Journal of Mathematical Sciences: Advances and Applications
Volume 43, 2017, Pages 89-101
Available at http://scientificadvances.co.in
DOI: http://dx.doi.org/10.18642/jmsaa_7100121761

ON ARCHIMAX COPULAS AND MULTIVARIATE GPD MODELS

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Abstract

This paper develops an extension to three dimensional study some properties of Archimax copulas. Moreover, multivariate generalized Pareto distributions are characterized by a pseudo-dependence function while usual multivariate extremal models are given with the corresponding dependence function.

1. Introduction

The concept of Archimedean copula is inherently related to Archimedean generator. So, the class of Archimedean copulas form a parameterized by real-valued functions. An n-dimensional copula C is one-dimensional generator functions (see [4]). Arising in the context of

2010 Mathematics Subject Classification: 62G32, 62N99, 62G20.
Keywords and phrases: Archimax copula, generator, extreme values, generalized Pareto distribution.
Received November 28, 2016

Laplace transform (Joe) Archimedean copulas form these last years an important family that allows multivariate dependence modelling involving one-dimensional generator functions (see [1]). An n-dimensional copula C is an Archimedean copula if there exists a continuous, strictly decreasing and convex function $\varphi : [0, 1] \to [0, +\infty]$, completely monotone on $[0; \infty]$, and which is $(n-2)$ times derivative and $(-1)^k (\varphi^{-1})^{(k)}(t) \geq 0$ for all $k = 1, 2, \ldots, n-2$, called the Archimedean generator function of C, such as for all $(u_1, \ldots, u_n) \in [0, 1]^n$,

$$C(u_1, \ldots, u_n) = \varphi[\varphi^{-1}(u_1) + \ldots + \varphi^{-1}(u_n)], \tag{1.1}$$

where the generalized inverse of $\varphi^{-1}(y) = \inf\{t \in [0, 1] : \varphi(t) \leq y\}$ (see [6]).

Among the most usual properties of Archimedean copulas that they present analytic form given by the generator, their exchangeability and positivity and positive and quadrant dependence of the underlying bivariate random vectors (see [1]). However in multivariate analysis applied in portfolio management, for example, we need non-exchangeable models. That is why authors use Laplace transforms to derive more flexible extension of this family of copulas by nesting of generators allowing different degrees of positive dependence in bivariate margins. By denoting an m-dimensional construction defined interactively for all $(u_1, \ldots, u_m) \in [0, 1]^m$ by

$$C_\psi(u_1, \ldots, u_m) = \psi_0[\psi_0^{-1}(u_1) + \psi_1^{-1}(C_{\Psi_1}(u_1, \ldots, u_m))]. \tag{1.2}$$

This generalisation leads to the class of nested Archimedean copulas, an asymmetric subfamily of this class of copulas. Nested Archimedean copulas allow different levels of dependence between the components of the underlying random vector.

The main contribution of this study is to construct a new class of Archimax copulas in a three-dimensional study. Multivariate Pareto models are characterized and particular distributions are given in both in three dimension and higher cases.

2. Three Dimensional Extension of Archimax Copulas

A bivariate member of this class with generator φ is given, for all $(u, v) \in [0, 1]^2$ by

$$C_{\varphi, A}(u, v) = \varphi^{-1}\left[\min\left(\varphi(0), (\varphi(u) + \varphi(v))A\left(\frac{\varphi(u)}{\varphi(u) + \varphi(v)}\right)\right)\right]; \tag{2.1}$$

where A is a Pickands ([8]) dependence function mapping the unit simplex (see [1])

$$S_n = \left\{x \in \mathbb{R}^n, \sum_{i=1}^{n} \|x\|_1 = 1, \|x\|_1 = \sum_{i=1}^{n} x_i\right\} \text{ of } \mathbb{R}^n, \tag{2.2}$$

to $\left[\frac{1}{n}, 1\right]$ satisfying $\max(t; 1-t) \leq A(t) \leq 1$ for all $t \in [0; 1]$.

