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TESTING EXPONENTIALITY AGAINST $NBRU_{rp}$ BASED ON GOODNESS OF FIT APPROACH

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Abstract

The concept of residual probability plays an important role in reliability and life testing. In the current investigation, we presented a new test for testing exponentiality against new better than renewal used in the RP order $NBRU_{rp}$ based on the goodness of fit approach. The Pitman asymptotic efficiency (PAE), the Pitman asymptotic relative efficiency (PARE) relative to $NBRU_{rp}$ test given in the work of Kayid et al. [12] are studied. Power and critical values of this test are calculated to assess the performance of the test. Finally, a test of exponentiality versus $NBRU_{rp}$ for right censored data, the power estimates of this test are also simulated for some commonly used distributions in reliability and sets of real data are used as examples to elucidate the use of the proposed test statistic for practical problems.

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1. Introduction and Motivation

In contrast to goodness of fit problems, where the test statistic is based on a measure of departure from that depends on H_0 both H_0 and H_1, most tests in life testing setting, including those referenced above do not use the null distribution in devising the test statistics, this resulted in test statistics that are often difficult to work and require programming to calculated. Alternatively, we demonstrate in current work that in cooperating into the measure of departure from it can lead to simpler test statistics that are easy to work with are asymptotically equivalent in distribution to those cited above and may have equal or higher efficiency than the classical procedures. They also may have better finite sample behaviours.

In literature, we found that, based on a goodness of fit approach studied by Ahmad [2] for testing exponentiality against major life testing IFR, NBU, NBUC, NBUE, and HNBUE, El-Arishy et al. [8] for testing exponentiality against NRBU, Diab and Mahmoud [6] for testing exponentiality versus HNRBUE, Mahmoud and Diab [16] for testing exponentiality versus DVRL, Kayid et al. [11] for testing exponentiality against NBU(2). Finally Diab [5] for testing exponentiality against NBUL among other.

In reliability, various ageing classes of life distributions have been introduced to describe several types of improvement that accompany ageing. The residual probability (RP) function is a well-known reliability measure which has applications in many disciplines such as reliability theory, survival analysis, and actuarial studies.

Let X and Y be two random life times representing the life times of two systems with distribution functions F and G and survival functions $\overline{F}(x) = 1 - F(x)$ and $\overline{G}(y) = 1 - G(y)$, respectively. The systems can be considered as the products of two different branches of a company. Then the quantity $P(X > Y)$ gives the reliability of X relative to Y. In case of both X and Y being distributed as Weibull, Brown and Rutemiller [4] have pointed out that to design as long-lived a product as possible one can consider the quantity $P(X > Y)$ and then choose X or Y when this probability is greater or less than 0.5, respectively. However, if the systems are known to have a survival age t, it is important to take into account the age, when we compare the remaining lifetimes. Let $X_t = [X - t | X > t]$ and $Y_t = [Y - t | Y > t]$ denote the additional residual lifetime of X and Y given that the systems have survived up to age t. The RP function is defined as

$$R(t) = P(X_t > Y_t), \quad t > 0.$$

Kayid et al. [12] defined the $NBRU_{rp}$ and investigated the probabilistic characteristics of this class of life distribution.

Definition 1. The random variable X is said to be smaller than Y in the residual probability order (denoted by $X \leq_{rp} Y$) if

$$\int_t^{\infty} [f(x)\overline{G}(x) - g(x)\overline{F}(x)]dx \geq 0, \quad \forall t \geq 0.$$

Definition 2. A random life X is said to be new better than renewal used in the RP order $(NBRU_{rp})$, if $X^* \leq_{rp} X$, or equivalently,

$$\int_t^{\infty} \left[\overline{F}^2(x) - f(x)\int_x^{\infty} \overline{F}(u)du\right]dx \geq 0, \quad \forall t \geq 0. \tag{1.1}$$

As the dual version, new worse than renewal used in the RP order $(NWRU_{rp})$, may be defined through $X^* \geq_{rp} X$.

