

A JOINT ALGORITHM OF EQUALIZER AND DETECTOR BASED ON CODED-OFDM SYSTEMS IN HIGH-SPEED CHANNELS

LEI LIN

School of Mathematics and Systems Science

Guangdong Polytechnic Normal University

Guangzhou 510665

P. R. China

e-mail: linlei2@mail2.sysu.edu.cn

Abstract

In this letter, we present a joint algorithm of equalization and detector based on block Markov superposition transmission (BMST) codes for OFDM systems (called BMST-OFDM) over high-speed channels. In the BMST-OFDM systems, the equalizer can exchange soft messages with the demapper and BMST decoder in an iterative fashion. Simulation results show that: (1) our proposed equalization algorithm can perform better than the conventional algorithms in the BMST-OFDM systems, although the proposed equalizer has a higher error floor in the un-coded systems, (2) our proposed equalization algorithm has lower computation complexity than the conventional algorithms, which require to construct a huge state space or perform the inversion of matrix, and (3) the performances of the proposed scheme match well with the lower bounds in the high signal-to-noise ratio (SNR) region.

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1. Introduction

Orthogonal frequency division multiplexing (OFDM) is an effective technique for mitigating the effect of delay spread introduced by the multipath channel. It is widely used in the existing wireless communication standards such as: digital video broadcasting (DVB), worldwide interoperability for microwave access (WiMAX), long term evolution (LTE) and LTE advanced (LTE-A) fourth generation (4G) mobile communications [1]. Recently, the fifth generation (5G) network has been introduced to enable to support high-mobility communication scenarios up to 500km/h [2]. In the high-mobility channels, the rapidly time-variation (TV) of the channel over an OFDM block corrupts the orthogonality among each subcarrier and induces intercarrier interference (ICI), which results in an irreducible error floor in conventional equalizers.

There have been many previous works devoted to counteract such ICI effects in un-coded OFDM systems [3, 4] or the coded OFDM systems [5-10]. This paper focuses on the latter. In [6], the reduced state maximum a posteriori (MAP) equalization was presented by exploiting the banded structure of the frequency-domain channel matrix. In [7], the optimal maximum likelihood (ML) receiver joint equalizer and decoding was proposed, which is based on space alternating generalized expectation maximization (SAGEM) technique. In [8], two low-complexity MAP equalizations are proposed based on sequential detection with optimal ordering (SDOO) and sequential detection with successive cancellation (SDSC). Recently, a soft output algorithm based on zero forcing (ZF) equalization over the high-mobility channels was proposed in [9]. However, such equalization schemes has huge implementation complexity, due to a huge state space or the inversion of matrix. In [10], a reduced-complexity maximum-likelihood (RC-ML) equalization with ICI mitigation was proposed. Although this equalization scheme can reduce computation complexity, the performance degrades due to hard decision feedback in an iterative fashion.

In recent years, a coded scheme called block Markov superposition transmission (BMST) codes was proposed [11], whose performance can be predicted by a simple genie-aided bound. In addition, the design of BMST codes is flexible in the sense that any code of any rate with fast encoding algorithm and efficient SISO decoding algorithm can be taken as the basic code. In this letter, we present a simple soft-in soft-out (SISO) equalization based on BMST codes for OFDM systems (called BMST-OFDM) over high-mobility channels. In the BMST-OFDM systems, the SISO equalizer, the soft demapper, and the decoder exchange soft messages from each other in an iterative fashion. Simulation results show that the performance of the proposed equalization algorithm outperforms the conventional algorithms, while preserving lower implementation complexity.

