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COMPUTATION OF TOPOLOGICAL INDICES OF LINE GRAPH OF JAHANGIR GRAPH

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Abstract

In this paper, we compute first Zagreb index (coindex), second Zagreb index (coindex), third Zagreb index, first hyper-Zagreb index, atom-bond connectivity index, fourth atom-bond connectivity index, sum connectivity index, Randic connectivity index, augmented Zagreb index, Sanskruti index, geometric-arithmetic connectivity index and fifth geometric-arithmetic connectivity index of line graph of Jahangir graph.

Keywords: topological indices, line graph, coindices.

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1. Introduction and Preliminaries

Let $G = (V; E)$ be a simple graph, with vertex set V and edge set $E \in V \times V$, on $p = |V|$ vertices and $q = |E|$ edges. The complement of a graph G, denoted by G, is a simple graph on the same set of vertices $V(G)$ in which two vertices u and ν are connected by an edge $u\nu$, if and only if they are not adjacent in G. Obviously, $E(G) \cup E(\overline{G}) = E(K_p)$, where K_p is complete graph of order p, and $|E(\overline{G})| = \frac{p(p-1)}{2} - q$. The degree d_ν of a vertex ν is the number of vertices joining to ν and the degree d_e of an edge $e \in E(G)$, where d_e is the number of its adjacent vertices in $V(L(G))$, where $L(G)$ is the simple graph whose vertices are the edges of G, with $u\nu \in E(L(G)$ when u and ν have a common end point in G. In structural chemistry, line graph of a graph G is very useful. The first topological indices on the basis of line graph was introduced by Bertz in 1981 (see [1]). For more details on line graph, see the articles [2-7].

Topological indices are the numerical quantities which represent the structure of any simple finite graph. They are invariant under the graph isomorphisms. The idea of topological index appears from work done by Wiener (see [8]) in 1947 although he was working on boiling point of para n. He called this index as Wiener index and then theory of topological index started. The Wiener index of graph G is defined as follows:

$$W(G) = \frac{1}{2} \sum_{(u,\nu)} d(u, \nu), \tag{1}$$

where $(u; \nu)$ is any ordered pair of vertices in G and $d(u; \nu)$ is $u - \nu$ geodesic.

The Zagreb indices were first introduced by Gutman in [9], they are important molecular descriptors and have been closely correlated with many chemical properties (see [10]) and defined as:

$$M_1(G) = \sum_{u \in V(G)} d_u^2, \tag{2}$$

$$M_2(G) = \sum_{uv \in E(G)} d_u d_v. \tag{3}$$

In fact, one can rewrite the first Zagreb index as

$$M_1(G) = \sum_{uv \in E(G)} [d_u + d_v].$$

The third Zagreb index, introduced by Fath-Tabar in [11]. This index is defined as follows:

$$M_3(G) = \sum_{uv \in E(G)} |d_u - d_v|. \tag{4}$$

The hyper-Zagreb index was first introduced in [12]. This index is defined as follows:

$$HM(G) = \sum_{uv \in E(G)} (d_u + d_v)^2. \tag{5}$$

The first Zagrab coindex is defined as:

$$\overline{M_1} = \overline{M_1}(G) = \sum_{uv \notin E(G)} [d_u + d_v].$$

The second Zagrab coindex is defined as:

$$\overline{M_2} = \overline{M_2}(G) = \sum_{uv \notin E(G)} d_u d_v.$$

The degree distance index for graphs developed by Dobrynin and Kochetova in [13] and Gutman in [14] as a weighted version of the Wiener index. The degree distance of G, denoted by $DD(G)$, is defined as follows:

$$DD(G) = \sum_{\{u, v\} \subseteq V(G)} d(u, v)[d_u + d_v]. \tag{6}$$

Randic index introduced by Randic in 1975 (see [15]). This index is defined as follows:

$$R(G) = \sum_{u\nu \in E(G)} \frac{1}{\sqrt{d_u d_\nu}}. \tag{7}$$

Later, this index was generalized by Bollobas and Erdos (see [16]) to the following form for any real number, and named the general Randic index:

$$R_\alpha(G) = \sum_{u\nu \in E(G)} d_u d_\nu{}^{\alpha}. \tag{8}$$

The atom-bond connectivity index (ABC), introduced by Estrada et al. in [17] which has been applied up until now to study the stability of alkanes and the strain energy of cycloalkanes. The ABC index of G is defined as:

$$ABC(G) = \sum_{u\nu \in E(G)} \sqrt{\frac{d_u + d_\nu - 2}{d_u d_\nu}}. \tag{9}$$