On the other hand, reliability analysts and statisticians have shown a growing interest in modelling survival data using classifications of life distributions by means of various stochastic orders. These categories are useful for maintenance, modelling situations, biometry, and inventory theory (cf. Barlow and Proschan [3]).

The exponential distribution represents the lifetime of the units that never ages due to wear and tear. Some nonparametric classes have come up in the literature testifying to how a lifetime component or/and a system ages over the time. A natural question to ask is which ageing class a real data set belongs to. Thus, the problem of testing exponentiality against various nonparametric classes may be of some interest in reliability or survival analysis (cf. Lai and Xie [15]).

The rest of the article is organized as follows. In Section 2, we present a test statistic based on a U-statistic for testing H_0 : is exponential against H_1 : is $NBRU_{rp}$ but not exponential. In Section 3, the Pitman asymptotic efficiencies (PAE) is studied for some commonly used distributions in reliability. The Pitman asymptotic relative efficiencies (PARE) are calculated based on U-statistic that proposed by Kayid et al. [12]. Monte Carlo null distribution critical points are simulated for sample sizes $n = 5(1)30$, 40, 50 and the power estimates of this test are also calculated at the significant level $\alpha = 0.05$ for some common alternatives distribution in Section 4. In Section 5, we considered case of right-censored data, and the critical values and the power estimates of this test are tabulated. Finally, in Section 6, we discuss some applications to elucidate the usefulness of the proposed test in reliability analysis for censored and un-censored data.

2. Hypothesis Testing Problem Against $NBRU_{rp}$ Class for Non-censored Data

Our goal in this section is to present a test statistic based on goodness-of-fit approach for testing $H_0 : F$ is exponential against an alternative that $H_1 : F$ is belongs to $NBRU_{rp}$ class but not exponential. We propose the following measure of departure:

$$\Delta_{rp} = E\left[2\int_t^\infty \overline{F}^2(x)dx - \overline{F}(t)\int_t^\infty \overline{F}(x)dx\right]$$

$$= \int_0^\infty \left[2\int_t^\infty \overline{F}^2(x)dx - \overline{F}(t)\int_t^\infty \overline{F}(x)dx\right]dF_0(t),$$

where $F_0(t) = 1 - e^{-t}$ under H_0.

The following lemma is essential for the development of our test statistic.

Lemma 2.1. *If F is $NBRU_{rp}$, then a measure of the deviation from the null hypothesis H_0 is $\Delta_{rp} > 0$, where*

$$\Delta_{rp} = E[\min(X_1, X_2)] - E[1 - e^{-\min(X_1, X_2)}]$$
$$-\frac{1}{2}\left(\int_0^\infty \left[x + (2+x)e^{-x}\overline{F}(x) - e^{-x}\left(\int_0^\infty tI(t > x)dF(t)\right)\right]dF_0(x)\right) + \frac{1}{2}. \tag{2.1}$$

Proof. Since F is $NBRU_{rp}$, then from (1.1)

$$\int_t^\infty \overline{F}^2(x)dx \geq \int_t^\infty f(x)\left[\int_x^\infty \overline{F}(u)du\right]dx$$
$$\geq \int_t^\infty \overline{F}(u)\left[\int_t^u f(x)dx\right]du$$
$$\geq \int_t^\infty \left[\overline{F}(t)\overline{F}(u) - \overline{F}^2(u)\right]du,$$

which can be written in the form

$$2\int_t^\infty \overline{F}^2(x)dx \geq \overline{F}(t)\int_t^\infty \overline{F}(x)dx.$$

Take the integral with respect to $F_0(t)$, then take $I_1 \geq \frac{1}{2}I_2$, set

$$I_1 = \int_0^\infty e^{-t}\int_t^\infty \overline{F}^2(x)dxdt$$
$$= E\left[\min(X_i, X_j)\right] - E[1 - e^{-\min(X_i, X_j)}], \tag{2.2}$$

and

$$I_2 = \int_0^\infty e^{-t}\overline{F}(t)\int_t^\infty \overline{F}(x)dxdt$$
$$= \left(\int_0^\infty \left[x + (2+x)e^{-x}\overline{F}(x) - e^{-x}\left(\int_0^\infty tI(t > x)dF(t)\right)\right]dF(x)\right) - 1. \tag{2.3}$$

Hence, from (2.2) and (2.3), the result follows.

