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ADDITIONAL PROPERTIES AND CHARACTERIZATIONS OF T_0, *Qo*, WEAKLY *Qo*, AND MAXIMAL, PROPER, DENSE, *Qo OXTO* SUBSPACES IN WEAKLY *Qo* AND "Not-T_0" SPACES

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Abstract

Within this paper, recent discoveries in the study of weakly *Po* spaces and properties are used to further characterize each of T_0, *Qo*, and weakly *Qo* and to expand known results for *OXTO* subsets concerning maximal, proper, dense subspaces to all spaces that are weakly *Qo* and "not-T_0".

1. Introduction and Preliminaries

In the 1936 paper [11], for a space (X, T), an externally generated, strongly (X, T) related space, called the T_0-identification space of (X, T), was introduced and used to further characterize each of metrizable and pseudometrizable.

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Definition 1.1. Let (X, T) be a space, let R be the equivalence relation on X defined by xRy iff $Cl(\{x\}) = Cl(\{y\})$, let X_0 be the set of R equivalence classes of X, let $N : X \to X_0$ be the natural map, and let $Q(X, T)$ be the decomposition topology on X_0 determined by (X, T) and the map N. Then $(X_0, Q(X, Y))$ is the T_0-identification space of (X, T).

Theorem 1.1. *A space (X, T) is pseudometrizable iff $(X_0, Q(X, T))$ is metrizable.*

T_0-identification spaces were cleverly created to add T_0 to the externally generated, strongly (X, T) related T_0-identification space of (X, T) [11], which, when combined with the fact that metrizable and (pseudometrizable and T_0) are equivalent, was used to establish the result above.

The characterization of metrizable given above raised the following questions: What property, if any, together with T_i would be equivalent to T_{i+1}, $i = 0, 1$, respectively?, which led to the introduction and investigation of the R_0 and R_1 separation axioms.

The R_0 separation axiom was introduced in the 1943 paper [10].

Definition 1.2. A space (X, T) is R_0 iff for each closed set C and each $x \notin C$, $C \cap Cl(\{x\}) = \phi$.

In the 1961 paper [1], the R_0 separation axiom was rediscovered and used to resolve the question above for T_1 and the R_1 separation axiom was introduced and used to resolve the question above for T_2.

Definition 1.3. A space (X, T) is R_1 iff for x and y in X such that $Cl(\{x\}) \neq Cl(\{y\})$, there exist disjoint open sets U and V such that $x \in U$ and $y \in V$.

Theorem 1.2. *A space is T_i iff it is $(R_{i-1}$ and $T_{i-1})$; $i = 1, 2$, respectively* [1].

In the 1961 paper [1], it was shown that R_1 implies R_0, which was used with the result above to show a space is T_2 iff it is $(R_1$ and $T_0)$. Thus the questions of whether T_0-identification spaces could be used to further characterize R_i and $T_{i+1}, i = 0, 1,$ as in the case of pseudometrizable and metrizable given above arose.

In the 1975 paper [9], T_0-identification spaces were used to further characterize R_1 and T_2.

Theorem 1.3. *A space (X, T) is R_1 iff its T_0-identification space $(X_0, Q(X, T))$ is T_2* [9].

Within the paper [2], T_0-identification spaces proved to be a useful tool to further characterize T_0 : A space (X, T) is T_0 iff the natural map $N : (X, T) \rightarrow (X_0, Q(X, T))$ is a homeomorphism.

Within the paper [3], the metrizable and Hausdorff properties were generalized to weakly *Po* properties.

Definition 1.4. Let *P* be topological properties such that *Po* = (*P* and T_0) exists. Then a space (X, T) is weakly *Po* iff its T_0-identification space $(X_0, Q(X, T))$ has property *P*. A topological property *Po* for which weakly *Po* exists is called a weakly *Po* property [3].

