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NEW APPROACH OF (G'/G)-EXPANSION METHOD AND NEW APPROACH OF GENERALIZED (G'/G)-EXPANSION METHOD FOR THE COUPLED RAMANI EQUATION

GUIYING CHEN

College of Mathematics Science
Liaocheng University
Liaocheng 252059
Shandong Province
P. R. China
e-mail: chengquiying@126.com.

Abstract

By using two extended (G'/G)-expansion methods, new approach of (G'/G)-expansion method and new approach of generalized (G'/G)-expansion method, the coupled Ramani equation is studied, new families of exact solutions with arbitrary parameters are obtained including the hyperbolic functions, the trigonometric functions and the rational functions. In methods, the nonlinear auxiliary equation is implemented. When the parameters in the auxiliary equation are taken as special values, the new obtained solutions are degenerated to the known solutions appear in the previous literatures. This show that these two methods are effective and powerful.

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1. Introduction

Nonlinear partial differential equations (NLPDEs) are used to model complex phenomena in various scientific fields, such as, mechanics, biology, solid state physics, nonlinear optics and so on. Therefore, construction of the analytic exact solutions to NLPDEs plays a vital role in the study of nonlinear science. In the past decades, a variety of powerful methods have been developed to generate exact solutions to NLPDEs. For example, the homogeneous balance method [1], the sine-cosine method [2], the tanh method [3, 4], the Jacobi elliptic function expansion method [5], the (G'/G)-expansion method [6], the Lie group analysis method [7, 8, 9], the Darboux transformation method [10], and so on.

Recently, Naher and Abdullah [11] extended the (G'/G)-expansion method and proposed new approach of (G'/G)-expansion method and new approach of generalized (G'/G)-expansion method to obtain travelling wave solutions for KdV equation. Later on, many NLPDEs were investigated by these two methods, such as the $(3+1)$-dimensional potential-YTSF equation and the $(2+1)$-dimensional modified Zakharov-Kuznetsov equations [12], the ZKBBM equation [13], the $(2+1)$-dimensional breaking soliton equation [14], the KP-BBM equation [15], the Klein-Gordon equation [16], etc. To the best of our knowledge, no previous work has been done by these two methods for the coupled Ramani equation.

In this article, we would like to investigate the following celebrated coupled Ramani equation:

$$\frac{1}{5}u_{xxxxxx} + 3u_{xx}u_{xxx} + 3u_x u_{xxxx} + 9u_x^2 u_{xx} - (u_{xxxt} + 3u_{xx}u_t + 3u_x u_{xt})$$

$$- u_{tt} + \frac{18}{5}v_x = 0,$$

$$v_t - v_{xxx} - 3v_x u_x - 3vu_{xx} = 0. \tag{1.1}$$

Equation (1.1) was first introduced in [17] as an integrable extansion of the Ramani equation, many interesting results have been obtained. For example, the Lax pairs, Bäcklund transformations and solition solutions are investigated by bilinear equations in [17], the multi-solition solutions are derived by using Pfaffians in [18], the bilinear Bäcklund and Lax pair are obtained from its bilinear form in [19], the multiple soliton solutions are studied by employing the Hirota's bilinear method in [20]. In the present work, by using new approach of (G'/G)-expansion method and new approach of generalized (G'/G)-expansion method, we get many new exact travelling wave solutions of Equation (1.1) which generalizes the previous results by choice of special parametric values.

The rest of this paper is organized as follows: In Section 2, we give new families of exact solutions including the solitons, the hyperbolic, the trigonometric and the rational forms by the new approach of (G'/G)-expansion method and new approach of generalized (G'/G)-expansion method. In the last section, we give the conclusion.

2. New Travelling wave Solutions

In this part, we would construct exact solutions for Equation (1.1) by the new approach of (G'/G)-expansion method and new approach of generalized (G'/G)-expansion method.