Note that under $H_0 : \Delta_{rp} = 0$, while under $H_1 : \Delta_{rp} > 0$.

2.1. Empirical test statistic for $NBRU_{rp}$ alternative

To estimate Δ_{rp}, let $X_1, X_2, \ldots, X_n$ be a random sample from F. Let $\overline{F}_n(x)$ denote the empirical distribution of the survival function $\overline{F}(x)$, where

$$\overline{F}_n(x) = \frac{1}{n}\sum_{j=1}^{n} I(X_j > x),$$

and let $\hat{\Delta}_{rp}$ be the empirical estimate of Δ_{rp}, where

$$\hat{\Delta}_{rp} = E[\min(X_1, X_2)] - E[1 - e^{-\min(X_1, X_2)}]$$

$$-\frac{1}{2}\left(\int_0^\infty\left[x + 2e^{-x}\overline{F}_n(x) + xe^{-x}\overline{F}_n(x)\right.\right.$$

$$\left.\left.- e^{-x}\left(\int_0^\infty tI(t > x)dF_n(t)\right)\right]dF_n(x)\right) + \frac{1}{2}. \tag{2.4}$$

With

$$I(t > x) = \begin{cases} 1 & t > x, \\ 0 & \text{otherwise}, \end{cases}$$

$\hat{\Delta}_{rp}$ can be written as,

$$\hat{\Delta}_{rp} = \frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{j=1}^{n}\{\min(X_i, X_j) - (1 - e^{-\min(X_i, X_j)})$$

$$-\frac{1}{2}[X_i + (2e^{-X_i} + X_i e^{-X_i} - X_j e^{-X_i})I(X_j > X_i)] + \frac{1}{2}\}. \tag{2.5}$$

To make the test $\hat{\Delta}_{rp}$ scale invariant, we let

$$\hat{\delta}_{rp} = \frac{\hat{\Delta}_{rp}}{\overline{X}}. \tag{2.6}$$

Now set

$$\phi(X_1, X_2) = \min(X_1, X_2) - (1 - e^{-\min(X_1, X_2)})$$
$$- \frac{1}{2}[X_1 + (2e^{-X_1} + X_1 e^{-X_1} - X_2 e^{-X_1})I(X_2 > X_1)] + \frac{1}{2}, \tag{2.7}$$

and define the symmetric kernel as

$$\psi(X_1, X_2) = \frac{1}{2!}\sum_R \phi(X_i, X_j),$$

where the sum is over all arrangements of X_i and X_j. This shows that $\hat{\delta}_{rp}$ is equivalent to U_n-statistic given by

$$U_n = \frac{1}{\binom{n}{2}}\sum_R \phi(X_i, X_j).$$

The following theorem summarizes the asymptotic normality of $\hat{\delta}_{rp}$:

Theorem 2.1. *As* $n \to \infty$, $\sqrt{n}\left(\hat{\delta}_{rp} - \delta_{rp}\right)$ *is asymptotically normal with mean* 0 *and variance*

$$\sigma^2 = \operatorname{Var}\left[2\int_0^{X_1} \overline{F}(x)dx - 2\int_0^{X_1} e^{-x}\overline{F}(x)dx - \frac{1}{4}\left(X_1 - \frac{1}{2}e^{-2X_1} + \frac{1}{2}\right)\right].$$

Under H_0, *the variance* σ^2 *reduces to*

$$\sigma_0^2 = \operatorname{Var}\left[\frac{9}{8}e^{-2X_1} - 2e^{-X_1} - \frac{1}{4}X_1 + \frac{7}{8}\right]. \tag{2.8}$$

