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MATHEMATICAL ANALYSIS OF AN ECOLOGICAL MODEL FOR ASSESSING THE EMISSION OF AIR POLLUTANTS

MD. ABDUL ALIM[1], MILON ROY[2], MD. SIRAJUL ISLAM[2], TAHERA PARVIN[3] and MD. HAIDER ALI BISWAS[3]

[1]Department of Mathematics
University of Chittagong
Chittagong-4331
Bangladesh

[2]Department of Mathematics
BSMRSTU
Gopalganj-8100
Bangladesh

[3]Mathematics Discipline
Khulna University
Khulna-9208
Bangladesh
e-mail: mhabiswas@yahoo.com

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Abstract

Recently, air pollution has become a major threat problem and a threat for human and with the increasing environmental awareness much attention has been paid to the environmental toxic. Mathematical models play an important role for understanding the ecological mechanisms. From this point of view, emission of pollutants dynamics has become a target for understanding the nature of air pollution. In this paper, we propose a non linear mathematical model to assess the emission of air pollutants, produced from the coal based power plant, from vehicles and from oil and fuel burning. We study the process using a four compartmental ecological model which is generally used for modelling dynamic ecosystems and process to assess the concentration of greenhouse gases like CO_2. The model has been analyzed both analytically and numerically. The analytical findings have been validated with the numerical simulations.

1. Introduction

Air pollution may cause diseases, allergies and even death to humans. Both human activity and natural processes can generate air pollution. Global warming and climate change are the contemporary threats to ecosystem services and biodiversity that has a huge impact on the environment, livelihood of communities and economics across the world. With rapid urbanization, industrialization, increased population and significant change in lifestyle, the burning of coal, oil and natural gas has emitted approximately 500 billion tons of carbon dioxide, around half of which remains in the atmosphere [1, 18-20]. In 2014, according to World Health Organization report, air pollution in 2012 caused the deaths of around 7 million people worldwide. Worldwide ambient air pollution accounts for: (a) 29% of all deaths and disease from lung cancer, (b) 17% of all deaths and disease from acute lower respiratory infection, (c) 24% of all deaths from stroke, (d) 25% of all deaths and disease from ischemic heart disease (e) 43% of all deaths and disease from chronic obstructive pulmonary disease. National Ambient Air Quality Standards (NAAQS) were established by the Clean Air Act Amendments of 1970, which required EPA to set air-quality standards for specific pollutants such as PM2.5 and ozone to protect the health of the general public and populations most at risk for pollutant-related adverse health outcomes. Air pollution has become a major health problem worldwide as it leads to 4.2 million deaths every year [2]. Burning coal is a leading cause of smog, acid rain, global warming, and air toxics. In an average year, a typical coal plant generates: 3,700,000 tons of carbon dioxide (CO_2), the primary human cause of global warming as much carbon dioxide as cutting down 161 million trees; 10,000 tons of sulphur dioxide (SO_2), which causes acid rain that damages forests, lakes, and buildings, and forms small airborne particles that can penetrate deep into lungs; 500 tons of small airborne particles, which can cause chronic bronchitis, aggravated asthma, and premature death, as well as haze obstructing visibility; 10,200 tons of nitrogen oxide (NO), as much as would be emitted by half a million late-model cars. NO leads to formation of ozone, which inflames the lungs, burning through lung tissue making people more susceptible to respiratory illness; 720 tons of carbon monoxide (CO), which causes headaches and place additional stress on people with heart disease.

2. Basic Assumptions and Mathematical Formulation

We assume that the total air pollutants emitted from the different sources are distributed into several ways. As the environment around the sources is considered as the first initial compartment for storing the air pollutants which is named by emission, population, motor vehicles and factories are the other main components, we can define the distribution of the emission, concentration and absorption of air pollutants as shown in Figure 2.1.