2. System Model

2.1. OFDM system

We consider a coded OFDM system with N subcarriers over the high-mobility channels depicted in Figure 1. At the transmitter side, the information bit sequence \mathbf{u} is transformed into a coded sequence \mathbf{c} by the encoder. Then the sequence \mathbf{c} , one block of the sequence is directly mapped into the vector of N -coded frequency-domain symbols $\mathbf{X} = [X(0), \dots, X(N-1)]^T$ according to the modulation. The symbol $X(k)$ is chosen from a complex signal constellation \mathcal{S} with $|\mathcal{S}| = 2^{M_c}$, and each symbol carries M_c coded bits. After inverse discrete Fourier transform (IDFT) and cyclic prefix (CP) insertion, the time-domain signal can be obtained. Then the time-domain symbol is transmitted in the high-mobility channel, in which the CP length is greater than the maximum delay spread of channel L . At the receiver side, after CP removal and the discrete Fourier transform (DFT), the received vector can be expressed as

$$\mathbf{Y} = \mathbf{GX} + \mathbf{W}, \quad (1)$$

where \mathbf{Y} is the $N \times 1$ received vector, \mathbf{W} is $N \times 1$ noise vector, whose entries are independent and identically distributed (i.i.d.) according to $\mathcal{CN}(0, 1)$. The symbol \mathbf{G} presents the $N \times N$ frequency-domain channel matrix. Due to the TV nature of the high-mobility channel, \mathbf{G} is no longer diagonal, and each diagonal element is associated with a discrete Doppler spread, which results in ICI.

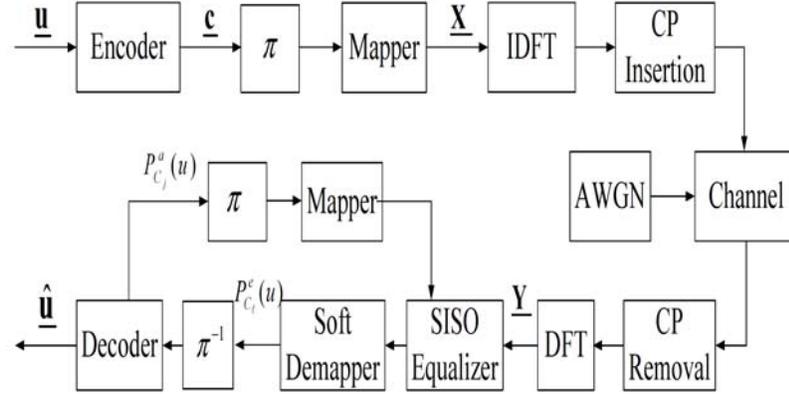


Figure 1. The block diagram of the coded OFDM system.

2.2. Channel model

As mentioned above, the high-mobility channels are considered as rapidly TV multipath fading channels. Since time variation implies time-selective fading, while multipath implies frequency selective fading, we actually consider discrete-time doubly-selective fading channels. The impulse response of the discrete time channel in the ℓ -th tap can be written as

$$h(n, \ell) = \sum_{\ell=0}^{L-1} \gamma_0(nT_s, \ell T_s), \quad (2)$$

where the continuous-time channel $\gamma_0(t, \tau)$ is wide-sense stationary with uncorrelated scattering (WSSUS), τ is the delay spread between the transmitted and received signals, T_s is the sampling period. We adopt the classical Jakes model [12], whose autocorrelation is given by

$$\mathbb{E}[h(n, \ell)h^*(n + \Delta n, \ell + \Delta \ell)] = R_h(\Delta n T_s) \sigma_\ell^2 \delta(\Delta \ell), \quad (3)$$

where $R_h(\Delta n T_s) = J_0(2\pi\Delta n f_d T_s)$ is the normalized time-correlation function, $J_0(\cdot)$ is the zeroth-order Bessel function of first kind, $\sigma_\ell^2 R_h(0) = \sigma_\ell^2$ is the average power of the ℓ -th tap. The normalized maximum Doppler frequency shift can be defined as $\vartheta_{\max} = \frac{f_c}{F} \frac{\nu}{c_0}$, where f_c is the carrier frequency (Hz), ν is the speed of the mobile terminal, and c_0 is the speed of light, F is the subcarrier space. With the Jakes Doppler spectrum, the normalized ICI power is described as [6]. It shows that the symbol energy leakage caused by ICI is most concentrated in the neighbouring. For instance, more than 99.6% of the symbol energy collects in the 3 neighbouring subcarriers at the speed of 600km/h. This makes us aware that the energy of the useful subcarrier is still dominant in the receiver, so we can design the corresponding equalization to eliminate the ICI in the high-mobility channels.