Substitution of the travelling wave transformation $u = t + F_1(\zeta)$, $v = F_2(\zeta)$ (where $\zeta = x - at$) into Equation (1.1) results in the following ODEs:

$$F_1^{(6)} + 15F_1''F_1''' + 15F_1'F_1^{(4)} + 45F_1'^2F_1'' + 5aF_1^{(4)} - 5(3 + a^2)F_1'' + 30aF_1'F_1'' + 18F_2' = 0,$$

$$aF_2' + F_2''' + 3F_1'F_2' + 3F_2F_1'' = 0, \tag{2.1}$$

where a prime again denotes differentiation with respect to ζ, and $F_1^{(6)}$ denotes six order derivatives with respect to ζ.

2.1. New approach of (G'/G)-expansion method for Equation (1.1)

Step 1. Suppose that the travelling wave solution of Equation (2.1) can be expressed as follows:

$$F_1 = \sum_{i=0}^{m} a_i\left(\frac{G'}{G}\right)^i + \sum_{i=1}^{m} b_i\left(\frac{G'}{G}\right)^{-i}, \quad F_2 = \sum_{j=0}^{n} c_j\left(\frac{G'}{G}\right)^j + \sum_{j=1}^{n} d_j\left(\frac{G'}{G}\right)^{-j}, \tag{2.2}$$

where a_i, $b_i (i = 1, \ldots, m)$, c_j, $d_j (j = 1, \ldots, n)$ are arbitrary constants to be determined later, a_m and b_n can not be zero at a time, and G satisfies the following nonlinear auxiliary ordinary differential equation:

$$AGG'' - BGG' - C(G')^2 - EG^2 = 0, \tag{2.3}$$

where prime denotes derivative with respect to ζ, A, B, C, and E are real parameters.

Remark 1. If $A = 1$, $B = -\lambda$, $C = 0$, $E = -\mu$, Equation (2.3) coincides with linear existing ordinary differential equation

$$G'' + \lambda G' + \mu G = 0.$$

That is, the new approach of (G'/G)-expansion method extends the (G'/G)-expansion method. By means of the (G'/G)-expansion method the travelling solutions of Equation (1.1) have been obtained in [21]. Now, we would present many new travelling solutions for Equation (1.1).

Remark 2. The general solutions of Equation (2.3) given in [11] are as follows:

(1) When $B \neq 0$, $\psi = A - C$, and $\Omega = B^2 + 4E(A - C) > 0$,

$$\frac{G'}{G} = \frac{B}{2\psi} + \frac{\sqrt{\Omega}}{2\psi}\frac{C_1 \sinh\left(\frac{\sqrt{\Omega}}{2\psi}\zeta\right) + C_2 \cosh\left(\frac{\sqrt{\Omega}}{2\psi}\zeta\right)}{C_1 \cosh\left(\frac{\sqrt{\Omega}}{2\psi}\zeta\right) + C_2 \sinh\left(\frac{\sqrt{\Omega}}{2\psi}\zeta\right)}. \tag{2.4}$$

(2) When $B \neq 0$, $\psi = A - C$, and $\Omega = B^2 + 4E(A - C) < 0$,

$$\frac{G'}{G} = \frac{B}{2\psi} + \frac{\sqrt{\Omega}}{2\psi} \frac{C_1 \sin(\frac{\sqrt{\Omega}}{2\psi}\zeta) + C_2 \cos(\frac{\sqrt{\Omega}}{2\psi}\zeta)}{C_1 \cos(\frac{\sqrt{\Omega}}{2\psi}\zeta) + C_2 \sin(\frac{\sqrt{\Omega}}{2\psi}\zeta)}. \tag{2.5}$$

(3) When $B \neq 0$, $\psi = A - C$, and $\Omega = B^2 + 4E(A - C) = 0$,

$$\frac{G'}{G} = \frac{B}{2\psi} + \frac{C_2}{C_1 + C_2\zeta}. \tag{2.6}$$

Step 2. Taking the homogeneous balance between the highest order derivatives and the highest order nonlinear terms in Equation (2.2), we obtain $m = 1$, $n = 2$. Therefore, the travelling wave solutions of Equation (2.2) is as follows:

$$F_1 = a_0 + a_1(\frac{G'}{G}) + b_1(\frac{G'}{G})^{-1}, \quad F_2 = c_0 + c_1(\frac{G'}{G}) + c_2(\frac{G'}{G})^2 + d_1(\frac{G'}{G})^{-1} + d_2(\frac{G'}{G})^{-2}. \tag{2.7}$$