Proof. Recall the definition of $\phi(X_1, X_2)$ from (2.7) then,

$$\eta_1(X_1) = E[\phi(X_1, X_2)|X_1]$$

$$= \int_0^{X_1} \overline{F}(x)dx - \int_0^{X_1} e^{-x}\overline{F}(x)dx - \frac{1}{2}(X_1 + e^{-2X_1} - 1),$$

next,

$$\eta_2(X_1) = E[\phi(X_2, X_1)|X_1]$$

$$= \int_0^{X_1} \overline{F}(x)dx - \int_0^{X_1} e^{-x}\overline{F}(x)dx + \frac{1}{4}\left(X_1 + \frac{5}{2}e^{-2X_1} - \frac{5}{2}\right),$$

considering,

$$\psi(X_1) = \eta_1(X_1) + \eta_2(X_1)$$

$$= 2\int_0^{X_1} \overline{F}(x)dx - 2\int_0^{X_1} e^{-x}\overline{F}(x)dx - \frac{1}{4}\left(X_1 - \frac{1}{2}e^{-2X_1} + \frac{1}{2}\right).$$

Since,

$$\sigma^2 = \mathrm{Var}[\psi(X_1)].$$

Under H_0 the variance reduces to (2.8), after calculation $\sigma_0^2 = \frac{1}{120}$.

3. The Pitman Asymptotic Relative Efficiency

In order to asses how good our proposed family of tests relative to others in the literature, we employ the concept of "Pitman's asymptotic relative efficiency" (PARE) of proposed test. To present this, we need to evaluate the "Pitman's asymptotic efficiency" (PAE) for our tests and then compare this (by taking ratios) to the PAEs of other tests to get the (PARE). Let us first evaluate the (PAE) for our proposed family of tests $\hat{\Delta}_{rp}$ which is defined in (2.1). It is known that Pitman's asymptotic efficiency (PAE) which is defined as Pitman [18] is given by

$$PAE\,(\,\hat{\Delta}_{rp}\,(\theta)\,) = \frac{1}{\sigma_0}\,|\,\frac{d}{d\theta}\,\hat{\Delta}_{rp}\,(\theta)\,|_{\theta\to\theta_0}.$$

We calculate the Pitman asymptotic efficiency (PAE) of $NBRU_{rp}$ test statistic. These calculations are done using the following common alternatives in reliability theory:

(i) Linear failure rate family, $\overline{F}_1(x) = \exp(-x - \theta x^2 / 2), \quad x \geq 0,\ \theta \geq 0,$

(ii) Makeham family, $\overline{F}_2(x) = \exp(-x - \theta(x + e^{-x} - 1)), \quad x \geq 0,\ \theta \geq 0,$

(iii) Weibull family, $\overline{F}_3(x) = \exp(-x^{\theta}), \quad x \geq 0,\ \theta \geq 1,$

(iv) Gamma family, $\overline{F}_3(x) = \int_x^{\infty} e^{-u} u^{\theta-1} du / \Gamma(\theta), \quad x > 0,\ \theta \geq 0.$

Note that H_0 (the exponential distribution) is attained at $\theta_0 = 0$ in (i), (ii), and at $\theta = 1$ in (iii), (iv).

Since

$$\hat{\Delta}_{rp}\,(\theta) = \int_0^{\infty} (1 - e^{-x}\,)\,\overline{F}_{\theta}^2\,(x)\,dx - \frac{1}{2}\{\int_0^{\infty} (\,x + 2e^{-x}\,\overline{F}_{\theta}\,(x) + xe^{-x}\,\overline{F}_{\theta}\,(x)$$

$$- e^{-x}\left(\int_X^{\infty} t dF_{\theta}(t)\right)) dF_{\theta}(x)\} + \frac{1}{2}.$$

The $PAE\,(\,\hat{\Delta}_{rp}\,(\theta)\,)$ can be written as

$$PAE\left(\hat{\Delta}_{rp}\,(\theta)\right) = \frac{1}{\sigma_0}\left|\left\{\int_0^{\infty} 2\,(1 - e^{-x}\,)\,\overline{F}_{\theta}\,(x)\,\overline{F}'_{\theta}\,(x)\,dx\right\}\right.$$

$$- \frac{1}{2}\left\{\int_0^{\infty}\left[x + 2e^{-x}\overline{F}_{\theta}(x) + xe^{-x}\overline{F}_{\theta}(x) - e^{-x}\left(\int_X^{\infty} t dF_{\theta}(t)\right)\right] dF'_{\theta}(x)\right\}$$

$$\left. - \frac{1}{2}\left\{\int_0^{\infty}\left[2e^{-x}\overline{F}'_{\theta}(x) + xe^{-x}\overline{F}'_{\theta}(x) - e^{-x}\left(\int_X^{\infty} t dF'_{\theta}(t)\right)\right] dF_{\theta}(x)\right\}\right|.$$

