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THE (3 + 1)-DIMENSIONAL FRACTIONAL MODIFIED KdV-ZAKHAROV-KUZNETSOV EQUATION BY USING THE IMPROVED GENERALIZED TANH-COTH METHOD

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Abstract

In this paper, the improved generalized tanh-coth method is used to establish exact travelling wave solutions of the (3 + 1)-dimensional fractional modified KdV-Zakharov-Kuznetsov equation. The fractional derivative version of Yang modified, linked with fractional complex transform is employed to reduce fractional differential equations into the corresponding ordinary differential equations. The results confirm that proposed method is reliability and effectiveness of the current method.

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1. Introduction

Fractional differential equations (FDEs) arise in numerous problems of science, control theory, engineering, biology, physics, mathematics, chemistry and other areas. Therefore, exact solution methods of the fractional differential equations have become more important. Many researchers have used diverse methods to get exact solutions, such as, the functional variable method, Lie symmetries, Riccati-sub equation method, first integral method, improved fractional sub-equation method, the exp-function method and so on [1-6].

Another powerful method has been presented by Maliet [7], who had customized the tanh technique and called the tanh method. In 2002, Fan and Hona [8] extended the tanh method which is called the extended tanh method. In 2006, Wazwaz [9] extended and improved this method which is called the tanh-coth method. In 2008, Gomez and Salas [10] improved and generalized this method which is called the improved generalized tanh-coth method. After wards, several researchers applied this method to obtain exact solutions for nonlinear PDEs [10-13].

The (3 + 1)-dimensional fractional modified KdV-Zakharov-Kuznetsov equation can be derived for the behaviour of weakly nonlinear ion-acoustic waves with components of two negative ions of different temperatures. It is the aim of this work, we use the improved generalized tanh-coth method to obtain exact travelling wave solutions of the (3 + 1)-dimensional fractional modified KdV-Zakharov-Kuznetsov equation.

The remainder of this paper is organized as follows. The formal definitions and properties of local fractional derivative have been explained in Section 2 and we briefly describe the extended and improved this method in Section 3. In Section 4, the improved generalized tanh-coth method is applied to the (3 + 1)-dimensional fractional modified KdV-Zakharov-Kuznetsov equation. Some conclusions are given in the last section.

2. Local Fractional Derivative and its Properties

The summary of local fractional derivative of order which is used further in this paper is defined by the following expression [14, 15]:

$$D^{\alpha} f(t_0) = \frac{d^{\alpha} f(t)}{dt^{\alpha}} = \lim_{t \to t_0} \frac{\Delta^{\alpha}(f(t) - f(t_0))}{(t - t_0)^{\alpha}}, \quad 0 < \alpha \leq 1, \tag{1}$$

in which $\Delta^{\alpha}(f(t) - f(t_0))$ is the $\Delta\alpha$ function defined by

$$\Delta^{\alpha}(f(t) - f(t_0)) \cong \gamma(1 + \alpha)(f(t) - f(t_0)). \tag{2}$$

Some important properties of the local derivative famous formula can be listed as follows:

$$D^{\alpha}(f_1(t) \pm f_2(t)) = D^{\alpha}(f_1(t)) \pm D^{\alpha}(f_2(t)), \tag{3}$$

$$D^{\alpha}(cf(t)) = cD^{\alpha}(f(t)), \quad c = \text{constant},$$

$$D^{\alpha} t^{\beta} = \frac{\Gamma(1 + \beta)}{\Gamma(1 + \beta - \alpha)} t^{\beta - \alpha}, \quad \beta \geq \alpha > 0,$$

$$D^{\alpha}(f(g(t))) = D^{\alpha}(g(t)) g^{(1)}(t). \tag{4}$$

3. The Improved Generalized Tanh-Coth Method

We suppose that given nonlinear partial differential equation for $u(x, y, z, t)$ to be in from

$$F(u, u_x, u_t, u_y, u_z, \ldots) = 0, \tag{5}$$

which can be converted to an ODE.