Step 3. Substituting Equation (2.7) and Equation (2.3) into Equation (2.1), we obtain polynomials in G'/G. Setting all the coefficients of like powers of $(G'/G)^i (i = -2, -1, 0, 1, \ldots, 7)$ to zero yields a system of algebraic equations, solving these equations, we get

Case 1:

$$a_1 = \frac{2\psi}{A}, \; b_1 = 0, \; d_1 = 0, \; d_2 = 0,$$

$$c_0 = -[(5a^3 + 15a)A^6 - 6aA^2B^4 + 15A^4B^2 - B^6 + 32E^3\psi^3 - 18aA^2B^2E\psi + 24aA^2E^2\psi^2 - 30(a^2 + 1)A^4E\psi - 6B^4E\psi)]/(54A^6),$$

$$c_1 = -B\psi[\Omega^2 + 5aA^2\Omega - (5a^2 + 15)A^4]/(9A^6),$$

$$c_2 = \psi^2[\Omega^2 + 5aA^2\Omega - (5a^2 + 15)A^4]/(9A^6), \tag{2.8}$$

a_0 and d are arbitrary constants.

Case 2:

$$a_1 = 0,\ b_1 = \frac{2E}{A},\ c_1 = 0,\ c_2 = 0,$$

$$c_0 = -[(5a^3 + 15a)A^6 - 6aA^2B^4 + 15A^4B^2 - B^6 + 32E^3\psi^3 - 18aA^2B^2E\psi$$
$$+ 24aA^2E^2\psi^2 - 30(a^2 + 1)A^4E\psi - 6B^4E\psi)]/(54A^6),$$

$$d_1 = BE[\Omega^2 + 5aA^2\Omega - (5a^2 + 15)A^4[/(9A^6),$$

$$d_2 = E^2[\Omega^2 + 5aA^2\Omega - (5a^2 + 15)A^4]/(9A^6), \tag{2.9}$$

a_0 and d are arbitrary constants.

For Case 1: Substituting Equation (2.8) into Equation (2.7), along with Equation (2.4) yields the following hyperbolic function solution:

$$u1(x, t) = t + a_0 + \left(\frac{B}{A} + \frac{\sqrt{\Omega}}{A}\frac{C_1 \sinh(\frac{\sqrt{\Omega}}{2\psi}\zeta) + C_2 \cosh(\frac{\sqrt{\Omega}}{2\psi}\zeta)}{C_1 \cosh(\frac{\sqrt{\Omega}}{2\psi}\zeta) + C_2 \sinh(\frac{\sqrt{\Omega}}{2\psi}\zeta)}\right),$$

$$v1(x, t) = -[(5a^3 + 15a)A^6 - 6aA^2B^4 + 15A^4B^2 - B^6 + 32E^3\psi^3$$
$$- 18aA^2B^2E\psi + 24aA^2E^2\psi^2$$
$$- 30(a^2 + 1)A^4E\psi - 6B^4E\psi]/(54A^6)$$
$$+ \left[-\frac{B^2}{4} + \frac{\Omega}{4}\left(\frac{C_1 \sinh(\frac{\sqrt{\Omega}}{2\psi}\zeta) + C_2 \cosh(\frac{\sqrt{\Omega}}{2\psi}\zeta)}{C_1 \cosh(\frac{\sqrt{\Omega}}{2\psi}\zeta) + C_2 \sinh(\frac{\sqrt{\Omega}}{2\psi}\zeta)}\right)^2\right]$$
$$\times (\Omega^2 + 5aA^2\Omega - (5a^2 + 15)A^4)/(9A^6). \tag{2.10}$$