Since for a space, its T_0-identification space has property T_0, then, for a topological property *P* for which *Po* exists, a space is weakly *Po* iff its T_0-identification space has property *Po*.

Within the paper [3], it was shown that for a weakly *Po* property *Qo*, a space is weakly *Qo* iff its T_0-identification space is weakly *Qo*, which led to the introduction and investigation of T_0-identification *P* properties [4].

Definition 1.5. Let S be a topological property. Then S is a T_0-identification P property iff both a space and its T_0-identification space simultaneously shares property S.

In the introductory weakly Po property paper [3], it was shown that R_0 = weakly $(R_0)o$ = weakly T_1 and that weakly Po is neither T_0 nor "not-T_0", where "not-T_0" is the negation of T_0. The need and use of "not-T_0" revealed "not-T_0" as a useful topological property and tool, motivating the inclusion of the long-neglected properties "not-P", where P is a topological property for which "not-P" exists, as important properties for investigation and use in the study of topology. As a result, within a short time period, many new, important, fundamental, foundational, never before imagined properties have been discovered, expanding and changing the study of topology forever.

In past studies of weakly Po spaces and properties, for a classical topological property Qo, a special topological property W was sought such that for a space with property W, its T_0-identification space has property Qo, which then implied the initial space has property W, with no certainty that such a topological property W exists. As given above, the study of weakly Po spaces and related properties has been a productive study, but, if the past search process continued, the study of weakly Po spaces and properties would continue to be uncertain, tedious, and never ending. To make the process more certain, the question of precisely which topological properties are weakly Po properties arose leading to answers in the 2017 paper [5].

Answer 1.1. Let Q be a topological property for which both Qo and (Q and "not-T_0") exist. Then Q is a T_0-identification P property that is weakly Po and Q = weakly Qo = (Qo or (Q and "not-T_0")) [5].

Answer 1.2. $\{Qo \mid Q$ is a T_0- identification P property $\} = \{Qo \mid Qo$ is a weakly Po property$\} = \{Qo \mid Q$ is a topological property and Qo exists$\}$ [5].

Thus, major progress was achieved in the study of weakly Po and related properties. If Q is a topological property for which both Qo and (Q and "not-T_0") exist, Answer 1.1 quickly and easily gives weakly Qo. If Q is a topological property for which $Q = Qo$, then $Q = Qo$ is a weakly Po property, but $Q = Qo$ is not a T_0-identification P property or weakly Po. Within the paper [5], a topological property W that can be both T_0 and "not-T_0" was shown to exist that is a T_0-identification P property that is weakly Po such that W = weakly Qo, again making the search process certain, but, just knowing such a W exists, gave little insight into the precise, needed topological property W, raising the question of whether the known information could somehow be used to more precisely determine W for the fixed Qo.

The investigation of that question led to the introduction and investigation of $OXTO$ subsets and the corresponding subspace for each space (X, T) [6].

Definition 1.6. Let (X, T) be a space and for each $x \in X$, let C_x be the T_0-identification space equivalence class containing x. Then Y is an $OXTO$ is a subset of X iff Y contains exactly one element from each equivalence class C_x.

Within the paper [6], it was shown that for each $OXTO$ subset Y of X, (Y, T_Y) is homeomorphic to $(X_0, Q(X, T))$ Since, as stated earlier, the T_0-identification space of each space is T_0 and T_0 is a topological property, then for each $OXTO$ subset Y of X in a space (X, T), (Y, T_Y) is T_0. Also, within the paper [6], it was shown that for each topological property Q for which Qo exists, a space (X, T) is weakly Qo iff for each

OXTO subset Y of X, (Y, T_Y) has property *Qo*, which can be, and has been, used to precisely determine weakly *Qo* [6]. Thus, the study of weakly *Po* and weakly *Po* properties has been completely internalized and greatly simplified by the use of *OXTO* subsets and the corresponding subspaces and the earlier results, replacing many, earlier uncertainties with certainties.