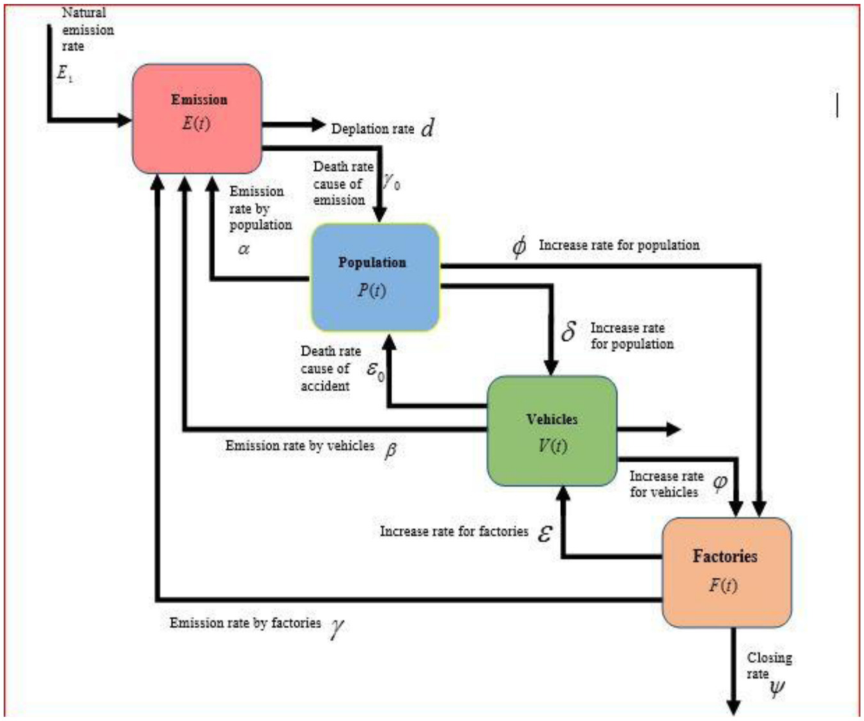

Figure 2.1. Schematic distribution diagram of the emission, concentration and absorption of air pollutants from different sources.

3. Model Formulation

Mathematical models play important roles in describing and assessing the complex behaviors and insights of dynamical systems. We refer readers to [3-11] for some recent developments on modeling in diverse applications. From these motivational works, we now formulate our proposed model. Considering the assumptions described in Figure 2.1. We can describe this process by a four-compartmental mathematical model in terms of a system of nonlinear ordinary differential equations as follow:

$$\frac{dE}{dt} = E_1 + \alpha PE + \beta VE + \gamma FE - dE, \tag{3.1}$$

$$\frac{dP}{dt} = \eta P - \gamma_0 PE - \varepsilon_0 PV, \tag{3.2}$$

$$\frac{dV}{dt} = \delta PV + \varepsilon FV - fV, \tag{3.3}$$

$$\frac{dF}{dt} = \phi FP + \varphi VF - \psi F, \tag{3.4}$$

with initial conditions

$$E(0) = E_0 \geq 0;\ P(0) = P_0 \geq 0;\ V(0) = V_0 \geq 0;\ F(0) = F_0 \geq 0. \tag{3.5}$$

The variables and parameters used in the model (3.1)-(3.5) are summarized in Table 3.1.

Table 3.1. Description of symbols and parameters used in this model

Symbols	**Descriptions**
$E(t)$	Total emission
$P(t)$	Total population
$V(t)$	Total number of vehicles
$F(t)$	Total number of factories
E_1	Natural emission rate
α	Emission rate due to population
β	Emission rate due to vehicle
γ	Emission rate due to factories
d	Natural depletion rate
δ	Increasing rate of vehicles due to population
ε	Increasing rate of vehicles due to factories
f	Decreasing rate of vehicles due to accidental and damaging
η	Natural growth rate of population
γ_0	Death rate due to emission
ε_0	Death rate due to vehicle
ϕ	Increasing rate of factories due to population
φ	Increasing rate of factories due to vehicle
ψ	Closing rate of factories

4. Model Analysis

The objective of this analysis is to find and verify the amount of total emission produced from population, vehicles and factories, for that a closed set has been considered as $\Phi = \left\{(E(t),\ P(t),\ V(t),\ F(t)) \in \boldsymbol{R}_4^+\right\}$, with the initial conditions $E(0) \geq 0,\ P(0) \geq 0,\ V(0) \geq 0,\ F(0) \geq 0$.

4.1. Analytical analysis

Now through the positivity of solutions of the model in the Theorem 1, it is easy to prove that all the variables in the model of system of equations are positive.

