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OPEN PROBLEMS ON INTEGRAL SUM LABELLINGS

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Abstract

Let $G = (V, E)$ be a simple graph and Z be the set of all integers. An integral sum graph of a set S of integers as the graph $G^+(S)$ having S as its vertex set, with two vertices adjacent whenever their sum is in S. A graph $G^+(S)$ so obtained is called an integral sum graph. In other words, an integral sum graph $G^+(S)$ of a finite subset $S \subset Z$ is the graph (S, E) with $uv \in E$ if and only if $u + v \in S$. And S is called an integral sum labelling of $G^+(S)$. In the paper, many obtained conclusions are summarized and relevant open problem is raised.

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1. Introduction

Let $G = (V, E)$ be a simple graph with the vertex set V and edge set E. Let Z and N denote the set of all integers and the set of all positive integers, respectively. The concepts of the integral sum graph and the sum graph were introduced by Harary ([2], [3]). The related research is mainly applied in data storage and algorithm acceleration of computer.

An integral sum graph of a set S of integers as the graph $G^+(S)$ having S as its vertex set, with two vertices adjacent whenever their sum is in S. A graph $G^+(S)$ so obtained is called an integral sum graph. In other words, an integral sum graph $G^+(S)$ of a finite subset $S \subset Z$ is the graph (S, E) with $uv \in E$ if and only if $u + v \in S$. And the set S is called an integral sum labelling of $G^+(S)$. If we replace Z with N, the notions of the sum graph, the sum labelling and the sum number are got respectively.

To simplify the notations, throughout this paper we may assume that the vertices of G are identified with their labels. All other notations and terminologies are referred to [1].

In this paper, we mainly talk about the integral sum labelling of a graph.

2. Main Results

Let P_n and C_n denote Path and Cycle with n vertices, respectively.

Example 1. P_4 is one integral sum graph (see Figure 1).

Figure 1. P_4.

Example 2. C_5 is one integral sum graph (see Figure 2).

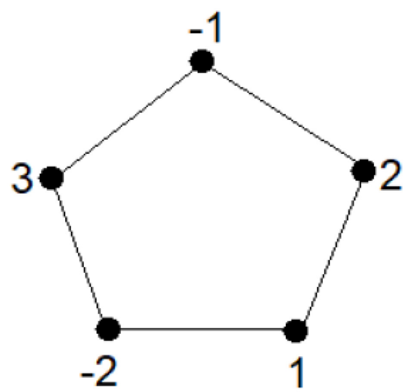

Figure 2. C_5.

According to the definitions above, $\{1, 2, -1, 3\}$ and $\{3, -2, 1, 2, -1\}$ are the corresponding integral sum labels of P_4 and C_5. Thus, two graphs in Figures 1 and 2 are the integral sum graphs.

As we know, it is difficult to determine whether a graph is an integral sum graph. Even a graph is known to be an integral sum graph, its topological properties are very important and it is hard to obtain for us. These questions are still open.

Problem 1. How to determine if a graph is an integral sum graph.

Problem 2. Characterize the integral sum graph.

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