$$O(u, u', u'', u''', \ldots) = 0, \tag{6}$$

by the travelling wave transformation is given by $u = u(\xi)$, $\xi = ax + by + cz - vt$. We seek the exact solution of Equation (6) that can be expressed in the following form:

$$u(\xi) = \sum_{n=0}^{M} A_n \phi(\xi)^n + \sum_{n=M+1}^{2M} A_n \phi(\xi)^{M-n}, \tag{7}$$

where M is a positive integer that will be determined by balancing the highest order derivative term with the highest order nonlinear term. The coefficients A_n are constants $(A_M \neq 0 \text{ and } A_{-M} \neq 0)$ that are determined later while the new variable $\phi(\xi)$ is the solution to the generalized Riccati equation

$$\phi'(\xi) = g_0 + g_1\phi(\xi) + g_2(\phi(\xi))^2, \tag{8}$$

where g_0, g_1, and g_2 are constants. The solutions of generalized Riccati equation are given by [10].

Case 1 (Exponential function solution)**:** When $g_0 = 0$,

$$\phi(\xi) = \frac{g_1}{-g_2 + g_1 e^{-g_1\xi}}. \tag{9}$$

Case 2 (Trigonometric and hyperbolic function solutions)**:** When $g_1 = 0$,

$$\phi(\xi) = \begin{cases} \frac{\sqrt{g_0 g_2}}{g_2} \tan(\sqrt{g_0 g_2}\xi), & g_0 > 0,\ g_2 > 0, \\ \frac{\sqrt{g_0 g_2}}{g_2} \tanh(\sqrt{g_0 g_2}\xi), & g_0 > 0,\ g_2 < 0, \\ \frac{\sqrt{-g_0 g_2}}{g_2} \tanh(-\sqrt{-g_0 g_2}\xi), & g_0 < 0,\ g_2 > 0, \\ \frac{\sqrt{g_0 g_2}}{g_2} \tan(-\sqrt{g_0 g_2}\xi), & g_0 < 0,\ g_2 < 0. \end{cases} \tag{10}$$

Case 3 (Exponential function solution)**:** When $g_2 = 0$,

$$\phi(\xi) = \frac{-g_0 + g_1 e^{g_1 \xi}}{g_1}. \tag{11}$$

Case 4 (Rational function solution)**:** When $g_0 = 0$, $g_1 = 0$,

$$\phi(\xi) = -\frac{1}{g_2 \xi}. \tag{12}$$

Case 5 (Rational function solution)**:** When $g_1^2 \neq 0$ and $g_1^2 = 4g_2 g_0$,

$$\phi(\xi) = -\frac{2g_0(g_1\xi + 2)}{g_1^2 \xi}. \tag{13}$$

Case 6 (Trigonometric function solution)**:** When $g_1^2 < 4g_0 g_2$ and $g_2 \neq 0$,

$$\phi(\xi) = \frac{\sqrt{4g_0 g_2 - g_1^2}\tan\left(\frac{1}{2}\sqrt{4g_0 g_2 - g_1^2}\,\xi\right) - g_1}{2g_2}. \tag{14}$$

Case 7 (Hyperbolic function solution)**:** When $g_1^2 > 4g_0 g_2$ and $g_2 \neq 0$,

$$\phi(\xi) = \frac{\sqrt{g_1^2 - 4g_0 g_2}\tanh\left(\frac{1}{2}\sqrt{g_1^2 - 4g_0 g_2}\,\xi\right) - g_1}{2g_2}. \tag{15}$$

We then substitute Equation (7) into Equation (6) and collect all terms with the same order of $\phi^j(\xi)$ together. We can get a polynomial in $\phi(\xi)$. Equating each coefficient of the polynomial to zero yields a system of algebraic equations involving the parameters A_i, g_0, g_1, and g_2. Solving the equation system, we can construct a variety of exact solutions of Equation (5).

4. Application of the Improved Generalized Tanh-Coth Method

Consider the (3 + 1)-dimensional fractional modified KdV-Zakharov-Kuznetsov equation ([2-6], [16])

$$D_t^{\alpha}u + \delta u^2 D_x^{\alpha}u + D_x^{3\alpha}u + D_x^{\alpha}D_y^{2\alpha}u + D_x^{\alpha}D_z^{2\alpha}u = 0, \tag{16}$$

where $0 < \alpha \leq 1$. We use the wave transformation $u(x, y, z, t) = U(\xi)$, $\xi = \frac{ax^{\alpha}}{\Gamma(1+\alpha)} + \frac{by^{\alpha}}{\Gamma(1+\alpha)} + \frac{cz^{\alpha}}{\Gamma(1+\alpha)} - \frac{vt^{\alpha}}{\Gamma(1+\alpha)}$ to reduce (16) to following ODE:

$$-vU' + \delta aU^2U' + (a^3 + ab^2 + ac^2)U''' = 0. \tag{17}$$

Integrating the resultant once, we obtain

$$-vU + \frac{\delta a}{3}U^3 + (a^3 + ab^2 + ac^2)U'' = 0. \tag{18}$$

Balancing the highest order term U'' with the highest order nonlinear term U^3 in (13), we have $M + 2 = 3M$, then $M = 1$. Consequently, we set

$$U(\xi) = A_0 + A_1\phi(\xi) + A_2\phi(\xi)^{-1}. \tag{19}$$

Substituting Equation (19) into Equation (18) and equating all the coefficients of power of $\phi(\xi)$ to be zero, we obtain a system of algebraic equations involving the parameters A_0, A_1, A_2, and v.

$$\phi(\xi)^{-3} : 2a^3A_2\alpha^2 + \frac{1}{3}\delta aA_2{}^3 + 2b^2aA_2\alpha^2 + 2c^2aA_2\alpha^2 = 0, \tag{20}$$

$$\phi(\xi)^{-2} : 3a^3A_2\alpha\beta + 3b^2aA_2\alpha\beta + 3c^2aA_2\alpha\beta + \delta aA_2{}^2A_0 = 0, \tag{21}$$

$$\phi(\xi)^{-1} : 2A_2a^3\alpha\gamma + A_2a^3\beta^2 + 2A_2ab^2\alpha\gamma + A_2ab^2\beta^2 + 2A_2ac^2\alpha\gamma + A_2ac^2\beta^2 + A_0{}^2A_2a\delta + A_1A_2{}^2a\delta - A_2v = 0, \tag{22}$$

$$\phi(\xi)^0 : a^3 A_1 \beta\alpha + b^2 a A_1 \beta\alpha + c^2 a A_1 \beta\alpha + \frac{1}{3}\delta a A_0{}^3 + b^2 a A_2 \beta\gamma$$

$$+ c^2 a A_2 \beta\gamma - v A_0 + a^3 A_2 \beta\gamma + 2\delta a A_2 A_0 A_1 = 0, \tag{23}$$

$$\phi(\xi)^1 : 2A_1 a^3 \alpha\gamma + A_1 a^3 \beta^2 + 2A_1 a b^2 \alpha\gamma + A_1 a b^2 \beta^2 + 2A_1 a c^2 \alpha\gamma$$

$$+ A_1 a c^2 \beta^2 + A_0{}^2 A_1 a \delta + A_1{}^2 A_2 a \delta - A_1 v = 0, \tag{24}$$

$$\phi(\xi)^2 : 3A_1 a^3 \beta\gamma + 3A_1 a b^2 \beta\gamma + 3A_1 a c^2 \beta\gamma + A_0 A_1{}^2 a\delta = 0, \tag{25}$$

$$\phi(\xi)^3 : 2a^3 A_1 g_2{}^2 + \frac{1}{3}\delta a A_1{}^3 + 2c^2 a A_1 \gamma^2 + 2b^2 a A_1 \gamma^2 = 0. \tag{26}$$

We solve the system of algebraic equations with the aid of Maple, using Equations (20)-(26), that obtain following:

First set

$$A_0 = \pm\frac{1}{2}\sqrt{-\frac{6a^2 + 6b^2 + 6c^2}{\delta}}\beta, \; A_1 = 0,$$

$$A_2 = \pm\alpha\sqrt{-\frac{6a^2 + 6b^2 + 6c^2}{\delta}}, \; v = \frac{a}{2}(a^2 + b^2 + c^2)(4g_2\alpha - \beta^2).$$