Theorem 1. *If* $E(0) \geq 0,\ P(0) \geq 0,\ V(0) \geq 0,\ F(0) \geq 0,$ *then the solutions* $E(t), P(t), V(t)$ *and* $F(t)$ *of the model* (3.1)-(3.5) *presented by the system of equations are positive.*

4.2. Stability analysis

By applying Jaciobian Technique, the system of nonlinear ordinary differential equations (3.1)-(3.5) has three equilibrium points (i) Vehicle free equilibrium point, $E_V\left(\frac{\eta}{\gamma_0}, \frac{\psi}{\phi}, 0, \frac{1}{\gamma\eta\phi}(d\eta\phi - E_1\gamma_0\phi - \alpha\psi\eta)\right)$ is stable, (ii) Industries or factories free equilibrium point, $E_F\left(E^*, \frac{f}{\delta}, \frac{\eta - \gamma_0 E^*}{\varepsilon_0}, 0\right)$ is asymptotically stable if $d < \frac{\alpha f}{\delta}$ and $\psi < \frac{\phi f}{\delta}$, (iii) co-existence equilibrium point, $E = E\left(\frac{1}{\gamma_0}\left(\eta - \frac{\psi\varepsilon_0}{\varphi} + \frac{f\varepsilon_0\phi}{\delta\varphi} - \left(\frac{\phi\varepsilon_0\gamma}{\delta}\right)M\right), \frac{f}{\delta} - \left(\frac{\gamma\varphi}{\delta}\right)M,\right.$ $\left.\frac{\psi}{\varphi} - \frac{f\phi}{\delta\varphi} + \left(\frac{\phi\gamma}{\delta}\right)M, \left(\frac{\gamma\varphi}{\varepsilon}\right)M\right)$ is stable if $\phi = \frac{\alpha\varphi}{\beta}$, $d \geq \frac{\beta\psi}{\varphi}$ and

$$\lambda_1 = \alpha \left(\frac{f}{\delta} - \left(\frac{\gamma\varphi}{\delta}\right)M\right) + \beta\left(\frac{\psi}{\varphi} - \frac{f\phi}{\delta\varphi} + \left(\frac{\phi\gamma}{\delta}\right)M\right) + \gamma\left(\frac{\gamma\varphi}{\varepsilon}\right)M - d < 0,$$

$$\lambda_2 = \delta\phi\left(\frac{f}{\delta} - \left(\frac{\gamma\varphi}{\delta}\right)M\right) + \varepsilon\phi\left(\frac{\gamma\varphi}{\varepsilon}\right)M - f\phi - \delta\varphi\left(\frac{\psi}{\varphi} - \frac{f\phi}{\delta\varphi} + \left(\frac{\phi\gamma}{\delta}\right)M\right) < 0,$$

where

$$E^* = \frac{-B \pm \sqrt{B^2 + 4C}}{2},$$

$$B = \frac{d\varepsilon_0}{\gamma_0\beta} - \frac{\eta}{\gamma_0} - \frac{\alpha f\varepsilon_0}{\gamma_0\beta\delta}, \; C = \frac{\varepsilon_0 E_1}{\gamma_0\beta},$$

$$M = \frac{\beta\eta\varepsilon - \alpha f\psi + \beta f\psi\varepsilon}{\alpha\eta\delta\varphi - \beta\phi\eta\gamma}.$$

5. Numerical Simulation

We now solve the model numerically based on the respective parameters present in the system of equations (3.1-3.5). The simulations are carried out using MATLAB programming language by the set of parameter values and initial conditions. The purpose of this simulation is to investigate the behaviorus of our model and describe the dynamics of the emission of air pollutants. Figure 5.1 shows that all class of variable at the natural growth rate of population and natural emission rate are $\eta = 0.012$ and $E_1 = 0.05$.

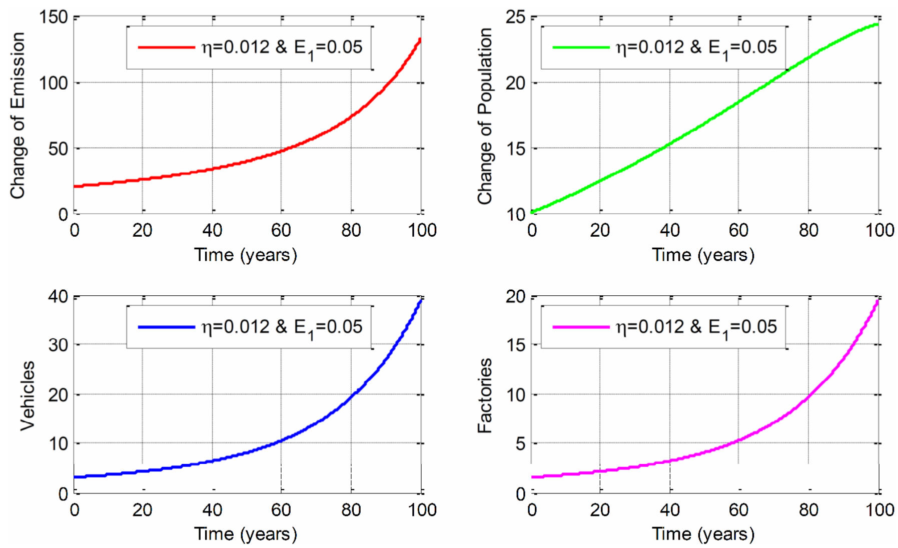

Figure 5.1. Profile of dynamic variables at $\eta = 0.012$ and $E_1 = 0.05$.

