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THE EQUIVALENCE OF T_0, T_1, T_2, URYSOHN, AND T_3 AND INFINITELY MANY NEW CHARACTERIZATIONS OF THE T_3 PROPERTY

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Abstract

The (regular and T_1) topological property, commonly denoted by T_3, is a long studied, widely used separation axiom. It is well-known that T_3 implies Urysohn, which implies T_2, which implies T_1, which implies T_0 and examples are known showing the implications are not reversible. Thus questions concerning topological properties for which the five separation axioms are equivalent arise. In this paper, a new category of topological properties is introduced and used to give infinitely many topological properties for which the five separation axioms are equivalent, and results from the study of weakly *Po* spaces and properties are used to give infinitely many new characterizations of the T_3 separation axiom.

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1. Introduction and Preliminaries

In classical topological studies of separation axioms, the primary focus in the past has been on those that satisfy the T_0 separation axiom; T_0, T_1, T_2, Urysohn, T_3, $\ldots$, and metrizable. However, in the 1936 paper [13], with the introduction and use of T_0-identification spaces, the focus began to be shared with a separation that need not be T_0.

Definition 1.1. Let (X, T) be a space; R be the equivalence relation on X defined by xRy iff $Cl(\{x\}) = Cl(\{y\})$; X_0 be the set of R equivalence classes of X; $N : X \to X_0$ be the natural map; and $Q(X, T)$ be the decomposition topology on X_0 determined by (X, T) and the natural map N. Then $(X_0, Q(X, T))$ is the T_0-identification space of (X, T) [9].

Theorem 1.1. *A space* (X, T) *is pseudometrizable iff its* T_0*-identification space is metrizable.*

Thus, for a brief moment, pseudometrizable and metrizable jointly shared the focus of interest. Additional separation axioms that do not necessarily imply T_0 continued to be introduced, but the primary focus in the study of topological separation axioms continued to be on those separation axioms that imply T_0.

In the 1961 paper [1], the R_1 topological property was introduced to further characterize the T_2 separation axiom and the R_0 separation axiom was revisited to further characterize the T_1 separation axiom.

Definition 1.2. A space (X, T) is R_1 iff for x and y in X such that $Cl(\{x\}) \neq Cl(\{y\})$, there exist disjoint open sets U and V such that $x \in U$ and $y \in V$.

Definition 1.3. A space (X, T) is R_0 iff for each closed set C and each $x \notin C$, $C \cap Cl(\{x\}) = \phi$ [12].

Theorem 1.2. *A space is* T_i *iff it is* $(T_{i-1}$ *and* $R_{i-1})$; $i = 1, 2$, *respectively*.

Thus another connection between a separation axiom that need not be T_0 and a separation axiom that implies T_0 was established giving additional importance to separation axioms that need not be T_0 and providing greater understanding and insights into those that imply T_0.

In the 1975 paper [11], T_0-identification spaces were used once again to jointly share the focus of a classical separation axiom that implies T_0 with a non-T_0 axiom.

Theorem 1.3. *A space* (X, T) *is weakly Hausdorff iff its* T_0*-identification space is Hausdorff* [11].

In the 1975 paper [11], weakly Hausdorff was shown to be equivalent to the R_1 separation axiom.

Thus, as in the 1936 paper [13], in the 1975 paper [11], T_0-identification spaces were used to jointly focus attention on a separation axiom that need not be T_0 and a separation axiom that does imply T_0. Could T_0-identification spaces be used for other separation axiom to jointly share the focus of attention for a property that need not be T_0 with a property that is T_0? The question was resolved in the 2015 paper [2] in which weakly *Po* spaces and properties were introduced and investigated.

Definition 1.4. Let P be a topological property for which $Po = (P$ and $T_0)$ exists. Then (X, T) is weakly *Po* iff its T_0-identification space $(X_0, Q(X, T))$ has property P. A topological property *Qo* for which weakly *Qo* exists is called a *weakly Po property* [2].

The two models motivating the definition of weakly *Po* spaces and properties are, in fact, weakly *Po* spaces and properties with weakly metrizable = weakly (pseudometrizable)*o* = pseudometrizable and weakly T_2 = weakly $(R_1)o = R_1$ [2]. Thus metrizable was the first known weakly *Po* property and T_2 was the second known weakly *Po* property. As would have to be true to be a model of the relationships given between the properties above, for a topological property P for which weakly *Po* exists, weakly *Po* would not necessarily have to be T_0 and (weakly *Po*)*o* would have to be *Po*, which is exactly what happens [2].

In the 2015 paper [2], it was proven that for a topological property P for which weakly *Po* exists, weakly *Po* is a unique, topological property. Using the fact that the T_0-identification space for each space is T_0 [13], it was shown in the 2015 paper [2] that for a topological property Q for which weakly *Qo* exists, a space has property weakly *Qo* iff its T_0-identification space has property *Qo*. Also, the search for topological properties that are not weakly *Po* led to the need and use of T_0 and "not-T_0". For the most part, "not-P", where P is a topological property and "not-P" exists, had been ignored in the study of topology. The need and use of "not-T_0" in the investigation of weakly *Po* illustrated the importance of "not-P" in the study of topology leading to several investigations of "not-P" properties further illustrating the importance of "not-P" properties. Prior to the investigations of "not-P", the existence of the least of all topological property was not even imagined. However, within the study of "not-P" [3], it was proven that $L = (T_0$ or "not-T_0"$)$ is the least of all topological properties and that L can also be given by $L = (P$ or "not-P"$)$, where P is a topological property for which "not-P" exists. As shown in [3] and [4], the known existence of the least topological property necessitated a change in the definition of product properties and in the definition of subspace properties, respectively. Thus, as a result of the introduction and investigation of weakly *Po*, the importance of "not-P" was revealed leading to needed additions and changes in topology and, at the same time, greatly expanding the topics and properties to be included in the study of topology and other areas of mathematics.