As given above, the behaviour of R_0 and R_1 in T_0 spaces is long-known raising questions about their behaviour in "not-T_0" spaces. The investigation of those questions again revealed "not-T_0" as a strong, useful topological tool whose use continued to revealed additional never before imagined properties in the study of topology, including the results below.

Theorem 1.4. *Let* (X, T) *be a space. Then the following are equivalent*: (a) (X, T) *is "not-*T_0*"*, (b) (X, T) *has a maximal, proper, dense* T_0 *subspace, and* (c) *for each OXTO subset* Y *of* X, (Y, T_Y) *is a maximal, proper, dense* T_0 *subspace of* (X, T) [7].

Theorem 1.5. *Let* (X, T) *be a "not-*T_0*" space. Then* (X, T) *is* R_0 *iff for each OXTO subset* Y *of* X, (Y, T_Y) *is a maximal, proper, dense* T_1 *subspace of* (X, T) [7].

Theorem 1.6. *Let* (X, T) *be a "not-*T_0*" space. Then* (X, T) *is* R_1 *iff for each OXTO subset* Y *of* X, (Y, T_Y) *is a maximal, proper, dense* T_2 *subspace of* (X, T) [7].

The results above raise the following question: Can the results above be extended to all weakly *Qo* and "not-T_0" properties?

Within this paper, *OXTO* subsets and *OXTO* and related spaces for a space are used to further characterize T_0, weakly *Qo*, and *Qo*, and the answer to the question above is given.

2. Additional Characterizations of Each of T_0 and Weakly *Po*

Theorem 2.1. *Let* (X, T) *be a space and let Y be an OXTO subset of X. Then all of* (Y, T_Y), $(Y_0, Q(Y, T_Y))$, *and* $(X_0, Q(X, T))$ *are homeomorphic.*

Proof. By the results above, (Y, T_Y) and $(X_0, Q(X, T))$ are homeomorphic. Since (Y, T_Y) is T_0, then the natural map from (Y, T_Y) onto $(Y_0, Q(Y, T_Y))$ is a homeomorphism. Thus, both $(X_0, Q(X, T))$ and $(Y_0, Q(Y, T_Y))$ are homeomorphic to (Y, T_Y) and all of (Y, T_Y), $(X_0, Q(X, T))$, and $(Y_0, Q(Y, T_Y))$ are homeomorphic.

Theorem 2.2. *Let* (X, T) *be a space. Then each OXTO subset Y of X is minimal homeomorphic to* $(X_0, Q(X, T))$ *in that if Z is a subset of X containing Y, including Y itself, then* $(Z_0, Q(Z, T_Z))$ *is homeomorphic to each of* (Y, T_Y) *and* $(X_0, Q(X, T))$.

Proof. Let Y and Z be as above. Then Y is an *OXTO* subset of X and an *OZTO* subset of Z and (Y, T_Y) is homeomorphic to each of $(Z_0, Q(Z, T_Z))$ and $(X_0, Q(X, T))$.

In the paper [7], it was proven that for a space (X, T), all subspaces (Y, T_Y), where Y is an *OXTO* subset of X, are homeomorphic, which is used along with the results above to obtain additional characterizations of weakly *Po*.