Figure 5.2 shows that when the natural emission decreases from 0.05 to 0.02 the level for the amount of total emission of air pollution is decreases in small amount from 130 to 122 after 100 years. And the population is increases with small change from 24 to 25. And the vehicles and the factories also increases with small changes.

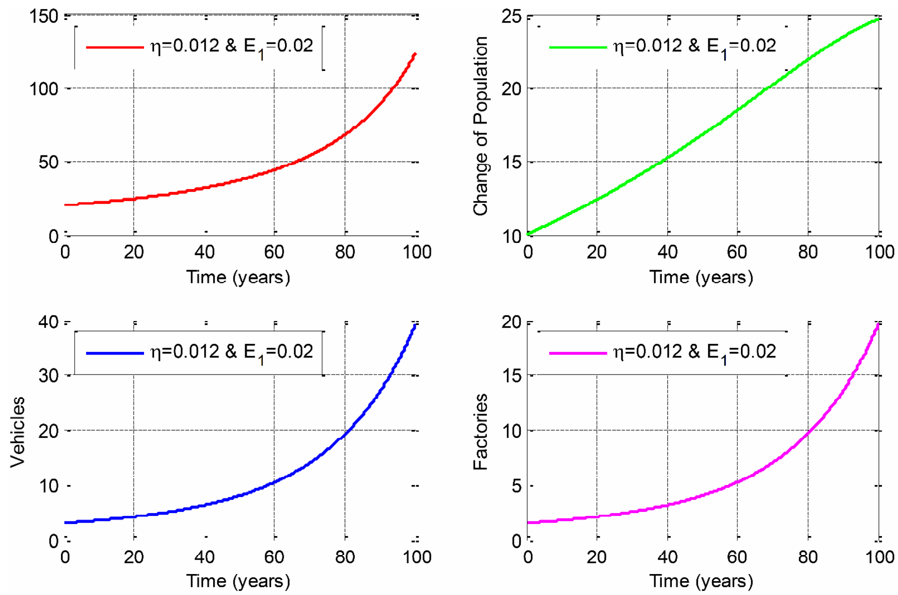

Figure 5.2. Profile of dynamic variables at $\eta = 0.012$ and $E_1 = 0.02$.

Figure 5.3 shows that when the natural emission is fixed and the natural growth rate of population is increases from 0.012 to 0.011 the population after 100 years become 22.5 and the corresponding emission also be decreases from 130 to 120. And the vehicles and the factories also be decreases. And Figure 5.4 shows that when the natural emission and the natural growth rate of population both are changes from higher rate to lower rate (0.012 to 0.011 for population and 0.05 to 0.02 for emission) then all the variables are become minimum stage.

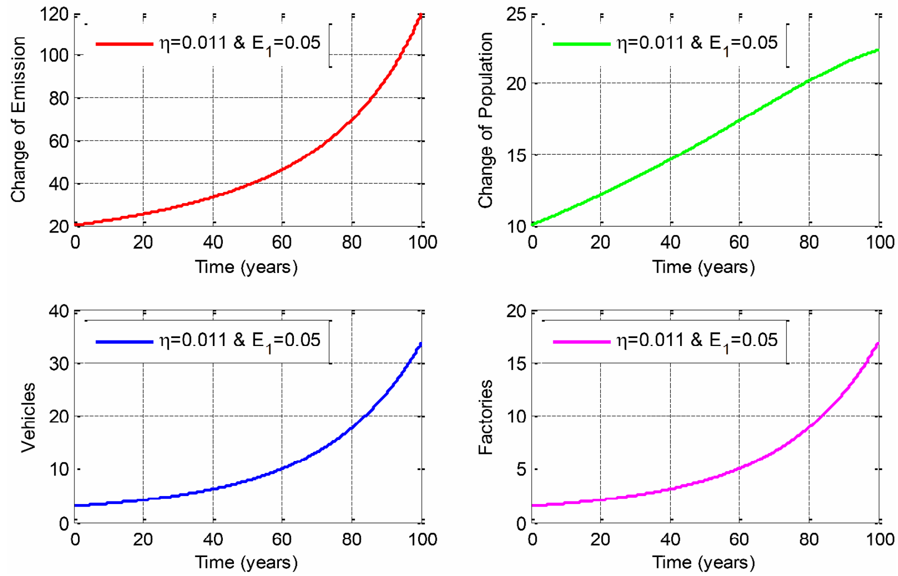

Figure 5.3. Profile of dynamic variables at $\eta = 0.011$ and $E_1 = 0.05$.