Substituting Equation (2.8) into Equation (2.7), along with Equation (2.5) yields the following trigonometric function solution:

$$u2(x, t) = t + a_0 + \left(\frac{B}{A} + \frac{\sqrt{\Omega}}{A} \frac{C_1 \sin\left(\frac{\sqrt{\Omega}}{2\psi}\zeta\right) + C_2 \cos\left(\frac{\sqrt{\Omega}}{2\psi}\zeta\right)}{C_1 \cos\left(\frac{\sqrt{\Omega}}{2\psi}\zeta\right) + C_2 \sin\left(\frac{\sqrt{\Omega}}{2\psi}\zeta\right)} \right),$$

$$\begin{aligned} v2(x, t) = &- [(5a^3 + 15a)A^6 - 6aA^2B^4 + 15A^4B^2 - B^6 + 32E^3\psi^3 \\ &- 18aA^2B^2E\psi + 24aA^2E^2\psi^2 \\ &- 30(a^2 + 1)A^4E\psi - 6B^4E\psi]/(54A^6) \\ &+ \left[-\frac{B^2}{4} + \frac{\Omega}{4} \left(\frac{C_1 \sin\left(\frac{\sqrt{\Omega}}{2\psi}\zeta\right) + C_2 \cos\left(\frac{\sqrt{\Omega}}{2\psi}\zeta\right)}{C_1 \cos\left(\frac{\sqrt{\Omega}}{2\psi}\zeta\right) + C_2 \sin\left(\frac{\sqrt{\Omega}}{2\psi}\zeta\right)} \right)^2 \right] \\ &\times (\Omega^2 + 5aA^2\Omega - (5a^2 + 15)A^4)/(9A^6). \end{aligned} \tag{2.11}$$

Substituting Equation (2.8) into Equation (2.7), along with Equation (2.4) yields the following rational form solution:

$$u_3(x, t) = t + a_0 + \frac{2\psi}{A} \left(\frac{B}{2\psi} + \frac{C_2}{C_1 + C_2\zeta} \right),$$

$$\begin{aligned} v3(x, t) = &- [(5a^3 + 15a)A^6 - 6aA^2B^4 + 15A^4B^2 - B^6 + 32E^3\psi^3 \\ &- 18aA^2B^2E\psi + 24aA^2E^2\psi^2 \\ &- 30(a^2 + 1)A^4E\psi - 6B^4E\psi]/(54A^6)\left[-\frac{B^2}{4} + \psi^2 \left(\frac{C_2}{C_1 + C_2\zeta} \right)^2 \right] \\ &\times (\Omega^2 + 5aA^2\Omega - (5a^2 + 15)A^4)/(9A^6), \end{aligned} \tag{2.12}$$

where $\zeta = x - at$.

For Case 2: We omit.

2.2. New approach of generalized (G'/G)-expansion method for the coupled Ramani equation

Step 1. Suppose that the travelling wave solution of Equation (2.1) can be expressed as follows:

$$F_1 = \sum_{i=0}^{m} a_i (d + \frac{G'}{G})^i + \sum_{i=1}^{m} b_i (d + \frac{G'}{G})^{-i},$$

$$F_2 = \sum_{j=0}^{n} c_j (d + \frac{G'}{G})^j + \sum_{j=1}^{n} d_j (d + \frac{G'}{G})^{-j}, \tag{2.13}$$

where a_i, $b_i (i = 1, \ldots, m)$, c_j, $d_j (j = 1, \ldots, n)$, d are arbitrary constants to be determined later, and G satisfies Equation (2.3).

Step 2. Taking the homogeneous balance between the highest order derivatives and the highest order nonlinear terms in Equation (2.13), we obtain $m = 1$, $n = 2$. Therefore, the travelling solutions of Equation (2.1) is as follows:

$$F_1 = a_0 + a_1 (d + \frac{G'}{G}) + b_1 (d + \frac{G'}{G})^{-1}, \; F_2 = c_0 + c_1 (d + \frac{G'}{G}) + c_2 (d + \frac{G'}{G})^2$$
$$+ d_1 (d + \frac{G'}{G})^{-1} + d_2 (d + \frac{G'}{G})^{-2}. \tag{2.14}$$

Step 3. Substituting Equation (2.14) and Equation (2.3) into Equation (2.1), we obtain polynomials in $d + \frac{G'}{G}$. Setting all the coefficients of like powers of $(d + \frac{G'}{G})^i$, $(i = -2, -1, \ldots, 7)$ to zero yields a system of algebraic equations, solving these equations, we get

