde{\mathscr{V}}^2_{H_p},\ \widetilde{\mathscr{V}}^3_{H_p})$ spec$_{p_{\mathbb{Z}}}m\theta$ extends to respectively:

$$\widetilde{\mathscr{V}}_{H_{p\mathbb{Z}}} : {}^{QSH_{p\mathbb{Z}}}/_{V_{H_{p\mathbb{Z}}}} \to (2^{\theta^{p-1}})^{p-1}$$

$$\widetilde{\mathscr{V}}^2_{H_{p\mathbb{Z}}} : {}^{QSH_{p\mathbb{Z}}}/_{\mathscr{V}^2_{H_{p\mathbb{Z}}}} \to (2^{\theta^{p-1}})^{2(p-1)}$$

$$\widetilde{\mathscr{V}}^3_{H_{p\mathbb{Z}}} : {}^{QSH_{p\mathbb{Z}}}/_{\mathscr{V}^3_{H_{p\mathbb{Z}}}} \to (2^{\theta^{p-1}})^{3(p-1)}$$

as follows.

If $\mathscr{V}_H(x_H) = n = \sigma_n = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ 0 \\ \vdots \\ \\ 0 \end{pmatrix}$ n-times, $\sigma_n \in 2^{\theta}$. Identifying 1 to

$$1_{I_p} = p-1, \text{thus } \sigma_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \dots, \sigma_{p-1} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} \in 2^{\theta}, n \in I_p \Leftrightarrow n_{p\mathbb{Z}}$$

$\in (I_p)_{p\mathbb{Z}} = \{0, 1_{p\mathbb{Z}}, \ldots, (p-1)_{p\mathbb{Z}}\}$. Define $\sigma_{n_{p\mathbb{Z}}}$ as $\sigma_{n_{p\mathbb{Z}}} = \begin{pmatrix} 1_{p\mathbb{Z}} \\ \vdots \\ 1_{p\mathbb{Z}} \\ 0 \\ \vdots \\ 0 \end{pmatrix}$ n-times

after what, observe that so written, $\sigma_{n_{p\mathbb{Z}}}$ economically should really be identified to $\sigma_{n_{p\mathbb{Z}}} = \begin{pmatrix} 1_{p\mathbb{Z}} \\ \vdots \\ 1_{p\mathbb{Z}} \end{pmatrix}$ n-times and this way

$$\sigma_{1_{p\mathbb{Z}}} = (1_{p\mathbb{Z}}) = (1, 2, \cdots, p-1)$$

$$= (\sigma_1, \sigma_2, \cdots, \sigma_{p-1}) \in 2^{\theta^{p-1}}$$

$$\sigma_{2_{p\mathbb{Z}}} = \begin{pmatrix} \sigma_1 & \cdots & \sigma_{p-1} \\ \sigma_1 & \cdots & \sigma_{p-1} \end{pmatrix} \in (2^{\theta^{p-1}})^2$$

$$\sigma_{n_{p\mathbb{Z}}} = \begin{pmatrix} \sigma_1 & \cdots & \sigma_{p-1} \\ \vdots & & \vdots \\ \sigma_1 & \cdots & \sigma_{p-1} \end{pmatrix} \in (2^{\theta^{p-1}})^n \subseteq (2^{\theta^{p-1}})^{p-1}.$$

Thus, $\forall n \in I_p, \sigma_{n_{p\mathbb{Z}}} \in (2^{\theta^{p-1}})^{p-1}$, indeed if $\widetilde{\mathscr{V}}_{H_p}(x_H) = n = \sigma_n \in 2^{\theta}$, then $\widetilde{\mathscr{V}}_{H_{p\mathbb{Z}}}(x_{H_{p\mathbb{Z}}}) = n_{p\mathbb{Z}} = \sigma_{n_{p\mathbb{Z}}} \in (2^{\theta^{p-1}})^{p-1}$.

If $\mathscr{V}_H^2 : QSH \to I_p^2 : \widetilde{\mathscr{V}}_{H_p}^2 \quad (x_H) = (n, l) = (\sigma_n, \sigma_l) \in 2^{\theta^2};\ 0 \le l \le n-1,$

then

$\widetilde{\mathscr{V}}_{H_{p\mathbb{Z}}}^2 (x_{H_{p\mathbb{Z}}}) = (\sigma_{n_{p\mathbb{Z}}}, \sigma_{l_{p\mathbb{Z}}})$

$$= \left(\begin{pmatrix} \sigma_1 & \sigma_{p-1} \\ \vdots & \vdots \\ \sigma_1 & \sigma_{p-1} \end{pmatrix} n\text{-times}, \begin{pmatrix} \sigma_1 & \sigma_{p-1} \\ \vdots & \vdots \\ \sigma_1 & \sigma_{p-1} \end{pmatrix} l\text{-times} \right) \in (2^{\theta^{p-1}})^{n+l} \subseteq$$

$(2^{\theta^{p-1}})^{2(p-1)}$.