The class of Archimax copulas contains a special subclass of all Archimedean copulas (when $A(t) \equiv 1$ see [7]) and all extreme values copulas, then $\varphi(t) = -\log(t)$. Particularly every extreme value model (copula or distribution) is associated to a dependence function A_C proposed by Pickands, subject to some convex constraint and defined on S_n (see [8]). Then, for all $(u_1, \dots, u_n) \in [0, 1]^n$,

$$C(u_1, \dots, u_n) = \exp\left\{\sum\nolimits_{i=1}^{n} \tilde{u}_i A_G\left(\frac{\tilde{u}_i}{\sum\nolimits_{i=1}^{n} \tilde{u}_i}, \dots \frac{\tilde{u}_{n-1}}{\sum\nolimits_{i=1}^{n} \tilde{u}_i}\right)\right\}, \tag{2.3}$$

where $\tilde{u}_i = \log u_i$; $i = 1, \dots, n$ (see [3] and [8]).

The following lemma introduces the main result of this section:

Lemma 1. *Let A be the Pickands dependence function and ϕ be a Laplace transform. Then, the univarite function defined such as $F_t(x) = \phi^{-1}(x)A(t_1, t_2)$ with $x \in \mathbb{R}$; $t_i \geq 0 \in [0, 1]$, where $t = (t_1, t_2) \in S_2$ is the distribution function of a random variable X_i.*

Proof. Since the function A is defined from the unit simplex to $[0, 1]$ and $0 \leq \phi^{-1}(x) \leq 1$ for all $x \in \mathbb{R}$. Therefore, for all $x > 0$, $0 \leq \psi(x) \leq 1$. Furthermore, one have $\lim_{x \to 0^+} \psi(x) = 0$ and $\lim_{x \to +\infty} \psi(x) = 1$.

Moreover, for all $x \in \mathbb{R}$,

$$\psi'(x) = (\phi^{-1})'(x) A(t_1, t_2) \geq 0.$$

So ψ is increasing and continuous. Finally, there exists a random variable X such that $\psi(x) = P(X \leq x).\psi'(x)$. □

The following result gives a trivariate extension of Archimax copulas:

Proposition 2. *For all bivariate Pickands dependence function A, i.e., satisfying* (3), *the parametric function defined, for all* $(u_1, u, u_3) \in [0, 1]^3$ *by*

$$C(u_1, u, u_3) = \phi^{-1}\left[\left(\sum_{i=1}^{3} \phi^{-1}(u_i)\right) A\left(\frac{\phi^{-1}(u_1)}{\sum_{i=1}^{m} \phi^{-1}(u_i)}, \frac{\phi^{-1}(u_2)}{\sum_{i=1}^{m} \phi^{-1}(u_i)}\right)\right] \tag{2.4}$$

is also a copula.

Proof. A sufficient condition for proving Proposition 2 is to build a continuous random vector (X_1, X_2, X_3) associated to copula C. For this end, let Y denote the univariate random with distribution with Laplace transform ψ, that is, $\psi(x) = \int_0^{+\infty} e^{-xt} d\psi(t)$.

Then, conditionally to Y let $X = (X_1, X_2, X_3)$ be three independent random variables. It follows that

$$P(X_1 \leq x_1, X_2 \leq x_2, X_3 \leq x_3) = E[P(X_1 \leq x_1, X_2 \leq x_2, X_3 \leq x_3 / Y)]$$

$$= E\left[\prod_{i=1}^{3} e^{-Y_\psi(x_i)}\right].$$

Then, we obtain

$$P(X_1 \leq x_1,\ X_2 \leq x_2,\ X_3 \leq x_3) = \phi(\psi(x_1) + \psi(x_2) + \psi(x_3)),$$

or

$$P(X \leq x) = \phi\left[\sum_{i=1}^{3} \phi^{-1}(u_i) A\left(\frac{\phi^{-1}(u_1)}{\sum_{i=1}^{3} \phi^{-1}(u_i)}, \frac{\phi^{-1}(u_1)}{\sum_{i=1}^{3} \phi^{-1}(u_i)}\right)\right].$$

Furthermore, by definition,

$$C_\phi(u_1,\ u,\ u_3) = P(X_1 \leq H_1^{-1}(u_1),\ X_1 \leq H_1^{-1}(u_1),\ X_1 \leq H_1^{-1}(u_1)).$$

Then, the H_i being the marginal distributions of H, by setting $H(x) = \phi(x)$, it comes that, for all $(u_1,\ u,\ u_3) \in [0, 1]^3$;

$$C_\psi(u_1,\ u,\ u_3) = \phi\left[\left(\sum_{i=1}^{3} \phi^{-1}(u_i)\right) A\left(\frac{\phi^{-1}(u_1)}{\sum_{i=1}^{m} \phi^{-1}(u_i)}, \frac{\phi^{-1}(u_2)}{\sum_{i=1}^{m} \phi^{-1}(u_i)}\right)\right].$$

So the relation (2.5) is obtained as asserted. □

The following result is a consequence of direct consequence.