Case 1:

$$a_1 = \frac{2\psi}{A},\ b_1 = 0,\ d_1 = 0,\ d_2 = 0,$$

$$\begin{aligned} c_0 = &-[(5a^3 + 15a)A^6 - 6aA^2B^4 + 15A^4B^2 - B^6 + 32E^3\psi^3 - 18aA^2B^2E\psi \\ &+ 24aA^2E^2\psi^2 \\ &- 30(a^2 + 1)A^4E\psi - 6B^4E\psi) + 6(dB\psi + d^2\psi^2)(\Omega^2 + 5aA^2\Omega \\ &- (5a^2 + 15)A^4)]/(54A^6), \end{aligned}$$

$$c_1 = -(B\psi + 2d\psi^2)[\Omega^2 + 5aA^2\Omega - (5a^2 + 15)A^4]/(9A^6),$$

$$c_2 = \psi^2[\Omega^2 + 5aA^2\Omega - (5a^2 + 15)A^4]/(9A^6), \tag{2.15}$$

a_0 and d are arbitrary constants.

Case 2:

$$a_1 = 0,\ b_1 = -\frac{2(d^2\psi^2 + dB - E)}{A},\ c_1 = 0,\ c_2 = 0,$$

$$\begin{aligned} c_0 = &-[(5a^3 + 15a)A^6 - 6aA^2B^4 + 15A^4B^2 - B^6 - 18aA^2B^2E\psi + 32E^3\psi^3 \\ &+ 24aA^2E^2\psi^2 \\ &- 30(a^2 + 1)A^4E\psi - 6B^4E\psi) + 6(dB\psi + d^2\psi^2)(\Omega^2 + 5aA^2\Omega \\ &- (5a^2 + 15)A^4)]/(54A^6), \end{aligned}$$

$$d_1 = -(B + 2d\psi)[\Omega^2 + 5aA^2\Omega - (5a^2 + 15)A^4](d^2\psi + dB - E)/(9A^6),$$

$$d_2 = [\Omega^2 + 5aA^2\Omega - (5a^2 + 15)A^4](d^2\psi + dB - E)^2/(9A^6), \tag{2.16}$$

a_0 and d are arbitrary constants

For Case 1: Substituting Equation (2.15) into Equation (2.14), along with Equation (2.4) yields the following hyperbolic function solution:

$$u4(x,t) = t + a_0 + \frac{2\psi}{A}\left(d + \frac{B}{2\psi} + \frac{\sqrt{\Omega}}{2\psi}\frac{C_1 \sinh(\frac{\sqrt{\Omega}}{2\psi}\zeta) + C_2 \cosh(\frac{\sqrt{\Omega}}{2\psi}\zeta)}{C_1 \cosh(\frac{\sqrt{\Omega}}{2\psi}\zeta) + C_2 \sinh(\frac{\sqrt{\Omega}}{2\psi}\zeta)}\right),$$

$$\begin{aligned} v4(x,t) = &-\big[(5a^3 + 15a)A^6 - 6aA^2B^4 + 15A^4B^2 - B^6 + 32E^3\psi^3 \\ &- 18aA^2B^2E\psi + 24aA^2E^2\psi^2 \\ &- 30(a^2+1)A^4E\psi - 6B^4E\psi + 6(dB\psi + d^2\psi^2)(\Omega^2 + 5aA^2\Omega \\ &- (5a^2+15)A^4)\big]/(54A^6) \\ &- \Big[d^2\psi^2 + dB\psi + \frac{B^2}{4} - \frac{\Omega}{4}\Big(\frac{C_1 \sinh(\frac{\sqrt{\Omega}}{2\psi}\zeta) + C_2 \cosh(\frac{\sqrt{\Omega}}{2\psi}\zeta)}{C_1 \cosh(\frac{\sqrt{\Omega}}{2\psi}\zeta) + C_2 \sinh(\frac{\sqrt{\Omega}}{2\psi}\zeta)}\Big)^2\Big] \\ &\times [\Omega^2 + 5aA^2\Omega - (5a^2+15)A^4]/(9A^6). \end{aligned} \tag{2.17}$$