If

$$\widetilde{\mathscr{V}}_{H_p}^3 (x_H) = (n, l, m),\ 0 \le l \le n-1;\ m = -l, -l+1, \ldots, -1, 0, 1, \ldots, l$$

$$= (\sigma_n, \sigma_l, \sigma_m) \in 2^{\theta^3},$$

then $\widetilde{\mathscr{V}}_{H_{p\mathbb{Z}}}^3 (x_{H_{p\mathbb{Z}}}) = (\sigma_{n_{p\mathbb{Z}}}, \sigma_{l_{p\mathbb{Z}}}, \sigma_{m_{p\mathbb{Z}}}) \in (2^{\theta^{p-1}})^{n+l+m}$, thus $\widetilde{\mathscr{V}}_{H_{p\mathbb{Z}}}^3$ $(x_{H_{p\mathbb{Z}}}) \in (2^{\theta^{p-1}})^{3(p-1)}$.

Theorem 4.1. *The following diagrams are commutative*:

$$\text{chrm}\theta \quad \begin{array}{ccc} QSH/\mathcal{V}_H & \xrightarrow{\widetilde{\mathcal{V}}_{H_p}} & 2^{\theta} \\ \big\downarrow spec_{p\mathbb{Z}} & & \big\downarrow spec_{p\mathbb{Z}} \\ QSH_{p\mathbb{Z}}/\mathcal{V}_{H_{p\mathbb{Z}}} & \xrightarrow[\widetilde{\mathcal{V}}_{H_{p\mathbb{Z}}}]{} & (2^{\theta^{p-1}})^{p-1} \end{array}$$

$$\text{chrm}\theta \quad \begin{array}{ccc} QSH/\mathcal{V}_H^2 & \xrightarrow{\widetilde{\mathcal{V}}_{H_p}^2} & 2^{\theta^2} \\ \big\downarrow spec_{p\mathbb{Z}} & & \big\downarrow spec_{p\mathbb{Z}} \\ QSH_{p\mathbb{Z}}/\mathcal{V}_{H_{p\mathbb{Z}}}^2 & \xrightarrow[\widetilde{\mathcal{V}}_{H_{p\mathbb{Z}}}^2]{} & (2^{\theta^{p-1}})^{2(p-1)} \end{array}$$

$$\text{chrm}\theta \quad \begin{array}{ccc} QSH/\mathcal{V}_H^3 & \xrightarrow{\widetilde{\mathcal{V}}_{H_p}^3} & 2^{\theta^3} \\ \downarrow {\scriptstyle spec_{p\mathbb{Z}}} & & \downarrow {\scriptstyle spec_{p\mathbb{Z}}} \\ QSH_{p\mathbb{Z}}/\mathcal{V}_{H_{p\mathbb{Z}}}^3 & \xrightarrow[\widetilde{\mathcal{V}}_{H_{p\mathbb{Z}}}^3]{} & \left(2^{\theta^{p-1}}\right)^{3(p-1)} \end{array}$$

Proof. Results from the fact $\widetilde{\mathscr{V}}_{H_p}, \widetilde{\mathscr{V}}^2_{H_p}, \widetilde{\mathscr{V}}^3_{H_p}, \widetilde{\mathscr{V}}_{H_{p\mathbb{Z}}}, \widetilde{\mathscr{V}}^2_{H_{p\mathbb{Z}}}, \widetilde{\mathscr{V}}^3_{H_{p\mathbb{Z}}}$ are the chrmθ isomorphisms.

Definition 4.1. $\forall x_H \in QSH$, $\forall x_{H_{p\mathbb{Z}}} \in QSH_{p\mathbb{Z}}$.

(1) Call $\mathscr{V}_{H_p}(x_H)(\mathscr{V}^2_{H_p}(x_H), \mathscr{V}^3_{H_p}(x_H))$ the quantum value (degree) of the quantum state x_H of $H : \mathscr{V}_{H_p}(x_H) \in 2^{\theta}, \mathscr{V}^2_{H_p}(x_H) \in 2^{\theta^2}, \mathscr{V}^3_{H_p}(x_H) \in 2^{\theta^3}$.