3. Detection/Equalization Algorithm

3.1. A soft-in soft-out equalization

In practical OFDM systems, when the velocity of the high-speed rail is 360km/h, the maximum normalized Doppler frequency $\nu_{\max} = 4 * 10^{-3}$ is small. The nondiagonal elements of the frequency-domain channel matrix \mathbf{G} are not large, and symbol energy leakage is still most concentrated in the neighbouring subcarriers. In order to obtain good performance with effective complexity, we propose a low-complexity SISO equalization based on treating the ICI as noise, which is

similar to [10]. When the number of subcarriers is normally large, we can take the ICI as Gaussian distributed and its power is determined by the variance. So the Equation (1) can be rewritten as

$$\begin{aligned} \mathbf{Y} &= \text{diag}(\mathbf{G})\mathbf{X} + \mathbf{G}'\mathbf{X} + \mathbf{W} \\ &= \text{diag}(\mathbf{G})\mathbf{X} + \hat{\mathbf{W}}, \end{aligned} \quad (4)$$

where $\text{diag}(\cdot)$ denotes a diagonal matrix with the main diagonal elements given by the matrix, \mathbf{G}' is matrix \mathbf{G} with the main diagonal elements set to zeros, $\hat{\mathbf{W}}$ is the equivalent noise vector, in which the k -th element is a complex Gaussian variable with mean $\mu_{\hat{W}}[k]$ and variance $\eta_{\hat{W}}[k]$.

$$\begin{aligned} \mu_{\hat{W}}[k] &= \sum_{i \neq k}^N \mathbf{g}'_{k,i} \mu_X[k], \\ \eta_{\hat{W}}[k] &= \sum_{i \neq k}^N |\mathbf{g}'_{k,i}|^2 \eta_X[k] + \sigma_w^2, \end{aligned} \quad (5)$$

where X is a random variable belonging to the complex signal constellation \mathcal{S} . Denote $\mathbf{G}' = [\mathbf{g}'_0, \mathbf{g}'_1, \dots, \mathbf{g}'_{N-1}]^T$, \mathbf{g}'_k represents the k -th row of matrix \mathbf{G}' . The symbols $\mu_X[k]$ and $\eta_X[k]$ are the mean and variance of the k -th subcarrier $X[k]$, respectively, which are computed through $\mu_X[k] = \sum_{X \in \mathcal{S}} X P^a\{X[k] = X\}$, $\eta_X[k] = \sum_{X \in \mathcal{S}} X^2 P^a\{X[k] = X\} - \mu_X^2[k]$, where P^a denotes a prior probability of a random variable X .

Based on the Gaussian approximation, we can compute the likelihood function $f(Y = Y[k]|X[k], \mathbf{G})$ to approximate $f(\mathbf{Y}|\mathbf{X}, \mathbf{G})$ for any symbol $X[k]$ in \mathbf{X} transmitted on one subcarrier. The symbol Y is a random variable corresponding to the k -th received symbol $Y[k]$. Then we can use the following reduced-complexity metric:

$$f(Y = Y[k]|X[k], \mathbf{G}) \propto$$

$$\frac{1}{\pi\eta_{\hat{W}}[k]} \exp\left(-\frac{|Y[k] - G(k, k)X[k] - \mu_{\hat{W}}[k]|^2}{\eta_{\hat{W}}[k]}\right). \quad (6)$$

The above proposed SISO equalization has similar implementation complexity as RC-ML equalization [10], but has lower computation complexity compared with ZF equalization [9]. In particular, this proposed algorithm can be suitable for large subcarriers and high-order modulation.