Direct calculations of (i) and (ii), we get the efficiencies of these families. Using MATHEMATECA 9 program to calculate the Pitman asymptotic efficiency (PAE) of $NBRU_{rp}$ test statistic in case of Weibull and Gamma family as alternatives and we get the following PAE values for the following Table 1, and compare this values to others that may be useful for this problem. Here we choose the tests $\hat{\delta}_\theta$ which represented by Kayid et al. [12].

Table 1. It shows that the asymptotic efficiencies $\hat{\Delta}_{rp}(\theta)$ and $\hat{\delta}_\theta$ test

Distribution	$\hat{\Delta}_{rp}(\theta)$	$\hat{\delta}_\theta$
Linear failure rate	0.91287	0.5708
Makeham	0.22823	0.2681
Weibull	0.78785	1.4263
Gamma	0.34142	–

It is clear from Table 1 that, the new test statistic $\hat{\Delta}_{rp}(\theta)$ for $NBRU_{rp}$ is higher than the efficiency of $\hat{\delta}_\theta$.

Table 2. It shows that the asymptotic relative efficiencies $\hat{\delta}_\theta(\lambda)$ test

Distribution	$PARE\,(\hat{\Delta}_{rp}(\theta), \hat{\delta}_\theta)$
Linear failure	1.59928
Makeham	0.85129
Weibull	0.55237

We can see from Table 2 that our test statistic $\hat{\Delta}_{rp}(\theta)$ for $NBRU_{rp}$ is more efficiently than $\hat{\delta}_\theta$ for all cases and also simpler.

4. Monte Carlo Null Distribution Critical Points

In practice, simulated percentiles for small samples are commonly used by applied statisticians and reliability analyst. We have simulated the upper percentile values for 90%, 95%, 98%, and 99%. Table 2 presented these percentile values of the statistics $\hat{\delta}_{rp}$. In (2.6) and the calculations are based on 5000 simulated samples of sizes $n = 5(1)30(10)50$.

Table 3. The upper percentile of $\hat{\delta}_{rp}$

n	90%	95%	98%	99%
10	0.0856381	0.0953402	0.107233	0.114546
15	0.0607192	0.0682655	0.0757513	0.0809333
20	0.0497829	0.0565534	0.0638284	0.0697333
25	0.0431859	0.0491471	0.0556706	0.0601939
30	0.0365525	0.0423996	0.0487393	0.0525652
35	0.032805	0.0380689	0.0436325	0.0481034
40	0.0304604	0.0355315	0.0409407	0.0439622
45	0.0274317	0.0321793	0.037968	0.0415453
50	0.0260057	0.0305405	0.0350498	0.0376607

In view of Table 3 and Figure 1, it is noticed that the critical values are increasing as the confidence level increasing and decreasing as the sample size increasing.