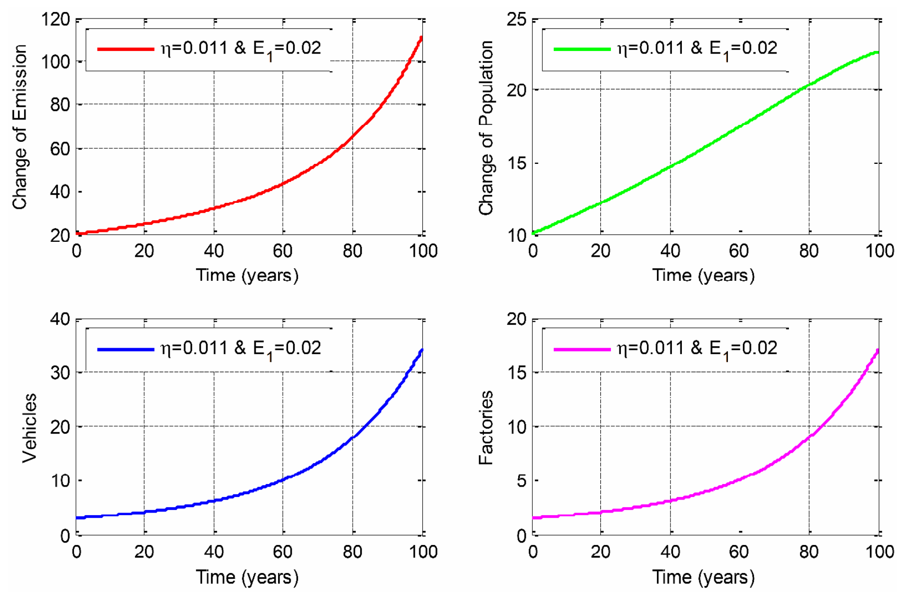

Figure 5.4. Profile of dynamic variables at $\eta = 0.011$ and $E_1 = 0.02$.

If we perform the numerical study by decreasing the natural growth rate of population then first it effect on the total population and then it also effect on the emission is shown in Figures 5.5 to 5.8.

Figure 5.5 shows that the value of natural growth rate of population decreases from 0.012 (globally) to 0.011 then the total amount of emission is small variation almost near 30 years but 30 to 60 years the total emission is medium variation and after 60 years the total emission is desreases highly. Finally after 100 years emission decreases from 132 to 119.

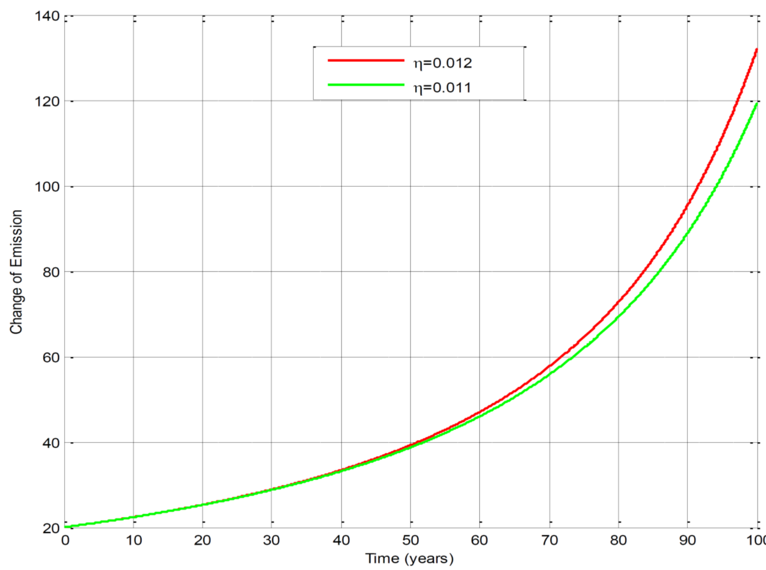

Figure 5.5. Profile of emission under various growth rate at the same $E_1 = 0.05$.