Corollary 2.1. *Let Q be a topological property and let* (X, T) *be a space. Then the following are equivalent*: (a) (X, T) *is weakly Qo*, (b) *for each OXTO subset Y of* X, $(Y_0, Q(Y, T_Y))$ *has property Qo*, (c) *for each OXTO subset Y of* X, $(Y_0, Q(Y, T_Y))$ *has property weakly Qo*, (d) *for each OXTO subset Y of* X, (Y, T_Y) *has property weakly Qo*, (e) *for each OXTO* *subset Y of X*, (Y, T_Y) *has property Q*, (f) *Qo exists*, (g) *there exists an OXTO subset Y of X such that* (Y, T_Y) *has property Qo*, (h) *there exists an OXTO subset Y of X such that* $(Y_0, Q(Y, T_Y))$ *has property Qo*, (i) *there exists an OXTO subset Y of X such that* $(Y_0, Q(Y, T_Y))$ *has property weakly Qo*, (j) *there exists an OXTO subset Y of X such that* (Y, T_Y) *has property weakly Qo*, (k) *for each subset Z of X containing an OXTO subset Y of X, including Y itself*, $(Z_0, Q(Z, T_Z))$ *has property Qo*, (l) *for each subset Z of X containing an OXTO subset Y of X, including Y itself*, $(Z_0, Q(Z, T_Z))$ *has property weakly Qo*, (m) *there exists a subset Z of X containing an OXTO subset Y of X such that* $(Z_0, Q(Z, T_Z))$ *has property Qo, and* (n) *here exists a subset Z of X containing an OXTO subset Y of X such that* $(Z_0, Q(Z, T_Z))$ *has property weakly Qo.*

Since Corollary 2.1 can be applied for each topological property Q for which Qo exists and there are many such properties, Corollary 2.1 has tremendous applications giving many new, never before imagined characterizations for each Qo.

Definition 2.1. Let (X, T) be a space, let Y be an *OXTO* subset of X, and for each $x \in X$, let $\{yx\} = Y \cap C_x$, where C_x is the T_0-identification class containing x. Then f_Y defined by $f_Y(x) = y_x$ for each $x \in X$ is the function from (X, T) onto (Y, T_Y).

Theorem 2.3. *Let* (X, T) *be a space. Then the following are equivalent*: (a) (X, T) *is* T_0, (b) *for each* $x \in X$, $C_x = \{x\}$, (c) *for each OXTO subset Y of X*, f_Y, *as defined above, is the identity function*, (d) *for each OXTO subset Y of X*, f_Y, *as defined above, is one-to-one*, (e) *for each OXTO subset Y of X*, (X, T) *is homeomorphic to* (Y, T_Y), *and* (f) *for each OXTO subset Y of X*, (X, T) *is homeomorphic to* $(Y_0, Q(Y, T_Y))$.

Proof. (a) implies (b): Let $x \in X$. Let $y \in X$, $y \neq x$. Then there exists an open set containing only one of x and y and $Cl(\{x\}) \neq Cl(\{y\})$. Thus $y \notin C_x$ and $C_x = \{x\}$.

Clearly (b) implies (c).

(c) implies (d): Let Y be an *OXTO* subset of X. Let $x, z \in X$ such that $f_Y(x) = f_Y(z)$. Since f_Y is the identify function on X, then $f_Y(x) = x$ and $f_Y(z) = z$ and $x = f_Y(x) = f_Y(z) = z$. Thus f_Y is one-to-one.

(d) implies (e): Suppose (X, T) is "not-T_0". Let x and y be distinct elements of X such that every open set containing one of x and y contains both x and y. Thus $Cl(\{x\}) = Cl(\{y\})$ and $f_Y(x) = f_Y(y)$, but $x \neq y$, which is a contradiction. Hence (X, T) is T_0 and f_Y is the identity function, which is a homeomorphism. Thus (X, T) and (Y, T_Y) are homeomorphic.

(e) implies (f): Since (Y, T_Y) is T_0, then the natural map from (Y, T_Y) onto $(Y_0, Q(Y, T_Y))$ is a homeomorphism. Hence (X, T) is homeomorphic to (Y, T_Y), which is homeomorphic to $(Y_0, Q(Y, T_Y))$, which implies (X, T) and $(Y_0, Q(Y, T_Y))$ are homeomorphic.

(f) implies (a): Since $(Y_0, Q(Y, T_Y))$ is T_0 and T_0 is a topological property, then (X, T) is T_0.

Within the paper [8], it was proven that $L = (T_0$ or "not-T_0"$)$ is the least of all topological properties and, since $Lo = T_0$ exists, then, by the results above, L is the least of all topological properties that are weakly Qo.