Substituting Equation (2.8) into Equation (2.7), along with Equation (2.5) yields the following hyperbolic function solution:

$$u5(x,t) = t + a_0 + \frac{2\psi}{A}\left(d + \frac{B}{2\psi} + \frac{\sqrt{\Omega}}{2\psi}\frac{C_1 \sin(\frac{\sqrt{\Omega}}{2\psi}\zeta) + C_2 \cos(\frac{\sqrt{\Omega}}{2\psi}\zeta)}{C_1 \cos(\frac{\sqrt{\Omega}}{2\psi}\zeta) + C_2 \sin(\frac{\sqrt{\Omega}}{2\psi}\zeta)}\right),$$

$$\begin{aligned} v5(x,t) = &-\big[(5a^3 + 15a)A^6 - 6aA^2B^4 + 15A^4B^2 - B^6 + 32E^3\psi^3 \\ &- 18aA^2B^2E\psi + 24aA^2E^2\psi^2 \\ &- 30(a^2+1)A^4E\psi - 6B^4E\psi + 6(dB\psi + d^2\psi^2)(\Omega^2 + 5aA^2\Omega \\ &- (5a^2+15)A^4)\big]/(54A^6) \end{aligned}$$

$$- [d^2\psi^2 + dB\psi + \frac{B^2}{4} - \frac{\Omega}{4}(\frac{C_1 \sin(\frac{\sqrt{\Omega}}{2\psi}\zeta) + C_2 \cos(\frac{\sqrt{\Omega}}{2\psi}\zeta)}{C_1 \cos(\frac{\sqrt{\Omega}}{2\psi}\zeta) + C_2 \sin(\frac{\sqrt{\Omega}}{2\psi}\zeta)})^2]$$

$$\times [\Omega^2 + 5aA^2\Omega - (5a^2 + 15)A^4]/(9A^6). \tag{2.18}$$

Substituting Equation (2.8) into Equation (2.7), along with Equation (2.6) yields the following hyperbolic function solution:

$$u6(x, t) = t + a_0 + \frac{2\psi}{A}(\frac{B}{2\psi} + \frac{C_2}{C_1 + C_2\zeta}),$$

$$v6(x, t) = - [(5a^3 + 15a)A^6 - 6aA^2B^4 + 15A^4B^2 - B^6 + 32E^3\psi^3$$

$$- 18aA^2B^2E\psi + 24aA^2E^2\psi^2$$

$$- 30(a^2 + 1)A^4E\psi - 6B^4E\psi + 6(dB\psi + d^2\psi^2)(\Omega^2 + 5aA^2\Omega$$

$$- (5a^2 + 15)A^4)]/(54A^6)$$

$$- [d^2\psi^2 + dB\psi + \frac{B^2}{4} - \psi^2(d + \frac{B}{2\psi} + \frac{C_2}{C_1 + C_2\zeta})^2]$$

$$\times (\Omega^2 + 5aA^2\Omega - (5a^2 + 15)A^4)/(9A^6), \tag{2.19}$$

where $\zeta = x - at$.

For Case 2: We omit.

Remark 3. It is obvious that if we consider $A = 1$, $B = -\lambda$, $C = 0$, $E = -\mu$, and $a = 1$, the new approach of (G'/G)-expansion method coincide with (G'/G)-expansion method, and the exact solution (2.10), (2.11), (2.12) comes out the existing hyperbolic function solution, the trigonometric function solution and the rational form solution in [21], respectively. Furthermore, if $d = 0$, the new approach of generalized (G'/G)-expansion method coincide with the new approach of (G'/G)-expansion method, and the exact solutions (2.17), (2.18), (2.19) turn out (2.10), (2.11), (2.12), respectively.

3. Conclusion

In this paper, new families of exact travelling wave solutions with arbitrary parameters of the coupled Ramani equation are obtained by means of new approach of (G'/G)-expansion method and new approach of generalized (G'/G)-expansion method. The solutions include the hyperbolic, the trigonometric, and the rational forms. In special cases, the new obtained solutions coincide with the known solutions appear in the previous literatures. This show that these two methods are effective and powerful.

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