Case 1 (Trigonometric and hyperbolic function solutions)**:** When $\beta = 0$,

$$u_{1,2} = \pm\frac{\gamma\alpha}{\sqrt{\gamma\alpha}}\sqrt{-\frac{6a^2 + 6b^2 + 6c^2}{\delta}}$$

$$\times \cot\left(\sqrt{\gamma\alpha}\left(\frac{ax^\alpha}{\Gamma(1+\alpha)} + \frac{by^\alpha}{\Gamma(1+\alpha)} + \frac{cz^\alpha}{\Gamma(1+\alpha)}\right.\right.$$

$$\left.\left. - (a^2\gamma + b^2 g_2 + c^2\gamma)\frac{2\alpha a t^\alpha}{\Gamma(1+\alpha)}\right)\right), \tag{27}$$

$$u_{3,4} = \pm\frac{\gamma\alpha}{\sqrt{-\gamma\alpha}}\sqrt{-\frac{6a^2+6b^2+6c^2}{\delta}}$$
$$\times\coth\left(\sqrt{\gamma\alpha}\left(\frac{ax^\alpha}{\Gamma(1+\alpha)}+\frac{by^\alpha}{\Gamma(1+\alpha)}+\frac{cz^\alpha}{\Gamma(1+\alpha)}\right.\right.$$
$$\left.\left.-(a^2\gamma+b^2g_2+c^2\gamma)\frac{2\alpha a t^\alpha}{\Gamma(1+\alpha)}\right)\right). \tag{28}$$

Case 2 (Exponential function solution)**:** When $\gamma = 0$,

$$u_{5,6} = \pm\sqrt{-\frac{6(a^2+b^2+c^2)}{\delta}}\beta$$
$$\times\left(\frac{\alpha}{-\alpha+\beta e^{\beta\left(\frac{ax^\alpha}{\Gamma(1+\alpha)}+\frac{by^\alpha}{\Gamma(1+\alpha)}+\frac{cz^\alpha}{\Gamma(1+\alpha)}+\frac{1}{2}(a^2\beta^2+b^2\beta^2+c^2\beta^2)\frac{at^\alpha}{\Gamma(1+\alpha)}\right)}}+\frac{1}{2}\right). \tag{29}$$

Case 3 (Trigonometric function solution)**:** When $\beta^2 < 4\alpha\gamma$ and $\gamma \neq 0$,

$$u_{7,8} = \pm\sqrt{-\frac{6a^2+6b^2+6c^2}{\delta(4\gamma\alpha-\beta^2)}}$$
$$\left(\frac{2\gamma\alpha}{\tan\left(\frac{1}{2}\sqrt{4g_2\alpha-\beta^2}\left(\frac{ax^\alpha}{\Gamma(1+\alpha)}+\frac{by^\alpha}{\Gamma(1+\alpha)}+\frac{cz^\alpha}{\Gamma(1+\alpha)}-\frac{1}{2}(a^2+b^2+c^2)(4\gamma\alpha-\beta^2)\frac{at^\alpha}{\Gamma(1+\alpha)}\right)\right)-\frac{\beta}{\sqrt{4\gamma\alpha-\beta^2}}}+\frac{\sqrt{4\gamma\alpha-\beta^2}\beta}{2}\right). \tag{30}$$

Case 4 (Hyperbolic function solution)**:** When $\beta^2 > 4\alpha\gamma$ and $\gamma \neq 0$,

$$u_{9,10} = \pm\sqrt{-\frac{6a^2 + 6b^2 + 6c^2}{\delta(-4\gamma\alpha + \beta^2)}}$$

$$\left(\frac{2\gamma\alpha}{\tanh\left(\frac{1}{2}\sqrt{-4\gamma\alpha + \beta^2}\left(\frac{ax^\alpha}{\Gamma(1+\alpha)} + \frac{by^\alpha}{\Gamma(1+\alpha)} + \frac{cz^\alpha}{\Gamma(1+\alpha)} - \frac{1}{2}(a^2 + b^2 + c^2)(4\gamma\alpha - \beta^2)\frac{at^\alpha}{\Gamma(1+\alpha)}\right)\right) - \frac{\beta}{\sqrt{-4\gamma\alpha+\beta^2}}} - \frac{\sqrt{4\gamma\alpha + \beta^2}\beta}{2}\right). \tag{31}$$

Second set

$$A_0 = \pm\frac{1}{2}\sqrt{-\frac{6a^2 + 6b^2 + 6c^2}{\delta}}\beta, \; A_1 = \pm\sqrt{-\frac{6a^2 + 6b^2 + 6c^2}{\delta}}\gamma,$$

$$A_2 = 0, \; v = \frac{a}{2}(a^2 + b^2 + c^2)(4\gamma\alpha - \beta^2). \tag{32}$$