Theorem 2.4. *Let (X, T) be a space and let Q be a topological property for which weakly Qo exists. Then the following are equivalent*: (a) (X, T) *has property Qo*, (b) (X, T) *is weakly Qo and for each $x \in X$, $C_x = \{x\}$*, (c) (X, T) *is weakly Qo and for each OXTO subset Y of*

X, (X, T) *and* (Y, T_Y) *are homeomorphic*, (d) (X, T) *is weakly Qo and for each OXTO subset of* X, (X, T) *and* $(Y_0, Q(Y, T_Y))$ *are homeomorphic*, (e) (X, T) *is weakly Qo and for each OXTO subset Y of* X, f_Y *is the identity function on X*, (f) (X, T) *is weakly Qo and for each OXTO subset of* X, f_Y *is one-to-one*, *and* (g) (X, T) *is weakly Qo and each OXTO subset of X equals X.*

The proof is straightforward using the results above and is omitted.

Thus, for each specific topological property *Qo*, by applying the result above, many additional characterizations of *Qo* are now known.

Theorem 2.5. *Let* (X, T) *be a space and let Q be a topological property for which Qo exists*. *Then the following are equivalent*: (a) (X, T) *is weakly Qo*, (b) *for each OXTO subset Y of* X, (Y, T_Y) *is Qo*, (c) *for each OXTO subset Y of* X, (Y, T_Y) *is weakly Qo*, (d) *for each OXTO subset Y of* X, $(Y_0, Q(Y, T_Y))$ *is weakly Qo*, (e) *for each OXTO subset Y of* X, $(Y_0, Q(Y, T_Y))$ *has property Qo*, (f) *for each subset Z of X containing an OXTO subset Y of* X, $(Z_0, Q(Z, T_Z))$ *is Qo*, (g) *for each subset Z of X containing an OXTO subset Y of* X, $(Z_0, Q(Z, T_Z))$ *is weakly Qo*, *and* (h) *for each subset Z of X containing an OXTO subset Y of* X, (Z, T_Z) *is weakly Qo*.

The proof is straightforward using the results above and is omitted.

Thus, for each specific topological property that is weakly *Qo*, by applying the result above, many additional characterizations of weakly *Qo* are now known.

3. Necessary and Sufficient Conditions for Each *OXTO* Subset to Give a Maximal, Proper, Dense, *Qo* Subspace

Definition 3.1. Let (X, T) be a space, let Y be a subspace of (X, T), and let Q be a topological property for which Qo exists. Then (Y, T_Y) is a maximal, proper, dense, Qo subspace of (X, T) iff (Y, T_Y) is a proper, dense, Qo subspace of (X, T) such that for each subspace $(Z, T_W))$ of (X, T), where Z properly containing Y, (Z, T_Z) is not Qo.

Theorem 3.1. *Let* (X, T) *be a space and let* Q *be a topological property for which* Qo *exists. Then* (a) *for each OXTO subset* Y *of* $X, (Y, T_Y)$ *is a maximal, proper, dense,* Qo *subspace of* (X, T) *iff* (b) (X, T) *is weakly* Qo *and "not-*T_0*".*

Proof. (a) implies (b): By definition, (X, T) has an *OXTO* subset Y. Then (Y, T_Y) is a maximal, proper, dense, Qo subspace of (X, T). If (X, T) is T_0, then $Y = X$, which is a contradiction. Thus (X, T) is "not-T_0". Since for every *OXTO* subset of X, (Y, T_Y) has property Qo, then (X, T) is weakly Qo. Thus (X, T) has property weakly Qo and "not-T_0".