Corollary 3. *For all Archimax triavariate copula* $C_{A,\varphi}$ *given by* (2.3) *with generator* φ. *Then, there exists a convex function* $D_{A,\varphi}$ *such as*

$$C(u_1,\ u,\ u_3) = \phi^{-1}\left[\left(\sum_{i=1}^{3} \phi^{-1}(u_i)\right) D_{A,\varphi}\left(\phi^{-1}(u_1),\ \phi^{-1}(u_1)\right)\right]. \tag{2.5}$$

3. A Characterization of Multivariate Generalized Pareto Models

Let recall some properties on multivariate generalized Pareto distributions (GPD) via the corresponding multivariate extreme values (MEV).

Definition 1 (see [5] and [10]). The multivariate generalized distribution of Pareto associated to a sample of random variables $X = (X_1, \ldots, X_n)$:

$$H(x) = 1 - \log\left(\frac{G(x)}{G(x \wedge 0)}\right) = \frac{-1}{\log G(x_0)} \log\left(\frac{G(x_0 + x)}{G(x \wedge x_0)}\right) \text{ if } x \geq 0, \tag{3.1}$$

with $x \wedge 0 = \min(x, 0)$ and $x_0 \in \text{support}(G)$, where G is the MEV distribution of the same.

The following result allows us to characterize MGP distributions. Consider Fréchet margin $Y_i = \frac{-1}{\log[F_i(X_i)]}$ of random variable X_i, that is, Y_i, i.e.,

$$P[Y_i \leq y_i] = \exp\left(\frac{-1}{y_i}\right); \; y_i > 0.$$

3.1. The pseudo-dependence function of multivariate GPD

Like the Pickands dependence function for the MEV distributions we propose convexe function which can characterize the family of GPD.

Proposition 2. *Let G be the multivariate generalized Pareto distribution associated to a sample of observations* $X = (X_1, \ldots, X_m)$ *then*

$$H(x) = 1 + \left\{-\sum_{i=1}^{m} y_i(x_i)\right\} B\left(\frac{y_1(x_1)}{\sum_{i=1}^{m} y_i(x_i)}, \ldots, \frac{y_{m-1}(x_{m-1})}{\sum_{i=1}^{m} y_i(x_i)}\right), \tag{3.2}$$

where B is the pseudo-Pickands' dependence function of H, defined on the unit simplex $S_{m-1} = \mathbb{R}^{m-1}$.

Proof. For a given $n \geq 1$, suppose $X_n = (X_{n1}, \ldots, X_{nm})$ are i.i.d random vectors with m-dimensional distribution F. Let

$$M_n = (M_{n1}, \ldots, M_{nm}) = \left(\max_{1 \leq i \leq n}(X_{i1}), \ldots, \max_{1 \leq i \leq n}(X_{im})\right).$$

be the compentwise maxima of the set $\{X_n, n \geq 1\}$. A multivariate extreme value (MEV) distributions is a continuous and non degenerated function G, for which there exists normalising constants vectors:

$$\sigma_n = (\sigma_{n1}, \ldots, \sigma_{nm}) \text{ with } \sigma_{nj} > 0 \text{ and } \mu_n = (\mu_{n1}, \ldots, \mu_{nm}),$$

with $\mu_{nj} \in \mathbb{R}$ for $j = 1, \ldots, m$ such that for all $x = (x_1, \ldots, x_m)$ in $\mathbb{R}^m$

$$\lim_{n \to +\infty} P\left(\frac{M_n - \mu_n}{\sigma_n} \leq x\right) = \lim_{n \to +\infty} F^n(\sigma_n x + \mu_n) = G(x).$$

Furthermore, the y_i are defined, for all $i = 1, \ldots, m$ by the transformations

$$y_i(x_i) = \left[1 + \xi_i\left(\frac{x_i - \mu_i}{\sigma_i}\right)\right]_+^{-\frac{1}{\xi_i}}, \text{ where } x_+ = \max(x, 0), \tag{3.3}$$