For more details, see the article [18]. In 2010, the general sum-connectivity index (G) has been introduced in [19]. For more detail on sum connectivity, we refer the articles [20, 21]. This index is defined as follows:

$$\chi(G) = \sum_{u\nu \in E(G)} \frac{1}{\sqrt{d_u + d_\nu}}. \tag{10}$$

Vukicevic and Furtula introduced the geometric arithmetic (GA) index in [22]. The GA index for G is defined by

$$GA(G) = \sum_{u\nu \in E(G)} \frac{2\sqrt{d_u d_\nu}}{d_u + d_\nu}. \tag{11}$$

Inspired by the work on the ABC index, Furtula et al. proposed the following modified version of the ABC index and called it as augmented Zagreb index (AZI) in [28]. This index is defined as follows:

$$AZI(G) = \sum_{u\nu \in E(G)} \left(\frac{d_u d_\nu}{d_u + d_\nu - 2} \right)^3. \tag{12}$$

The fourth member of the class of ABC index was introduced by Ghorbani et al. in [24-26] as:

$$ABC_4(G) = \sum_{uv \in E(G)} \sqrt{\frac{S_u + S_v - 2}{S_u S_v}}, \tag{13}$$

where S_u is the summation of degrees of all neighbours of vertex u in G. In other words, $S_u = \sum_{uv \in E(G)} d_v$. Similarly for S_v.

The 5-th GA index was introduced by Graovac et al. in [27] as:

$$GA_5(G) = \sum_{uv \in E(G)} \frac{2\sqrt{S_u S_v}}{S_u + S_v}. \tag{14}$$

The prediction power is better than the ABC index in the study of heat of formation for heptanes and octanes (see [27]).

The Sanskruti index $S(G)$ of a graph G is defined in [29] as follows:

$$S(G) = \sum_{uv \in E(G)} \left(\frac{S_u S_v}{S_u + S_v - 2} \right)^3. \tag{15}$$

Theorem 1.1 ([14]). *Let G be a graph of order p and size q. Then*

$$M_1(\overline{G}) = M_1(G) + p(p-1)^2 - 4q(p-1); \tag{16}$$

$$\overline{M_1}(G) = 2q(p-1) - M_1(G); \tag{17}$$

$$\overline{M_1}(\overline{G}) = 2q(p-1) - M_1(G). \tag{18}$$

Theorem 1.2 ([30]). *Let G be a graph of order p and size q. Then*:

$$M_2(\overline{G}) = \frac{1}{2} p(p-1)^3 - 3q(p-1)^2 + 2q^2 + \frac{2p-3}{2}, \tag{19}$$

$$M_1(G) - M_2(G);\ \overline{M_2}(G) = 2q^2 - \frac{1}{2} M_1(G) - M_2(G); \tag{20}$$

$$\overline{M_2}(\overline{G}) = q(p-1)^2 - (p-1)M_1(G) + M_2(G). \tag{21}$$

Theorem 1.3 ([23]). *Let G be a graph of order p and size q. Then*

$$\overline{M_1}(G) \geq 2W(G) - 2M_1(G) + 6q(p-1) - p^3 + p^2. \tag{22}$$

Theorem 1.4 ([23]). *Let G be a nontrivial graph of diameter $d \geq 2$. Then*

$$\overline{M_1}(G) \leq \frac{DD(G) - M_1(G)}{2}, \tag{23}$$

with equality if and only if $d = 2$.

The following lemma is helpful for computing the degree of a vertex of line graph.