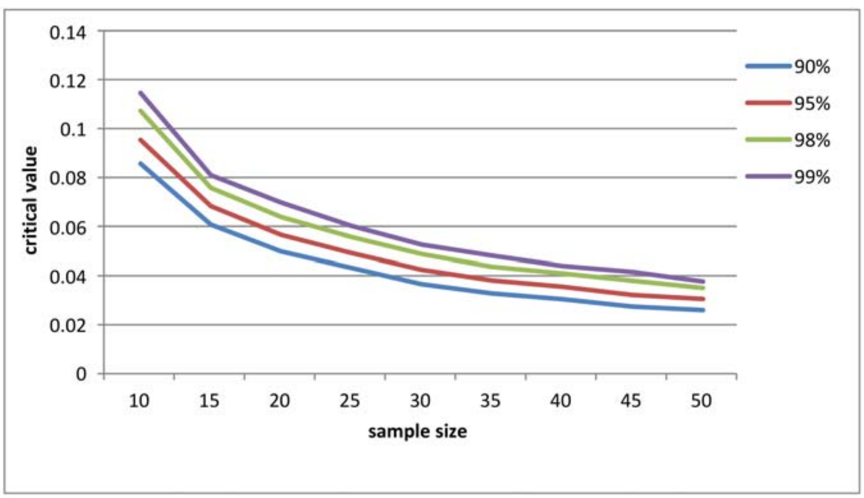

Figure 1. Relation between critical values, sample size and confidence levels.

4.1. The power estimates

Now, we present an estimation of the power estimate of the test statistic $\hat{\delta}_{rp}$ at the significance level $\alpha = 0.05$ using LFR, Weibull, and Gamma distribution. The estimates are based on 5000 simulated samples for sizes n = 10, 20, and 30 with parameter $\theta = 1, 2,$ and 3.

Table 4. Power estimates using $\alpha = 0.05$

Distribution	Parameter θ	Sample Size		
		$n = 10$	$n = 20$	$n = 30$
LFR family	1	1.000	1.000	1.000
	2	1.000	1.000	1.000
	3	1.000	1.000	1.000
Weibull Family	1	0.959	0.953	0.953
	2	1.000	1.000	1.000
	3	1.000	1.000	1.000
Gamma Family	1	0.956	0.949	0.951
	2	0.969	0.987	0.996
	3	0.996	1.000	1.000

From Table 4, it is noted that the power of the test increases by increases the value of the parameter θ and sample size n, and it is clear that our test has good powers.

5. Testing Against $NBRU_{rp}$ Class for Censored Data

The objective of this section, a test statistic is proposed to test H_0 versus H_1 with randomly right-censored data. Such a censored data is usually the only information available in a life-testing model or in a clinical study where patients may be lost (censored) before the completion of a study. This experimental situation can formally be modelled as follows.

Suppose n objects are put on test, and $X_1, X_2, \ldots, X_n$ denote their true life time. We assume that $X_1, X_2, \ldots, X_n$ be independent, identically distributed (i.i.d.) according to a continuous life distribution F. Let $Y_1, Y_2, \ldots, Y_n$ be (i.i.d.) according to a continuous life distribution G. Also we assume that X's and Y's are independent.

In the randomly right-censored model, we observe the pairs (Z_j, δ_j), $j = 1, \ldots, n$, where $Z_j = \min(X_j, Y_j)$ and

$$\delta_j = \begin{cases} 1 & \text{if} \quad Z_j = X_j \ (j\text{-th observation is un-censored}), \\ 0 & \text{if} \quad Z_j = Y_j \ (j\text{-th observation is censored}). \end{cases}$$

Let $Z(0) = 0 < Z(1) < Z(2) < \ldots < Z(n)$ denote the ordered Z's and $\delta_{(j)}$ is the δ_j corresponding to $Z_{(j)}$, respectively.

Using the censored data (Z_j, δ_j), $j = 1, \ldots, n$, Kaplan and Meier [10] proposed the product limit estimator.

$$\overline{F}_n(X) = 1 - F_n(x) = \prod_{[j: Z_{(j)} \leq X]} \{(n-j)/(n-j+1)\}^{\delta(j)}, \ X \in [0, Z_n].$$

Now, for testing $H_0 : \hat{\delta}_{rp} = 0$, against $H_1 : \hat{\delta}_{rp} > 0$, using the randomly right censored data, we propose the following test statistic:

$$\hat{\delta}_{rp}^{c} = \frac{1}{\mu}\{ E[\min(X_1, X_2)] - E[1 - e^{-\min(X_1, X_2)}]$$

$$- \frac{1}{2}\left(\int_0^\infty \left[x + (2+x)e^{-x}\overline{F}_n(x) - e^{-x}\left(\int_0^\infty tI(t > x)dF_n(t)\right)\right]dF_n(x)\right) + \frac{1}{2}\bigg\}.$$