$\{\mu_i \in \mathbb{R}\}$, $\{\xi_i \in \mathbb{R}\}$, and $\{\sigma_i > 0\}$ being respectively location, shape, and scale parameters of the univariate margins G_i of G. The generalized form of these margins G_i is given by:

$$G_i(x_i) = \begin{cases} \exp\left\{-\left[1 + \xi_i\left(\frac{x_i - \mu_i}{\sigma_i}\right)\right]_+^{-\frac{1}{\xi_i}}\right\} & \text{if } \xi_i \neq 0, \\ \exp\left\{-\exp\left[-\left(\frac{x_i - \mu_i}{\sigma_i}\right)_+\right]\right\} & \text{if } \xi_i = 0. \end{cases}$$

If (3.5) holds, F is said to belong to the max-domain of attraction of G. This is equivalent to say that there exists an MEV distribution G such as: $F(y_1, y_2, \ldots, y_m) = G_*(y_1, y_2, \ldots, y_m)$.

For all $t \in S_{m-1}$; μ being the angular measure on S_{m-1} (see [8])

$$G(y_1, y_2, \ldots, y_m) = \exp\left\{-\int_{S_m} \max\left(\frac{-q_1}{y_1}, \frac{-q_2}{y_2}; \ldots, \frac{-q_m}{y_m}\right)\mu d(q)\right\},$$

with $y_i > r_i$; $i = 1, 2, \ldots, m$, where r_i are thresholds. Then,

$$F(x_1, \ldots, x_m) = \exp\left[-\int_{S_m} \max\left(-q_1\left(\frac{-1}{\log[F_1(x_1)]}\right)^{-1}, \ldots, -q_m\left(\frac{-1}{\log[F_m(x_m)]}\right)^{-1}\right)\mu d(q)\right],$$

or

$$F(x) = \exp\left[-\int_{S_m} \max(q_1 \log[F_1(x_1)], \ldots, q_m \log[F_i(x_m)])\,\mu d(q)\right]. \quad (3.4)$$

Furthermore, suppose that F_i is a GPD.

Then, $F_i(x_i) = 1 - \lambda_i\left[1 + \xi_i\left(\frac{x_i - \mu_i}{\sigma_i}\right)\right]_+^{-\frac{1}{\xi_i}}$ if $t_i(x_i) = \left[1 + \xi_i\left(\frac{x_i - \mu_i}{\sigma_i}\right)\right]_+^{-\frac{1}{\xi_i}}$.

Then it follows that: $\log(F_i(x_i) = \log(1 - \lambda_i t_i(x_i)) \simeq \lambda_i t_i(x_i)$.

Then, $F(x) = \exp\left[-\int_{S_m} \max(q_1\lambda_1 t_1(x_1), \ldots, q_m\lambda_m t_m(x_m))\mu d(q)\right] + o$

$(\max(\lambda_i)$ if $B(x) = \int_{S_m} \max(q_1\lambda_1 t_1(x_1), \ldots, q_m\lambda_m t_m(x_m))\,\mu d(q)$, which is equivalent to

$$B(x) = \lambda\sum_{i=1}^{i=m} t_i(x_i)\int_{S_m} \max\left(\frac{q_1 t_1(x_1)}{\sum_{i=1}^{i=m} t_i(x_i)}, \ldots, q_m\left(1 - \frac{\sum_{i=1}^{i=m-1} t_i(x_i)}{\sum_{i=1}^{i=m} t_i(x_i)}\right)\right)\mu d(q),$$

where $B(x) = \lambda\sum_{i=1}^{i=m} t_i(x_i)A(\tilde{t}_1(x_1), \ldots, \tilde{t}_{m-1}(x_{m-1}), \tilde{t}_m(x_m))$ or $\tilde{t}_i(x_i) = \frac{t_i(x_i)}{\sum_{i=1}^{i=m} t_i(x_i)}$.

For the particular case of marginal distributions de Pareto standard parameter ξ (i.e., $\mu_i = 0$; $\sigma_i = 1$ and $\xi_i = \xi$, $\xi > 0$; $i = 1, 2, \dots, m$)

$$B(x) = B_{m,\xi,\lambda}(x) = \lambda \left[m + \xi \sum_{i=1}^{i=m} x_i^{-\xi} \right] A\left(\widetilde{t}_1(x_1), \dots, \widetilde{t}_{m-1}(x_{m-1}), \widetilde{t}_m(x_m)\right). \tag{3.5}$$

$\square$

3.2. Applications to usual some MEV distributions

Using the relation (3.5), it is easy to establish that to propose the pseudo-dependence function for MEV consiste simply in giving its corresponding Pickands functions. So, we recall here some usual models of MEV distributions with the corresponding Pickands dependence function (see [2] and [9]).