Lemma 1.5. *Let G be a graph with $u; \nu \in V(G)$ and $e = u\nu \in E(G)$. Then $d_e = d_u + d_\nu - 2$.*

Lemma 1.6 ([31]). *Let G be a graph of order p and size q, then the line graph L(G) of G is a graph of order p and size $1/2M_1(G) - q$.*

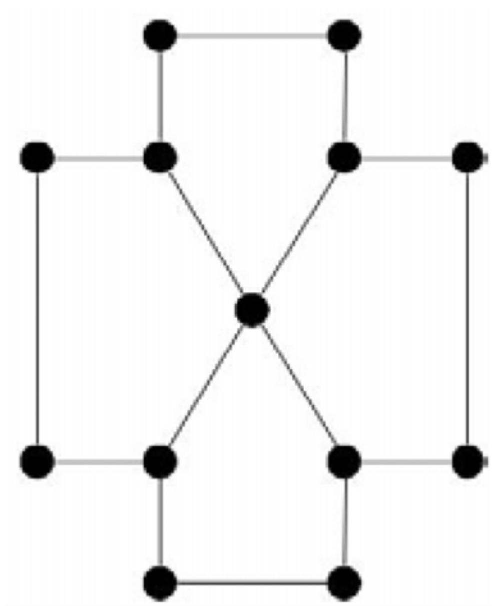

Figure 1. The Jahangir graph $J_{3;4}$.

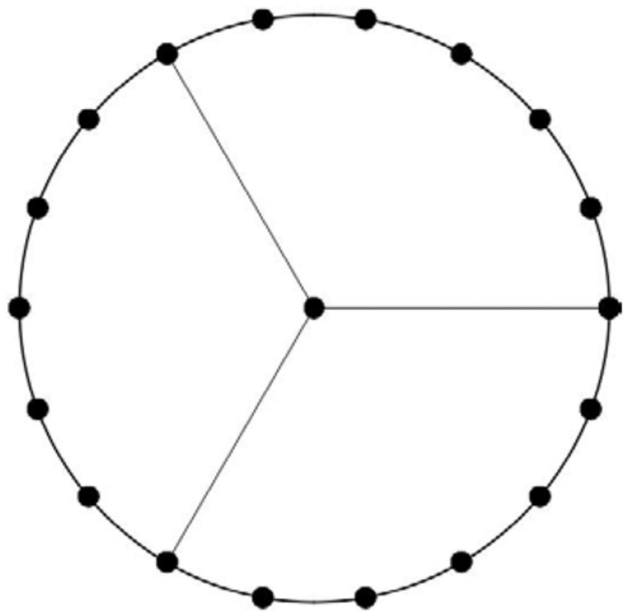

Figure 2. The Jahangir graph $J_{6;3}$.

2. Main Results and Discussions

Jahangir graph $J_{n;m}$ for $m \geq 3$, a graph consisting of a cycle C_{nm} with one additional vertex which is adjacent to m vertices of C_{nm} at distance n to each other on C_{nm} (see [32]). The J_{nm} has order $nm + 1$ and size $m(n + 1)$. The graphs $J_{3;4}$ and $J_{6;3}$ are shown in the Figures 1 and 2, respectively.

Theorem 2.1. *Let G be the line graph of the Jahangir graph* $J_{n;m}$. *Then*

(I) $M_3(G) = -2m^2 + 2m$;

(II) $HM(G) = 16mn + 38m + 2m(4 + m)^2 + 2m(m - 1)(m + 1)^2$;

(III) $\overline{M_1}(G) = \overline{M_1}(\overline{G}) = m^3n + 2m^2n^2 + 5m^2n - 6mn + 2m^3 + 4m^2 + 32m$;

(IV) $M_1(\overline{G}) = m^3n^3 + m^3n - 3m^2n^2 - 14m^2n - 4m^2 + 9mn + 18m;$

(V) $M_2(G) = 4mn + 6m^2 + 15m + \frac{1}{2}m(m-1)(m+1)^2;$

(VI) $$M_2(\overline{G}) = \frac{1}{2}m(n+1)(m(n+1)-1)^3 - \frac{3}{2}(m^2+3m+2mn)(mn+m-1)^2 + \frac{1}{2}(m^2+3m+2mn)^2 + 2(2mn+2m-3)mn - 4m^2 - 4m - 4mn + m^3 - \frac{1}{2}m(m-1)(m+1)^2;$$