For computational purposes, $\hat{\delta}_{rp}^{c}$ may be rewritten as

$$\hat{\delta}_{rp}^{c} = \frac{1}{\mu}\left(\eta - \frac{1}{2}\beta + \frac{1}{2}\right), \tag{5.1}$$

where

$$\mu = \sum_{i=1}^{n} \prod_{m=1}^{i-1} C_m^{\delta(m)} (Z_{(i)} - Z_{(i-1)}), \; \eta = \sum_{i=1}^{n} (1 - e^{-Z_{(i)}}) \left(\prod_{m=1}^{i-1} C_m^{\delta(m)} \right)^2 (Z_{(i)} - Z_{(i-1)}),$$

and

$$\beta = \sum_{i=1}^{n} \left\{ Z_{(i)} + (2 + Z_{(i)}) e^{-Z_{(i)}} \left(\prod_{p=1}^{i-1} C_p^{\delta(p)} \right) \right.$$

$$\left. - e^{-Z_{(i)}} \left(\sum_{j=i}^{n} Z_{(j)} \left[\prod_{q=1}^{j-2} C_q^{\delta(q)} - \prod_{q=1}^{j-1} C_q^{\delta(q)} \right] \right) \right\} \left(\prod_{l=1}^{i-2} C_l^{\delta(l)} - \prod_{l=1}^{i-1} C_l^{\delta(l)} \right),$$

where $C_k = \dfrac{n-k}{n-k+1}$.

Table 5 gives the critical values percentiles of $\hat{\delta}_{rp}^{c}$ test for sample sizes $n = 5(5)30(10)81$, 86, based on 5000 replications.

Table 5. Critical values for percentiles of $\hat{\delta}_{rp}^{c}$ test

n	90%	95%	98%	99%
5	0.499601	0.572898	0.655153	0.71178
10	0.302203	0.380319	0.484856	0.554815
15	0.224151	0.305659	0.421147	0.483003
20	0.181842	0.255692	0.357101	0.428505
25	0.161337	0.232022	0.323571	0.388901
30	0.139445	0.219249	0.295775	0.344373
40	0.11282	0.18023	0.266007	0.314739
50	0.10213	0.172359	0.244888	0.286549
60	0.0956138	0.151781	0.228875	0.273672
70	0.0904157	0.154129	0.214899	0.240948
81	0.0857939	0.140193	0.197087	0.228399
86	0.0817471	0.137208	0.185919	0.22613

It is noticed from Table 5 and Figure 2 that the critical values are increasing as the confidence level increasing and decreasing as the sample size increasing.

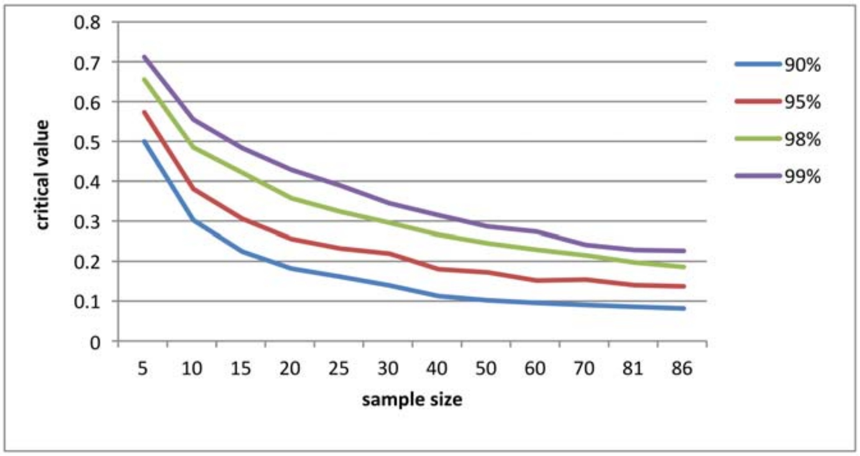

Figure 2. Relation between critical values, sample size and confidence levels.