- **Negative extension of logistic model of MEV distributions**

This model is given for all $x = (x_1, \dots, x_m) \in \mathbb{R}^m$ and $\theta = (\theta_1, \dots, \theta_m)$ by:

$$G_\theta(x) = \exp\left\{ -\sum_{i=1}^{m} x_i + \sum_{i<j} \left(x_i^{-\theta_{ij}} + x_j^{-\theta_{ij}} \right)^{\frac{-1}{\theta_{ij}}} + \sum_{k=3}^{m} (-1)^{k+1} \sum_{i=1}^{k} B_{1\dots k}(x_i x_k, \theta) \right\}.$$

The corresponding Pickands function is given, for $t = (t_1, \dots, t_{m-1}) \in S_m$ by

$$A_\theta(t) = \left[\sum_{i=1}^{m-1} t_i + \left(1 - \sum_{i=1}^{m-1} t_i \right) + \sum_{i=1, i<j}^{m-2} \left(t_i^{-\theta_{ij}} + t_j^{-\theta_{ij}} \right) \right.$$

$$\left. + \sum_{i=1}^{m-1} \left(t_i^{-\theta_{im}} + \left(1 - \sum_{i=1}^{m-1} t_i \right)^{-\theta_{im}} \right)^{\frac{-1}{\delta_{im}}} \right]^{\frac{1}{\theta}} + R(t_1, \dots, t_{m-1}, \delta_{12}, \dots, \delta_{1m});$$

where $R(t_1, \dots, t_{m-1}, \delta_{12}, \dots, \delta_{1m})$ is an integral rest.

• Logistic Gaussian model of MEV distribution

This model is given for all $x = (x_1, \ldots, x_m) \in \mathbb{R}^m$ and $\theta = (\theta_1, \ldots, \theta_m)$ by:

$$G_\theta(x) = \exp\left\{-\sum_{i=1}^{m} x_i + \sum_{1 \le i < j \le m} \left\{x_i + x_j - x_i \Phi\left(\frac{1}{\theta_{ij}} + \frac{\theta_{ij}}{2}\log\left(\frac{x_i}{x_j}\right)\right) - x_j \Phi\left(\frac{1}{\theta_{ij}} + \frac{\theta_{ij}}{2}\log\left(\frac{x_j}{x_i}\right)\right)\right\} + \sum_{S:|S| \ge 3} (-1)^{|S|+1} \int_0^{x_{i_k}} \overline{\Phi}_{k-1}\left(\log\left(\frac{x}{x_i}\right) + \frac{2}{\delta^2_{i_j, i_k}}\,;\, \Gamma\right) dx\right\};$$

where $\overline{\Phi}_{k-1}(., \Gamma)$ is the survival function of the Gaussian model $(k-1)$-multivariate with variance-covariance matrix Γ such as $\Gamma = \Gamma\left(\theta_{i_j, i_k}\right)$, a $(k-1)$-squared matrix where the elements (i, j') are all equal to $2\left(\theta^{-2}_{i_j, i_k} + \theta^{-2}_{i_{j'}, i_k} - \theta^{-2}_{i_j, i_{j'}}\right)$ with $1 \le j, j' \le k-1$ and $\theta^{-1}_{i,i} = 0$ (see [7]).

The corresponding Pickands function is given, for $t = (t_1, \ldots, t_{m-1}) \in S_m$ by

$$A_\theta(t) = 1 - \sum_{i=1}^{m-1} t_i - \left(1 - \sum_{i=1}^{m-1} t_i\right) + \sum_{i=1, i<j}^{m-2} \left(t_i + t_j - t_i \Phi\left(\frac{1}{\delta_{ij}} + \frac{\delta_{ij}}{2}\log\left(\frac{t_i}{t_j}\right)\right) - t_j \Phi\left(\frac{1}{\delta_{ij}} + \frac{\delta_{ij}}{2}\log\left(\frac{t_j}{t_i}\right)\right)\right)^{\frac{-1}{\delta_{ij}}} + \sum_{i=1}^{m-1}\left(t_i + t_m - t_i \Phi\left(\frac{1}{\delta_{im}} + \frac{\delta_{im}}{2}\log\left(\left(1 - \sum_{k=1}^{m-1} t_k\right)^{-1}\right)\right)\right)$$

$$-\left(1-\sum_{k=1}^{m-1} t_k\right)\Phi\left(\frac{1}{\delta_{ij}}+\frac{\delta_{ij}}{2}\log\left(\frac{1-\sum_{k=1}^{m-1} t_k}{t_i}\right)\right)\Bigg)\Bigg)$$

$$+ R(t_1, \ldots, t_{m-1}, \delta_{12}, \ldots, \delta_{1m});$$

where $R(t_1, \ldots, t_{m-1}, \delta_{12}, \ldots, \delta_{1m})$ is an integral rest.