Figure 5.6 shows that the value of natural growth rate of population decreases from 0.012 (globally) to 0.011 then the total population is small variation almost near 10 years but 10 to 20 years the total population is medium variation and after 20 years the total population is desreases highly. Finally after 100 years population decreases from 24.5 to 22.5.

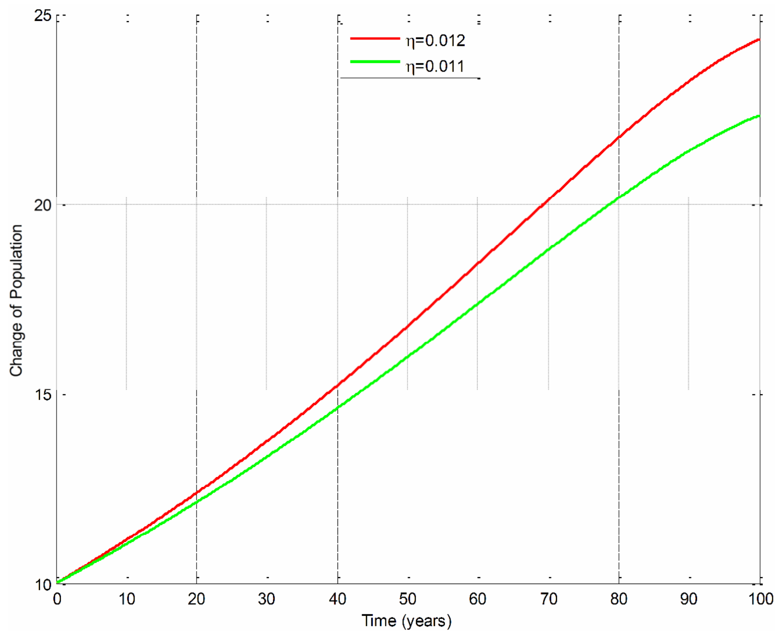

Figure 5.6. Profile of population under various growth rate at the same $E_1 = 0.05$.

Figure 5.7 shows that the value of natural growth rate of population decreases from 0.012 (globally) to 0.011 then the total amount of vehicles is small variation almost near 45 years but 45 to 70 years the total vehicles is medium variation and after 70 years the total vehicles is desreases highly. Finally after 100 years vehicles decreases from 38 to 33.

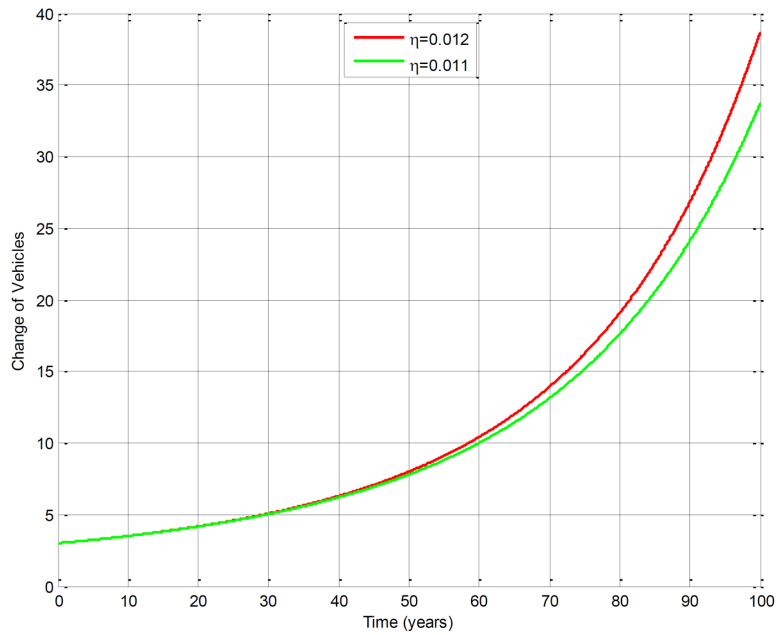

Figure 5.7. Profile of vehicles under various growth rate at the same $E_1 = 0.05$.

Figure 5.8 shows that the value of natural growth rate of population decreases from 0.012 (globally) to 0.011 then the total amount of factories is small variation almost near 45 years but 45 to 70 years the total factories is medium variation and after 70 years the total factories is desreases highly. Finally after 100 years factories decreases from 19 to 17.