3.2. SISO demapping algorithm

After the fore-mentioned SISO equalizer, the system requires an SISO demapping algorithm, which takes the *a priori messages* (from BMST) as input and delivers the extrinsic messages (to BMST) as output. We use C_ℓ to denote a random variable corresponding to c_ℓ (the ℓ -th component of $\tilde{\mathbf{c}}$). A bit sequence $\tilde{\mathbf{c}}$ is mapped to a symbol $X[k]$ and is denoted by $X[k] = \varphi(\tilde{\mathbf{c}})$. We denote the *a priori* message and the *extrinsic* message of a random variable C_ℓ as $P_{C_\ell}^a(u)$ and $P_{C_\ell}^e(u)$, $u \in \mathbb{F}_2$, respectively. The SISO algorithm described below is a key element of the decoding algorithm for BMST-OFDM systems in the high-mobility channels.

Algorithm 1. The SISO Demapping for BMST-OFDM

- **Input:** Take as inputs the received vector from the equalizers and the *a priori* messages $P_{C_\ell}^a(u)$, $u \in \mathbb{F}_2$ ($0 \leq \ell \leq M_c - 1$).

- **Output:** Deliver as outputs the extrinsic messages

$$P_{C_\ell}^e(u) \propto \sum_{\substack{\tilde{\mathbf{c}} \in \mathbb{F}_2^{M_c}, \\ c_\ell = u}} f(Y = Y[k]|\varphi(\tilde{\mathbf{c}})) \prod_{\substack{j=0, \\ j \neq \ell}}^{M_c-1} P_{C_j}^a(c_j), \quad u \in \mathbb{F}_2,$$

for $0 \leq \ell \leq M_c - 1$.

4. Encoding and Decoding Algorithm

4.1. BMST

In the following simulations, we will combine the BMST with the rapidly TV channel in the OFDM systems depicted in Section 3. Let $\mathcal{C}[n, k]$ be a binary linear *basic code* of length n and dimension k . We assume that $n = M_c N B$ for some positive integer B , where M_c is the bit number of the transmitted symbol per subcarrier. Let $\underline{\mathbf{u}}^{(t)} \in \mathbb{F}_2^k (0 \leq t \leq T - 1)$ be T sub-blocks of data to be transmitted. The encoding process with memory m is described as follows:

Algorithm 2. The Encoding for BMST-OFDM Systems

(1) **Initializing:** For $t < 0$, set $\underline{\mathbf{v}}^{(t)} = \underline{\mathbf{0}} \in \mathbb{F}_2^n$.

(2) **Loop:** For $t = 0, 1, \dots, T - 1$,

(a) *Encoding:* Encode $\underline{\mathbf{u}}^{(t)}$ into $\underline{\mathbf{v}}^{(t)} \in \mathbb{F}_2^n$ by the encoding algorithm of the basic code \mathcal{C} ;

(b) *Interleaving:* For $0 \leq j \leq m$, interleave $\underline{\mathbf{v}}^{(t-j)}$ by the j -th interleaver \prod_j into $\underline{\mathbf{w}}^{(j)}$;

(c) *Superposition:* Compute $\underline{\mathbf{c}}^{(t)} = \sum_{0 \leq i \leq m} \underline{\mathbf{w}}^{(i)}$;

(d) *Mapping:* Rewrite $\underline{\mathbf{c}}^{(t)} = [\mathbf{c}_0^{(t)} \mathbf{c}_1^{(t)} \dots \mathbf{c}_{B-1}^{(t)}]$, where $\mathbf{c}_j^{(t)} \in \mathbb{F}_2^{M_c N}$.

Map the sub-block $\underline{\mathbf{c}}^{(t)}$ into the transmitted signal vectors $\underline{\mathbf{X}}^{(t)} = [\mathbf{X}_0^{(t)} \mathbf{X}_1^{(t)} \dots \mathbf{X}_{B-1}^{(t)}]$.

(3) **Termination:** For $t = T, T + 1, \dots, T + m - 1$, set $\underline{\mathbf{u}}^{(t)} = \underline{\mathbf{0}} \in \mathbb{F}_2^k$ and compute $\underline{\mathbf{X}}^{(t)}$ following Step (2).