(VII) $\overline{M_2}(G) = 2m^3n + 6m^2n + 2mn^2 - 6mn + 2m^3 - 2m^2 - 20m;$

(VIII) $$\overline{M_2}(\overline{G}) = \frac{1}{2}m^4n^2 + m^3n^3 + \frac{7}{2}m^3n^2 + m^3n - 6m^2n^2 - 20m^2n + 9mn - 6m^2 + 27m;$$

(IX) $$W(G) \le \left(\frac{1}{2}m^2 + \frac{3}{2}m + mn\right)(m(n+1)-1) + 2mn + 18m + 3m^2 + \frac{3}{2}m(m^2-1) - \frac{3}{2}(m^2+3m+2mn)(mn+m-1) + \frac{1}{2}m^3(n+1)^3 - \frac{1}{2}m^2(n+1)^2;$$

(X) $$DD(G) \ge 4\left(\frac{1}{2}m^2 + \frac{3}{2}m + mn\right)(m(n+1)-1) - 4mn + 36m + 6m^2 + 3m(m^2-1);$$

(XI) $$ABC(G) = \frac{1}{2}(n-3)m\sqrt{2} + m\sqrt{2} + \frac{2}{3}m + \frac{2}{3}m\sqrt{3}\sqrt{\frac{2+m}{m+1}} + \frac{1}{2}m(m-1)\sqrt{2}\sqrt{\frac{m}{(m+1)^2}};$$

(XII) $R(G) = \frac{1}{2}(n-3)m + \frac{1}{3}m\sqrt{6} + \frac{1}{3}m + \frac{2m}{\sqrt{3m+3}} + \frac{1}{2}\frac{m(m-1)}{\sqrt{(m+1)^2}}$;

(XIII) $GA(G) = (n-3)m + \frac{4}{5}m\sqrt{6} + m + \frac{4m\sqrt{3m+3}}{4+m} + \frac{m(m-1)\sqrt{(m+1)^2}}{2m+2}$;

(XIV) $\chi(G) = \frac{1}{2}(n-3)m + \frac{2}{5}m\sqrt{5} + \frac{1}{6}m\sqrt{6} + \frac{2m}{\sqrt{4+m}} + \frac{1}{2}\frac{m(m-1)}{\sqrt{2m+2}}$;

(XV) $AZI(G) = 8(n-3)m + \frac{1753}{64}m + \frac{54m(m+1)^3}{(2+m)^3} + \frac{1}{16}\frac{m(m-1)(m+1)^6}{m^3}$.

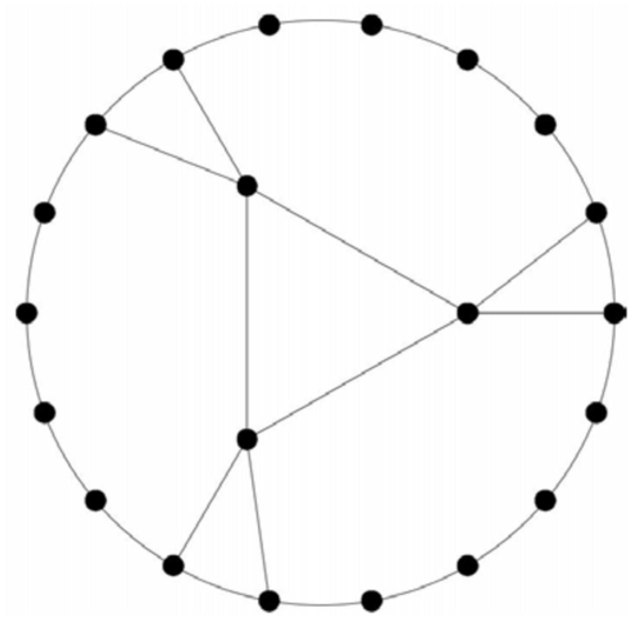

Figure 3. The line graph of Jahangir graph $J_{6;3}$.

Proof. The graph G for $n = 6$ and $m = 3$ is shown in Figure 3. By using Lemma 1.5, it is easy to see that the order of G is $m(n+1)$ out of which $2m$ vertices are of degree 3, m vertices are of degree $m+1$ and $(n-2)m$ vertices are of degree 2. Therefore by using Lemma 1.6, G has size

$\dfrac{m^2 + 2mn + 3m}{2}$. We partition the size of G into edges of the type $E_{(du;\, d\nu)}$, where $u\nu$ is an edge. In G, we get edges of the type $E_{(2;\,2)}$, $E_{(2;\,3)}$, $E_{(3;\,3)}$, $E_{(3;\,m+1)}$, and $E_{(m+1;\,m+1)}$. The number of edges of these types are given in the Table 1.