(2) Call $\mathscr{V}_{H_{p\mathbb{Z}}}(x_{H_{p\mathbb{Z}}})(\mathscr{V}^2_{H_{p\mathbb{Z}}}(x_{H_{p\mathbb{Z}}}), \mathscr{V}^3_{H_{p\mathbb{Z}}}(x_{H_{p\mathbb{Z}}}))$ the chrmθ quantum state $x_{H_{p\mathbb{Z}}}$ of $H_{p\mathbb{Z}}$. Therefore;

If $\mathscr{V}_H(x_H) = n \in I_p$, $\widetilde{\mathscr{V}}_{H_p}(x_H) \in 2^{\theta}$, then $\widetilde{\mathscr{V}}_{H_{p\mathbb{Z}}}(x_{H_{p\mathbb{Z}}}) \in (2^{\theta^{p-1}})^n \subseteq (2^{\theta^{p-1}})^{p-1}$.

If $\mathscr{V}^2_H(x_H) = (n, l) \in I^2_p, \widetilde{\mathscr{V}}^2_{H_p}(x_H) \in 2^{\theta^2}$, then $\widetilde{\mathscr{V}}^2_{H_{p\mathbb{Z}}}(x_{H_{p\mathbb{Z}}}) \in (2^{\theta^{p-1}})^{n+l} \subseteq (2^{\theta^{p-1}})^{2(p-1)}$.

If $\mathscr{V}^3_H(x_H) = (n, l, m) \in I^3_p$, $\widetilde{\mathscr{V}}^3_{H_p}(x_H) \in 2^{\theta^3}$, then $\widetilde{\mathscr{V}}^3_{H_{p\mathbb{Z}}}(x_{H_{p\mathbb{Z}}}) \in (2^{\theta^{p-1}})^{n+l+m} \subseteq (2^{\theta^{p-1}})^{3(p-1)}$.

(3) Define a chrmθ quantum bit as the atom of chrmθ quantum value (degree) of chrmθ quantum states: any σ of 2^{θ}.

(4) Define a chrmθ θ-quantum bit as any $\sigma \in 2^{\theta^{p-1}}$, with $\forall \alpha, \omega_\alpha^{p-1}(\sigma) \in 2^\theta$.

Observation

Let M_{QSH}^{θ} the Moisil θ-valent set of all quantum states of H and chrmθ QSH the set of chrmθ quantum states of H.

Obviously any quantum state is a chrmθ quantum state. Nevertheless any chrmθ quantum state that is not in M_{QSH}^{θ} is not a natural quantum state, but its modalities are quantum states.

There should be as number chrmθ quantum values all σ of any of $2^\theta, 2^{\theta^2}, 2^{\theta^3}, (2^{\theta^{p-1}})^n, (2^{\theta^{p-1}})^{n+l}, (2^{\theta^{p-1}})^{n+l+m}$; $0 \le l \le n-1$; $-l \le m \le l$; $n \in I_p$.

Definition 4.2. Let $q_{H_{p\mathbb{Z}}}$ be a chrmθ quantum state. One calls negation of $q_{H_{p\mathbb{Z}}}$, notation $\rceil q_{H_{p\mathbb{Z}}}$, any element of $QSH_{p\mathbb{Z}}$ with chrmθ quantum value $\rceil \widetilde{\mathscr{V}}_{H_{p\mathbb{Z}}}(q_{H_{p\mathbb{Z}}})(\rceil \widetilde{\mathscr{V}}^2_{H_{p\mathbb{Z}}}(q_{H_{p\mathbb{Z}}}), \rceil \widetilde{\mathscr{V}}^3_{H_{p\mathbb{Z}}}(q_{H_{p\mathbb{Z}}}))$.

Remark. Let $q_H \in M_{QSH}^{\theta}$, $\rceil q_H \in \text{chrm}\theta\, QSH \setminus M_{QSH}^{\theta}$: $\rceil q_H$ is not a natural quantum state.

Theorem 4.2 (Characterization of a chrmθ quantum state *x*). *Let* $x \in QSH_{p\mathbb{Z}}$,

(1) $x \in M_{QSH}^{\theta} \Rightarrow (\forall \alpha, \beta \in \{1, \dots, p-1\}, \alpha \le \beta \Rightarrow \omega_\beta(\mathscr{V}_{H_{p\mathbb{Z}}}(x)) \le \omega_\alpha(\mathscr{V}_{H_{p\mathbb{Z}}}(x)))$.

(2) $x \in QSH_{p\mathbb{Z}} \setminus M_{QSH}^{\theta} \Rightarrow (\exists \mu \lneqq \mu'; 1, \dots, p-1 : \omega_\mu(\mathscr{V}_{H_{p\mathbb{Z}}}(x)) \gneqq \omega_{\mu'}(\mathscr{V}_{H_{p\mathbb{Z}}}(x)))$.

Proof. It comes by the definition of 2^{θ} and of $(2^{\theta^{p-1}})^k$, $k \in \mathbb{N}^*$.