The decoding algorithm is taken the sliding-window decoding into account in [18].

4.2. Genie-aided lower bound

In this section, the genie-aided lower bound of the BMST-OFDM system is analyzed. Similar to the analyses in [11], for a given time t , if all the transmitted data $\underline{\mathbf{u}}^{(\ell)}$ ($0 \leq \ell \leq L-1$, $\ell \neq t$) but $\underline{\mathbf{u}}^{(t)}$ are known to the receiver, the BMST system is reduced to an equivalent system similar to that in Figure 3 of [13], where the only difference is that the high-order mapping ϕ is replaced by the signal mapping. Let $p = f_{Genie}(\gamma)$ and $p = f_{BMST-OFDM}(\gamma)$ be the performance functions of the equivalent system and the BMST-OFDM system, respectively, where p is the bit-error-rate (BER) and $\gamma \triangleq \frac{1}{\sigma^2}$ in dB. The $f_{BMST-OFDM}(\gamma)$ can be lower bounded as

$$f_{BMST-OFDM}(\gamma) \geq f(\gamma). \quad (7)$$

The above genie-aided lower bound is derived by assuming that $\underline{\mathbf{u}}^{(t)}$ is transmitted $(m+1)$ -times without interference from adjacent sub-block. The *asymptotic extra* coding gain of the BMST system over the basic system (with encoding memory of zero) can be roughly predicted as $10 \log_{10}(m+1)$ dB.

5. Numerical Results

This section presents the results of computer simulation of the proposed methods for rapidly time-varying channels [12]. The total bandwidth B_f is 1.9MHz, the number of subcarriers in each OFDM is $N = 128$, and corresponding subcarrier spacing F is 15kHz, carrier frequency $f_c = 2.0$ GHz, mobile terminal speed $\nu = 360$ km/h. The CP length is $N_{cp} = 8$. In the BMST-OFDM systems, the encoder uses

uniformly generated (but fixed) interleavers and terminates with every $T = 1000$ data blocks. The examples use the Cartesian product of the single parity-check (SPC) code as basic code $[3, 2]^{2048}$. The proposed SISO equalizer and the sliding window decoding algorithm perform with a maximum iteration number of 18 and an entropy stopping threshold of 10^{-5} .

Example 1. In this example, the quadrature phase-shiftkeying (QPSK) modulation is employed. The BER performance of the various schemes for QPSK modulation is depicted in Figure 2. For un-coded OFDM systems, we can observe that the proposed equalizer has a similar performance to the RC-ML equalizer, and is better than ZF equalizer in the low SNR region. However, the proposed equalizer and the RC-ML equalizer have higher error floors compared with the ZF equalizer in high SNR region, because the ICI effect exists in the high-mobility channels. In the BMST-OFDM systems, the proposed scheme and RC-ML scheme outperform the ZF scheme without iteration and with iteration at BER of 10^{-5} , and the proposed scheme has the best BER performance. On the other hand, it is observed that the iteration does not make any obvious deference for the proposed and the RC-ML schemes, having gains about 0.3dB at BER of 10^{-5} .