By using Formulas (1)-(12), Table 1 and by employing the Equations (16)-(23), we can obtain the required results. □

Table 1. The size partition of G

(d_u, d_ν), where $u\nu \in E(G)$	$(2, 2)$	$(2, 3)$	$(3, 3)$	$(3,\ m+1)$	$(m+1,\ m+1)$
Number of edges	$m(n-3)$	$2m$	m	$2m$	$\frac{m(m-1)}{2}$

Theorem 2.2. *Let G be the line graph of the Jahangir graph* $J_{n;m}$. *Then*

$$ABC_4(G) =$$

$$\begin{cases} m\sqrt{\dfrac{2m+10}{(m+6)^2}} + \dfrac{1}{2}m(m-1)\sqrt{\dfrac{2m^2+8}{(m^2+5)^2}} \\ \qquad + 2m\sqrt{\dfrac{m^2+m+9}{(m^2+5)(m+6)}} + \dfrac{1}{3}m\sqrt{6}\sqrt{\dfrac{m+10}{m+6}}, & \text{if } n = 3; \\ m\sqrt{\dfrac{2m+10}{(m+6)^2}} + \dfrac{1}{2}m(m-1)\sqrt{\dfrac{2m^2+8}{(m^2+5)^2}} \\ \qquad + 2m\sqrt{\dfrac{m^2+m+9}{(m^2+5)(m+6)}} + \dfrac{2}{5}m\sqrt{5}\sqrt{\dfrac{m+9}{m+6}}, & \text{if } n = 4; \\ \dfrac{1}{5}m\sqrt{35} + m\sqrt{\dfrac{2m+10}{(m+6)^2}} + \dfrac{1}{2}m(m-1)\sqrt{\dfrac{2m^2+8}{(m^2+5)^2}} \\ \qquad + 2m\sqrt{\dfrac{m^2+m+9}{(m^2+5)(m+6)}} + \dfrac{2}{5}m\sqrt{5}\sqrt{\dfrac{m+9}{m+6}}, & \text{if } n = 5; \\ \dfrac{1}{4}(n-5)m\sqrt{6} + \dfrac{1}{5}m\sqrt{35} + m\sqrt{\dfrac{2m+10}{(m+6)^2}} + \dfrac{1}{2}m(m-1)\sqrt{\dfrac{2m^2+8}{(m^2+5)^2}} \\ \qquad + 2m\sqrt{\dfrac{m^2+m+9}{(m^2+5)(m+6)}} + \dfrac{2}{5}m\sqrt{5}\sqrt{\dfrac{m+9}{m+6}}, & \text{if } n > 5. \end{cases}$$

Proof. We partition the size of G into edges of the type (S_u, S_v), where $uv \in E(G)$ as shown in Tables 2, 3, 4, and 5 for the case $n = 3$, $n = 4$, $n = 5$, and $n > 5$, respectively.

We know that

$$ABC_4(G) = \sum_{uv \in E(G)} \sqrt{\frac{S_u + S_v - 2}{S_u S_v}}.$$

Hence we get the required results by using Tables 2-5. □

Table 2. The size partition of G for $n = 3$

(S_u, S_v), where $uv \in E(G)$	$(m+6,\ m+6)$	$(m^2+5,\ m^2+5)$	$(m^2+5, m+6)$	$(m+6,\ 6)$
Number of edges	m	$\frac{m(m-1)}{2}$	$2m$	$2m$

Table 3. The size partition of G for $n = 4$

(S_u, S_v), where $uv \in E(G)$	$(m+6,\ m+6)$	$(m^2+5,\ m^2+5)$	$(m^2+5,\ m+6)$	$(m+6,\ 5)$
Number of edges	m	$\frac{m(m-1)}{2}$	$2m$	$2m$