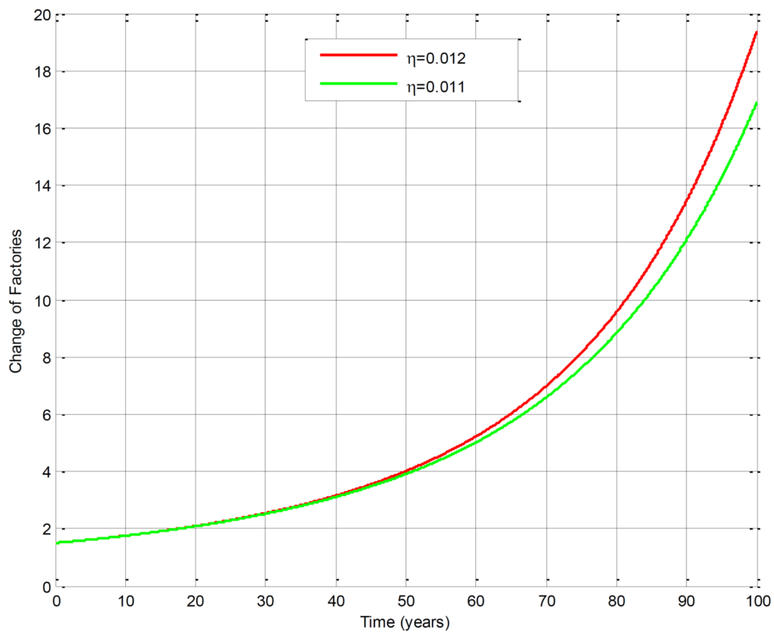

Figure 5.8. Profile of factories under various growth rate at the same $E_1 = 0.05$.

In the Figure 5.9, it is seen that, the variation in the value of changes the population dynamics logically. We found, according to the Figure 5.9, at the natural emission rate 0.05 after 100 years the total emission become to reach 6.5 times of initial amount of emission and after decreases the rate the emission reached to 122. Figure 5.10 we have to see that the effect of changing the natural emission rate on the population is small changes.

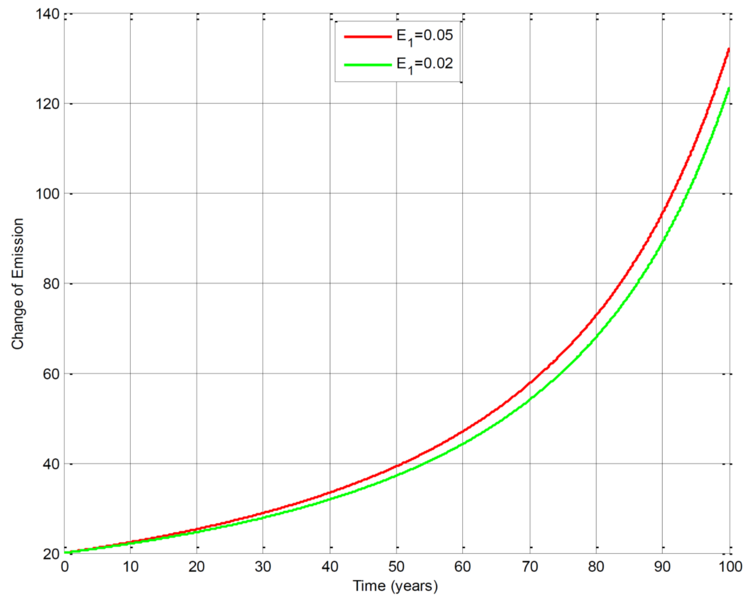

Figure 5.9. Profile of emission under different natural emission rate at the same $\eta = 0.012$.

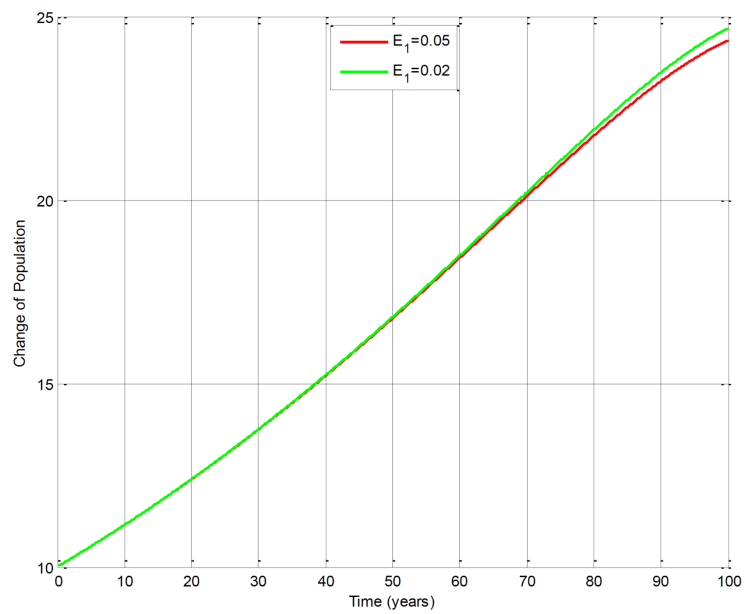

Figure 5.10. Profile of population under different natural emission rate at the same $\eta = 0.012$.