(b) implies (a): Since (X, T) is "not-T_0", then for each *OXTO* subset Y of $X, (Y, T_Y)$ is a maximal, proper, dense, T_0 subspace of (X, T). Since (X, T) is weakly Qo, then for each *OXTO* subset Y of X, (Y, T_Y) is Qo. If Z is a subset of X that properly contains Y, then (Z, T_Z) is "not-T_0", which implies (Z, T_Z) is "not-Qo". Thus, for each *OXTO* subset Y of X, (Y, T_Y) is a maximal, proper, dense, Qo subspace of (X, T).

Theorem 3.2. *Let* (X, T) *be a space let* Q *be a topological property for which* Qo *exists, and let* Z *be a subset of* X *that properly contains an OXTO subset* Y *of* X*. Then* (a) (Y, T_Y) *is a maximal, proper, dense,* Qo *subspace of* (Z, T_Z) *iff* (b) (Z, T_Z) *is weakly* Qo *and "not-*T_0*".*

Proof. Since Z is a subset of X that properly contains the *OXTO* subset Y of X, then Y is an *OZTO* subset of Z. Then by Theorem 3.1, (a) and (b) are equivalent.

Corollary 3.1. *Let (X, T) be a space, let Q be a topological property for which Qo exists, let (X, T) be weakly Qo and "not-T_0", and let Y be an OXTO subset of X. Then for each subset Z of X that properly contains Y, (Z, T_Z) is weakly Qo and "not-T_0".*

Thus, for a weakly Qo and "not-T_0" space (X, T) and an *OXTO* subset of X, all subspaces bigger than (Y, T_Y) share the same property; weakly Qo and "not-T_0", raising the question of whether all subspaces smaller than (Y, T_Y) share the property Qo with (Y, T_Y)?

As is known in the study of classical topology, there is a topological property Q for which Qo exists and Q = weakly Qo fails to be a subspace property [12]. Such a property is normal and in the book [12], a space that is (normal)$o = T_4$ and has a subspace that is not T_4 is given. Within the paper [5], a construction for each Po space (Z, S) was given that gave a weakly Po and "not-T_0" space (X, T) whose T_0-identification space $(X_0, Q(X, T))$ is homeomorphic to (Z, S). Below the results above are used to show the answer to the question above is no.

Theorem 3.3. *Let Q be a topological property for which Qo exists and Qo is not a subspace property, let (Z, S). be a space with property Qo that has a subspace that is not Qo, let (X, T) be a weakly Qo and "not-T_0" space whose T_0-identification space $(X_0, Q(X, T))$ is homeomorphic to (Z, S), and let Y be an OXTO subset of X. Then (Y, T_Y) has a proper subspace that does not share the property Qo with (Y, T_Y).*

Proof. Since (Y, T_Y) is homeomorphic to $(X_0, Q(X, T))$, which is homeomorphic to (Z, S), then (Y, T_Y) is homeomorphic to (Z, S). Since (Y, T_Y) has property Qo, (Z, S) has a proper subspace that in "not-Qo", and (Y, T_Y) is homeomorphic to (Z, S), then (Y, T_Y) has a proper subspace that is "not-Qo".

As established below, had Qo been a subspace property, then things would have been different.

Theorem 3.4. *Let Q be a topological property for which Qo exists and Qo is a subspace property, let (X, T) be a space with property weakly Qo and "not-T_0", and let Y be an OXTO subset of X. Then all subspaces of (X, T) bigger than (Y, T_Y) have property weakly Qo and "not-T_0" and all subspaces of (X, T) smaller or equal to (Y, T_Y) have property Qo.*

Proof. By results above all subspaces of (X, T) bigger than (Y, T_Y) have property weakly Qo and "not-T_0" and (Y, T_Y) has property $Qo = Q$ and T_0. Since both Q and T_0 are subspace properties, then each subspace of (Y, T_Y) has property Q and $T_0 = Qo$.

Thus, the continued investigation of weakly Po and use of the strong topological property "not-T_0" has given additional insights into the study of subspace and revealed other basic, foundational, never before imagined properties in the field of topology.

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