Table 4. The size partition of G for $n = 5$

(S_u, S_v), where $uv \in E(G)$	$(4,\ 5)$	$(m+6,\ m+6)$	$(m^2+5,\ m^2+5)$	$(m^2+5,\ m+6)$	$(m+6,\ 5)$
Number of edges	$2m$	m	$\frac{m(m-1)}{2}$	$2m$	$2m$

Theorem 2.3. *Let G be the line graph of the Jahangir graph $J_{n;m}$. Then*

$$GA_5(G)=\begin{cases} m+\dfrac{m(m-1)}{2}+\dfrac{4m\sqrt{(m^2+5)(m+6)}}{m^2+m+11} \\ \qquad +\dfrac{4m\sqrt{6m+36}}{m+12}, & if\ n=3; \\ m+\dfrac{m(m-1)}{2}+\dfrac{4m\sqrt{(m^2+5)(m+6)}}{m^2+m+11} \\ \qquad +\dfrac{4m\sqrt{5m+30}}{m+11}, & if\ n=4; \\ \dfrac{8}{9}m\sqrt{5}+m+\dfrac{m(m-1)}{2}+\dfrac{4m\sqrt{(m^2+5)(m+6)}}{m^2+m+11} \\ \qquad +\dfrac{4m\sqrt{5m+30}}{m+11}, & if\ n=5; \\ (n-5)m+\dfrac{8}{9}m\sqrt{5}+m+\dfrac{m(m-1)}{2} \\ \qquad +\dfrac{4m\sqrt{(m^2+5)(m+6)}}{m^2+m+11}+\dfrac{4m\sqrt{5m+30}}{m+11}, & if\ n>5. \end{cases}$$

Table 5. The size partition of G for $n > 5$

(S_u, S_v), where $uv \in E(G)$	$(4, 4)$	$(4, 5)$	$(m+6, m+6)$	(m^2+5, m^2+5)	$(m^2+5, m+6)$	$(m+6, 5)$
Number of edges	$m(n-5)$	$2m$	m	$\frac{m(m-1)}{2}$	$2m$	$2m$

Proof. We know that

$$GA_5(G) = \sum_{uv\in E(G)} \frac{2\sqrt{S_u S_v}}{S_u + S_v}.$$

Hence we get the required results by using Tables 2-5. □

Theorem 2.4. *Let G be the line graph of the Jahangir graph* $J_{n;m}$. *Then*

$$S(G)=\begin{cases} \dfrac{m(m+6)^6}{(2m+10)^3}+\dfrac{1}{2}\dfrac{m(m-1)(m^2+5)^6}{(2m^2+8)^3} \\ \qquad +\dfrac{2m(m^2+5)^3(m+6)^3}{(m^2+m+9)^3}+\dfrac{432m(m+6)^3}{(m+10)^3}, & if\ n=3; \\ \dfrac{m(m+6)^6}{(2m+10)^3}+\dfrac{1}{2}\dfrac{m(m-1)(m^2+5)^6}{(2m^2+8)^3} \\ \qquad +\dfrac{2m(m^2+5)^3(m+6)^3}{(m^2+m+9)^3}+\dfrac{250m(m+6)^3}{(m+9)^3}, & if\ n=4; \\ \dfrac{16000}{343}m+\dfrac{m(m+6)^6}{(2m+10)^3}+\dfrac{1}{2}\dfrac{m(m-1)(m^2+5)^6}{(2m^2+8)^3} \\ \qquad +\dfrac{2m(m^2+5)^3(m+6)^3}{(m^2+m+9)^3}+\dfrac{250m(m+6)^3}{(m+9)^3}, & if\ n=5; \\ \dfrac{512}{27}(n-5)m+\dfrac{16000}{343}m+\dfrac{m(m+6)^6}{(2m+10)^3}+\dfrac{1}{2}\dfrac{m(m-1)(m^2+5)^6}{(2m^2+8)^3} \\ \qquad +\dfrac{2m(m^2+5)^3(m+6)^3}{(m^2+m+9)^3}+\dfrac{250m(m+6)^3}{(m+9)^3}, & if\ n>5. \end{cases}$$

Proof. We know that

$$S(G) = \sum_{u\nu\in E(G)} \left(\frac{S_u S_\nu}{S_u + S_\nu - 2} \right)^3.$$

Hence we get the required results by using Tables 2-5.

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