• **Logistic mixed model of MEV distribution**

This model is given for all $x = (x_1, \ldots, x_m) \in \mathbb{R}^m$ and $\theta = (\theta_1, \ldots, \theta_m)$ by:

$$G_\theta(x) = \exp\left\{-\left[\sum_{1\le i<j\le m}\left(\left(p_i x_i^\theta\right)^{\theta_{ij}} + \left(p_j x_j^\theta\right)^{\theta_{ij}}\right)^{\frac{1}{\theta_{ij}}} + \sum_{i=1}^{m} \nu_i p_i x_i^\theta\right]^{\frac{1}{\theta}}\right\}.$$

The corresponding Pickands function is given, for $t = (t_1, \ldots, t_{m-1}) \in S_m$ by

$$A_\theta(t) = \left[\sum_{1\le i<j\le m-1}\left(\left(p_i t_i^\theta\right)^{\theta_{ij}} + \left(p_j t_j^\theta\right)^{\theta_{ij}}\right)^{\frac{1}{\theta_{ij}}} + \sum_{i=1}^{m} \nu_i p_i t_i^\theta\right.$$

$$\left.\sum_{1\le i\le m-1}\left(\left(p_i t_i^\theta\right)^{\theta_{ij}} + \left(p_j\left(1-\sum_{1\le k\le m-1} t_k\right)^\theta\right)^{\theta_{ij}}\right)^{\frac{1}{\theta_{ij}}} + \sum_{i=1}^{m} \nu_i p_i t_i^\theta\right]^{\frac{1}{\theta}}.$$

• **Logistic asymmetric model of MEV distribution**

This model is given for all $x = (x_1, \ldots, x_m) \in \mathbb{R}^m$ and $\theta = (\theta_1, \ldots, \theta_m)$ by:

$$G_\theta(y_t^{\breve{s}}) = \exp\left\{-\sum_{k=1}^{m-1} y_t^{(i)} + \sum_{S:|S|\geq 1}^{m} (-1)^{|S|+1}\left[\sum_{i\in S}^{m-1} y_t^{(k)\theta}\right.\right.$$

$$\left.\left. - \sum_{i<j;\, i,\, j\in S}\left[p^{-\theta_{ij}} y_t^{(i)} + p_j^{-\delta_{ij}} x_j^{\theta\theta_{Ij}}\right]^{\frac{-1}{\theta_{ij}}}\right]^{\frac{-1}{\theta}}\right\}.$$

The corresponding Pickands function is given, for $t = (t_1, \ldots, t_{m-1}) \in S_m$ by

$$A_\theta(t) = 1 - \sum_{S:2\leq|S|\leq m-1} (-1)^{|S|+1}\left[\sum_{i\in S} t_i^{-\theta} + - \sum_{i<j;\, i,\, j\in S}\left[p_i^{-\theta_{ij}} t_i^{\theta\theta_{Ij}} + p_j^{-\delta_{ij}} t_j^{\theta\theta_{Ij}}\right]^{\frac{-1}{\theta_{ij}}}\right]$$

$$+ (-1)^{m+1}\left[\left(1 - \sum_{1\leq i\leq m-1} t_i\right)^{\theta}\right.$$

$$\left. - \sum_{i<m}\left[p_i^{-\theta_{im}} t_i^{\theta\theta_{im}} + p_m^{-\theta_{ij}}\left(1 - \sum_{1\leq i\leq m-1} t_i\right)^{\theta\theta_{im}}\right]^{\frac{-1}{\theta_{im}}}\right].$$

4. Conclusion

The results of the study provides important characterizations of parametric max-stable processes. Especially, they show that the concept of Archimax copulas can also be extended to three-dimensional case. Moreover, multivariate generalized Pareto distributions are characterized by a pseudo-dependence function while usual multivariate extremal models are given with the corresponding dependence function.

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