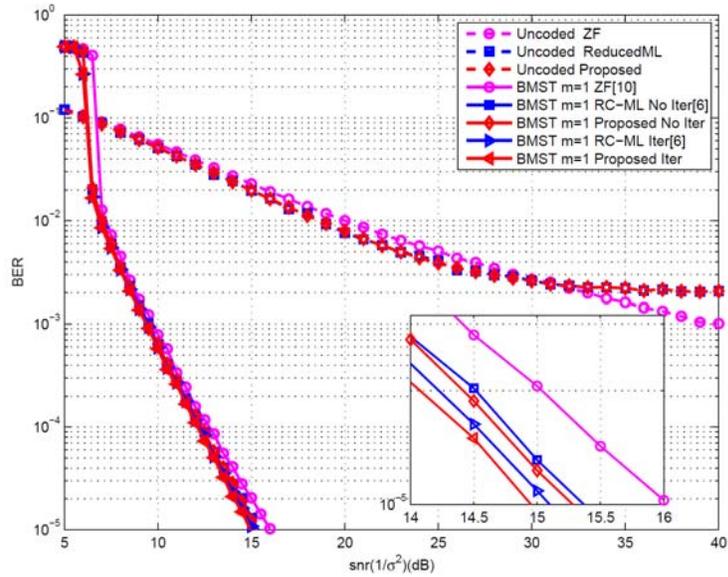


Figure 2. The BER performance of the proposed scheme and conventional schemes [9, 10] in the BMST-OFDM ($m = 1$) systems for QPSK modulation at the speed of 360km/h.

Example 2. In this example, the 16QAM modulation is used in Figure 3. In the un-coded systems, it is not surprising that the proposed equalizer still has a similar performance to the RC-ML equalizer, but is worse than ZF equalizer, because the ZF equalizer makes good use of the channel matrix elements.

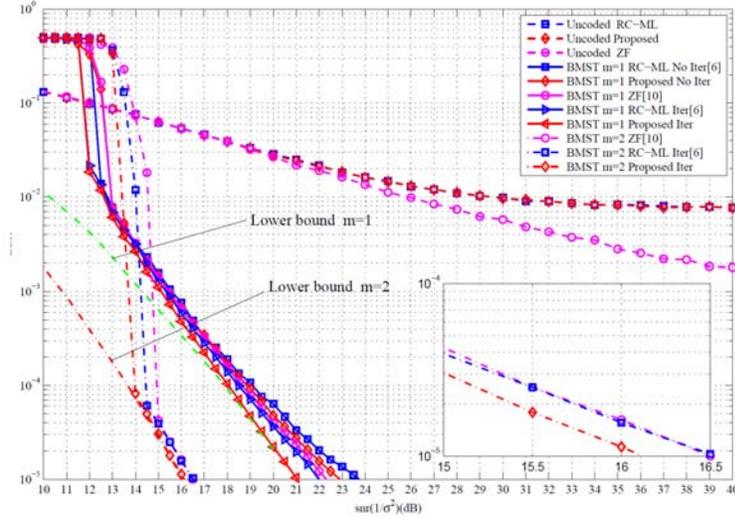


Figure 3. The BER performance of the proposed scheme and conventional schemes [9, 10] for 16QAM modulation at the speed of 360km/h, and the lower bounds in the BMST-OFDM ($m = 1, 2$) systems.

When the BMST codes with $m = 1$ is used, it shows that the proposed scheme outperforms the RC-ML scheme without iteration. Although the proposed scheme is still not better than the ZF scheme, the gap of their curves is about only 0.7dB, being far less than the gap in the un-coded systems. On the other hand, we can observe that both of the proposed scheme and RC-ML scheme outperform the ZF scheme in an iterative fashion. In particular, it is seen that our proposed scheme with iteration gains about 1.3dB over the ZF scheme at BER of 10^{-5} with lower computation complexity. Moreover, our proposed scheme gains about 0.9dB over the RC-ML scheme with iteration at BER of 10^{-5} , while preserving similar computation complexity. The performance difference between the proposed scheme and RC-ML scheme is increasing, particularly for high SNRs. As for the BMST with $m = 2$, our proposed scheme also has about 0.4dB gain compared with the RC-ML scheme and ZF scheme with iteration, whose performance matches well with the lower bounds in the high SNR region.

6. Conclusion

In this letter, we have presented a simple SISO equalization for BMST-OFDM systems in the high-mobility channels. In the BMST-OFDM systems, this simple SISO equalizer can exchange soft messages with the demapper and decoder in an iterative fashion. Simulation results showed that the proposed scheme with iteration outperforms the conventional schemes, while preserving effective complexity. In particular, the performances of the proposed algorithm match well with the lower bounds for the BMST-OFDM systems in the high SNR region.

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