According to Figures 5.11 and 5.12, we see that the effect on the vehicles and factories is also be small changes.

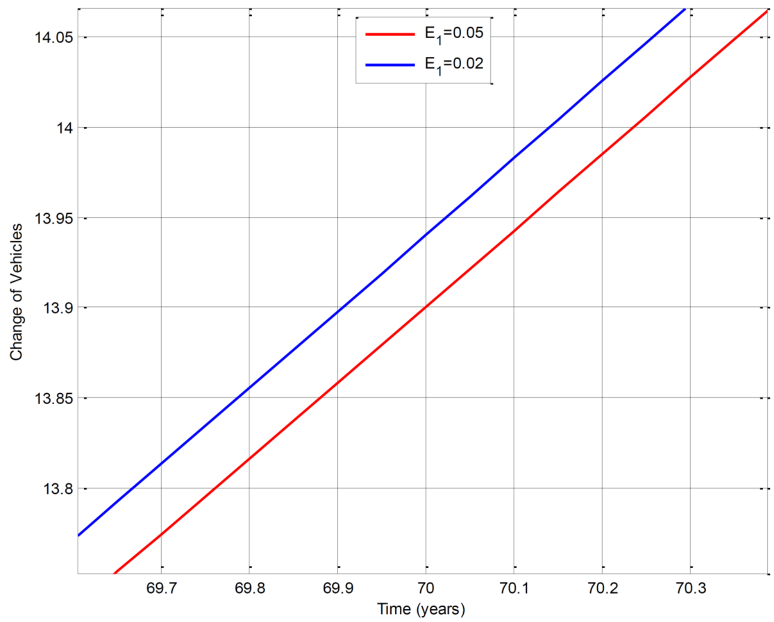

Figure 5.11. Profile of vehicles under different natural emission rate at the same $\eta = 0.012$.

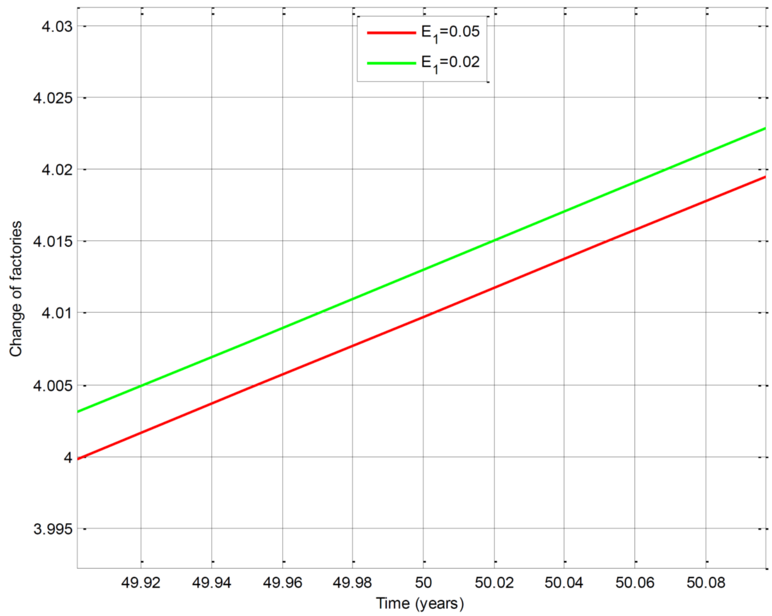

Figure 5.12. Profile of factories under different natural emission rate at the same η = 0.012.

6. Conclusion

From the above analysis, numerical simulations and figures, we can conclude that the amount of total emission will increase. We have to show, two ways can reduce the amount of emission (a) control natural emission rate and (b) control natural growth rate of population. If we decrease natural emission rate and natural growth rate of population then the total amount of emission will also decrease. In connection, they can effect on the other components: Population, vehicles and factories will increase in small amount for decreasing natural emission rate, and vehicles and factories will decrease for decreasing natural growth rate of population. Population growth rate is more effective than natural emission rate.

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