5.1. The power estimates for $\hat{\delta}_{rp}^{c}$

Here, we present an estimation of the power for testing exponentiality Versus $NBRU_{rp}$. Using significance level $\alpha = 0.05$ with suitable parameter values of θ at $n = 10$, 20, and 30, and for commonly used distributions in reliability such as LFR family, Weibull family, and Gamma family alternatives which include in Table 6.

Table 6. Power estimates for $\hat{\delta}_{rp}^{c}$ test

Distribution	Parameter θ	Sample size		
		$n = 10$	$n = 20$	$n = 30$
LFR family	1	0.993	1.000	1.000
	2	0.998	1.000	1.000
	3	0.999	1.000	1.000
Weibull family	1	0.943	0.957	0.950
	2	0.998	0.999	0.999
	3	1.000	1.000	1.000
Gamma family	1	0.953	0.959	0.952
	2	0.971	0.831	0.697
	3	0.996	0.8714	0.707

We notice from Table 6 that our test has a good power, and the power increases as the sample size increases.

6. Applications

In this section, we calculate some of a good real examples to illustrate the use of our test statistics $\hat{\delta}_{rp}$ in the case of non-censored and censored data at 95% confidence level.

6.1. Non-censored data

Data-set #1.

Consider the data set in Abouammoh et al. [1], these data represent set of 40 patients suffering from blood cancer (leukemia) from one of ministry of health hospitals in Saudi Arabia. In this case, we get $\hat{\delta}_{rp} = 0.164416$ and this value exceeds the tabulated critical value in Table 3. It is evident that at the significant level 95% this data set has $NBRU_{rp}$ property.

Data-set #2.

Consider the data set in Keating et al. [13], these data represent set on the time, in operating days, between successive failures of air conditioning equipment in an aircraft. In this case, it is found that $\hat{\delta}_{rp} = 0.020589,$ which is less than the critical value of Table 3, then we accept the null hypothesis. This means that the data set has the exponential property.

Data-set #3.

Consider the data set given in Grubbs [9]. This data set gives the times between arrivals of 25 customers at a facility. It is easily to show that $\hat{\delta}_{rp} = 0.27372,$ which is greater than the critical value of Table 3. Then we accept H_1 which states that the data set have $NBRU_{rp}$ property and not exponential.

Data-set #4.

Consider the data set given in Edgeman et al. [7] consists of 16 intervals in operating days between successive failures of air conditioning equipment in a Boeing 720 aircraft. We can see that the value of test statistic for the data set by (2.6) is given by $\hat{\delta}_{rp} = 0.032470,$ which is less than the critical value of Table 3. Then we accept the null hypothesis of exponentiality property.

Data-set #5.

Consider the data set in Kochar [14]. In an experiment at Florida state university to study the effect of methyl mercury poisoning on the life lengths of fish goldfish were subjected to various dosages of methyl mercury. At one dosage level the ordered times to death in day. We can see that the value of test statistic for the data set by (2.6) is given by $\hat{\delta}_{rp} = 0.366358$ and this value greater than the tabulated critical value in Table 3. This means that the set of data have $NBRU_{rp}$ property and not exponential.

6.2. Censored data

Data-set #6.

Consider the data from Susarla and Vanryzin [18], which represent 81 survival times (in months) of patients melanoma. Out of these 46 represents non-censored data.

Now, taking into account the whole set of survival data (both censored and un-censored). It was found that the value of test statistic for the data set by (5.1) is given by $\hat{\delta}_{rp}^{c} = 0.3437$ and this value greater than the tabulated critical value in Table 5. This means that the data set have the $NBRU_{rp}$ property and not exponential.

Data-set #7.

On the basis of right censored data for lung cancer patients from Pena [16]. These data consists of 86 survival times (in month) with 22 right censored.

Now account the whole set of survival data (both censored and un-censored), and computing the test statistic given by formula (5.1). It was found that $\hat{\delta}_{rp}^{c} = 0.616101,$ which is exceeds the tabulated value in Table 5. It is evident that at the significant level 0.95. Then this data set has $NBRU_{rp}$ property.

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