5. The Intrinsic Algebraic Structure of *H*

Let denote by (D_2) the following diagram:

$$\begin{array}{ccccccccccc}
\{0;1\} & \longrightarrow & \mathbb{N} & \longrightarrow & \mathbb{Z} & \longrightarrow & \mathbb{Q} & \longrightarrow & \mathbb{R} & \longrightarrow & \mathbb{C} \\
{\scriptstyle Spec_{p\mathbb{Z}}}\downarrow & & {\scriptstyle Spec_{p\mathbb{Z}}}\downarrow & & {\scriptstyle Spec_{p\mathbb{Z}}}\downarrow & & {\scriptstyle Spec_{p\mathbb{Z}}}\downarrow & & {\scriptstyle Sp_{p\mathbb{Z}}ec}\downarrow & & {\scriptstyle Spec_{p\mathbb{Z}}}\downarrow \\
2^{\theta} & \longrightarrow & \mathbb{N}_{p\mathbb{Z}} & \longrightarrow & \mathbb{Z}_{p\mathbb{Z}} & \longrightarrow & \mathbb{Q}_{p\mathbb{Z}} & \longrightarrow & \mathbb{R}_{p\mathbb{Z}} & \longrightarrow & \mathbb{C}_{p\mathbb{Z}}.
\end{array}$$

The commutativity of (D_2) is presented in [2] (P.1).

Let $q_H \in QSH$, thus $q_{H_{p\mathbb{Z}}} \in (QSH_{p\mathbb{Z}}, F_{\alpha})$. If $\mathcal{V}^{3}_{H_p}(q_H)) = (\sigma_n, \sigma_l, \sigma_m) \in 2^{\theta^3}$, $n \in I_p$; $l \in \{0, 1, \ldots, m-1\}$; $m = -l, \ldots, 0, 1, \ldots, l$, then $\mathcal{V}^{3}_{H_{p\mathbb{Z}}}(q_{H_{p\mathbb{Z}}})) = (\sigma_{n_{p\mathbb{Z}}}, \sigma_{l_{p\mathbb{Z}}}, \sigma_{m_{p\mathbb{Z}}}) \in (2^{\theta^{p-1}})^{n+l+m}$.

$\forall \alpha \in \{1, \ldots, p-1\}$, $F_{\alpha}(n_{p\mathbb{Z}})$ represents the quantum energy of an electron and the size of the orbital, $F_{\alpha}(l_{p\mathbb{Z}})$ represents the shape of an orbital with a particular principal quantum number $F_{\alpha}(n_{p\mathbb{Z}})$ and $F_{\alpha}(m_{p\mathbb{Z}})$ specifies the orientation in space of an orbital of a given energy $F_{\alpha}(n_{p\mathbb{Z}})$ and shape $F_{\alpha}(l_{p\mathbb{Z}})$.

It then appears that the following diagram:

$$\begin{array}{ccccccccc}
QSH_{\mathbb{N}} & \longrightarrow & QSH_{\mathbb{Z}} & \longrightarrow & QSH_{\mathbb{Q}} & \longrightarrow & QSH_{\mathbb{R}} & \longrightarrow & QSH_{\mathbb{C}} \\
{\scriptstyle Spec_{p\mathbb{Z}}}\downarrow & & {\scriptstyle Spec_{p\mathbb{Z}}}\downarrow & & \downarrow & & {\scriptstyle Spec_{p\mathbb{Z}}}\downarrow & & \downarrow \\
QSH_{\mathbb{N}_{p\mathbb{Z}}} & \longrightarrow & QSH_{\mathbb{Z}_{p\mathbb{Z}}} & \longrightarrow & QSH_{\mathbb{Q}_{p\mathbb{Z}}} & \longrightarrow & QSH_{\mathbb{R}_{p\mathbb{Z}}} & \longrightarrow & QSH_{\mathbb{C}_{p\mathbb{Z}}}
\end{array}$$

formally from a quantum point of view, may be taken as a quantum commutative diagram.

Definition 5.1. (1) Call chrmθ monoïd of chrmθ natural quantum states of the atom H the chrmθ structure $H_{p\mathbb{Z}} = (QSH_{p\mathbb{Z}}, F_{\alpha})$.

(2) Call chrmθ atom with the frame the atom H, the following couple of chrmθ structures $H_{\theta} = \left(\left(\left(2^{\theta^{p-1}}\right)^{3(p-1)}, \omega_{\alpha}^{3(p-1)^3}\right), H_{p\mathbb{Z}}\right)$.

6. Conclusion

According to all what proceeds we can conclude that the atom H has as an intrinsic natural logic, the chrmθ logic whose $(2^{\theta}, \omega_{\alpha})$ is its algebraic representation : its natural intrinsic quantum logic that induces $QSH_{p\mathbb{Z}}$ its chrmθ structure of quantum values.

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