Case 1 (Exponential function solution)**:** When $\alpha = 0$,

$$u_{11,12} = \pm\sqrt{-\frac{6a^2 + 6b^2 + 6c^2}{\delta}}\beta$$

$$\times\left(\frac{1}{2} + \frac{\gamma}{-\gamma + \beta e^{-\beta\left(\frac{ax^\alpha}{\Gamma(1+\alpha)} + \frac{by^\alpha}{\Gamma(1+\alpha)} + \frac{cz^\alpha}{\Gamma(1+\alpha)} + \frac{1}{2}(a^2\beta^2 + b^2\beta^2 + c^2\beta^2)\frac{at^\alpha}{\Gamma(1+\alpha)}\right)}}\right). \tag{33}$$

Case 2 (Trigonometric and hyperbolic function solutions)**:** When $\beta = 0$,

$$u_{13,14} = \pm\sqrt{-\frac{6a^2 + 6b^2 + 6c^2}{\delta}}\sqrt{\alpha\gamma}$$

$$\times \tan\left(\sqrt{\alpha\gamma}\left(\frac{ax^\alpha}{\Gamma(1+\alpha)} + \frac{by^\alpha}{\Gamma(1+\alpha)} + \frac{cz^\alpha}{\Gamma(1+\alpha)}\right.\right.$$

$$\left.\left. - (a^2\gamma + b^2 g_2 + c^2\gamma)\frac{2\alpha a t^\alpha}{\Gamma(1+\alpha)}\right)\right), \qquad (34)$$

$$u_{15,16} = \pm\sqrt{-\frac{6a^2 + 6b^2 + 6c^2}{\delta}}\sqrt{-\alpha\gamma}$$

$$\times \tanh\left(\sqrt{-\alpha\gamma}\left(\frac{ax^\alpha}{\Gamma(1+\alpha)} + \frac{by^\alpha}{\Gamma(1+\alpha)} + \frac{cz^\alpha}{\Gamma(1+\alpha)}\right.\right.$$

$$\left.\left. - (a^2\gamma + b^2 g_2 + c^2\gamma)\frac{2\alpha a t^\alpha}{\Gamma(1+\alpha)}\right)\right). \qquad (35)$$

Case 3 (Trigonometric function solution)**:** When $\beta^2 < 4\alpha\gamma$ and $\gamma \neq 0$,

$$u_{17,18} = \pm\frac{1}{2}\sqrt{-\frac{(6a^2 + 6b^2 + 6c^2)(4\alpha\gamma - \beta^2)}{\delta}}$$

$$\times \tan\left(\frac{1}{2}\sqrt{4\alpha\gamma - \beta^2}\left(\frac{ax^\alpha}{\Gamma(1+\alpha)} + \frac{by^\alpha}{\Gamma(1+\alpha)} + \frac{cz^\alpha}{\Gamma(1+\alpha)}\right.\right.$$

$$\left.\left. - \frac{1}{2}(a^2 + b^2 + c^2)(4\gamma\alpha - \beta^2)\frac{at^\alpha}{\Gamma(1+\alpha)}\right)\right). \qquad (36)$$

Case 4 (Hyperbolic function solution)**:** When $\beta^2 > 4\alpha\gamma$ and $\gamma \neq 0$,

$$u_{19,20} = \pm\frac{1}{2}\sqrt{-\frac{(6a^2+6b^2+6c^2)(-4\alpha\gamma-+\beta^2)}{\delta}}$$

$$\times\tanh\left(\frac{1}{2}\sqrt{-4\alpha\gamma+\beta^2}\left(\frac{ax^\alpha}{\Gamma(1+\alpha)}+\frac{by^\alpha}{\Gamma(1+\alpha)}+\frac{cz^\alpha}{\Gamma(1+\alpha)}\right.\right.$$

$$\left.\left.-\frac{1}{2}(a^2+b^2+c^2)(4\gamma\alpha-\beta^2)\frac{at^\alpha}{\Gamma(1+\alpha)}\right)\right). \tag{37}$$

5. Conclusion

We have exact travelling wave solutions for the (3 + 1)-dimensional fractional modified KdV-Zakharov-Kuznetsov equation by the improved generalized tanh-coth method. All exact travelling wave solutions put back into the original equation by main of Maple software. This is confirm the validity of the solutions obtained in this paper.

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