Because of the special role played by T_0 in the investigation and understanding of weakly *Po* spaces and properties, questions of what would happen if in the definition of weakly *Po*, T_0 is replaced by T_1 or T_2 arose, leading to the introduction and investigation of weakly $P1$ and weakly $P2$ spaces and properties.

Definition 1.5. Let P be a topological property for which $P1 = (P \text{ and } T_1)$ exists. Then a space (X, T) is weakly $P1$ iff its T_0-identification space $(X_0, Q(X, T))$ has property $P1$. A topological property $Q1$ for which weakly $Q1$ exists is called a *weakly $P1$ property* [5].

Definition 1.6. Let P be a topological property for which $P2 = (P \text{ and } T_2)$ exists. Then a space (X, T) is weakly $P2$ iff its T_0-identification space $(X_0, Q(X, T))$ has property $P2$. A topological property $Q2$ for which weakly $Q2$ exists is called a *weakly $P2$ property* [6].

Within a recent paper [7], the weakly P spaces and properties were expanded to include weakly P (Urysohn) spaces and properties.

Definition 1.7. A space (X, T) is Urysohn iff for distinct elements x and y in X, there exist open sets U and V such that $x \in U$, $y \in V$, and $Cl(U) \cap Cl(V) = \phi$ [14].

Definition 1.8. Let P be a topological property for which P(Urysohn) = (P and Urysohn) exists. Then a space (X, T) is weakly P(Urysohn) iff its T_0-identification space $(X_0, Q(X, T))$ is P(Urysohn). A topological property Q(Urysohn) for which weakly Q(Urysohn) exists is called a weakly P(Urysohn) property [7].

In similar manner to Urysohn above, in this paper the study of weakly P properties is expanded to include the T_3 separation axiom. Below results in this paper and in earlier papers will be used to give infinitely many new topological properties for which each of T_0, T_1, T_2, Urysohn, and T_3 are equivalent and infinitely many new characterizations of the long-studies, often-used T_3 separation axiom.

2. Infinitely Many Topological Properties Preserving the Equivalences

The regular and T_3 separation axioms were introduced in 1921 [15].

Definition 2.1. A space (X, T) is regular iff for each closed set C and each $x \notin C$, there exist disjoint open sets U and V such that $x \in U$ and $C \subseteq V$. A regular and T_1 space is denoted by T_3.

In the paper [8], it was shown that T_3 is a weakly Po property with weakly T_3 = weakly (regular)o = regular.

Definition 2.2. Let P be a topological property for which $P3 = (P$ and $T_3)$ exists. Then a space is weakly $P3$ iff its T_0-identification space is $P3$. A topological property $Q3$ for which weakly $Q3$ exists is called a *weakly* $P3$ *property*.

Theorem 2.1. *Let* Q *be a topological property for which weakly* $Q3$ *exists. Then weakly* $Q3$ *implies weakly* Q(*Urysohn*) *and* Q(*Urysohn*) *is a weakly* P(*Urysohn*) *property*, *weakly* $Q2$ *implies weakly* $Q2$ *and* $Q2$ *is a weakly* $P2$ *property, weakly* $Q3$ *implies weakly* $Q1$ *and* $Q1$ *is a weakly* $P1$ *property*, *and weakly* $Q3$ *implies weakly* Qo *and* Qo *is a weakly* Po *property*.

Proof. Let (X, T) be weakly $Q3$. Then $(X_0, Q(X, T))$ is $Q3$. Since T_3 implies Urysohn, then $(X_0, Q(X, T))$ is Q(Urysohn), which implies (X, T) is weakly (Urysohn). Thus weakly $Q3$ implies weakly Q(Urysohn) and Q(Urysohn) is a weakly P(Urysohn) property. Similarly, the remainder of the theorem follows.

Theorem 2.2. *Let Q be a topological property for which weakly $Q3$ exists. Then $Q3$ is a weakly Po, a weakly $P1$ property, a weakly $P2$ property, and a weakly P(Urysohn) property.*

Proof. Since $Q3 = (Q3)o$, then weakly $(Q3)o$ = weakly $Q3$ exists and $Q3 = (Q3)o$ is a weakly Po property. Similarly $Q3$ is a weakly $P1$, a weakly $P2$, and a weakly P(Urysohn) property.

Combining Theorem 2.2 with results above gives the following result.

Corollary 2.1. *Let Q be a topological property for which weakly $Q3$ exists. Then weakly $Q3$ is a unique topological property that is neither T_0 nor "not-T_0".*

Theorem 2.3. *Let Q be a topological property for which weakly $Q3$ exists. Then (weakly $Q3)o = Q3$.*

Proof. Since $(Q3) = (Q3)o$ and (weakly $Q3)o$ = (weakly $(Q3)o)o = (Q3)o$, then (weakly $Q3)o = Q3$.

In the 2015 paper [2], it was shown that for a weakly Po property Qo, a space is weakly Qo iff its T_0-identification space is weakly Qo, which when combined with the result above gives the following result.

Corollary 2.2. *Let $Q3$ be a weakly $P3$ property. Then a space is weakly $Q3$ iff its T_0-identification space is weakly $Q3$.*

In the paper [5], it was shown that for each weakly Po property Qo, weakly Qo = (Qo or ((weakly Qo) and "not-T_0")) is a decomposition of weakly Qo into two topological properties neither of which are weakly Po properties, which when combined with the results above give the following result.

Corollary 2.3. *Let $Q3$ be a weakly $P3$ property. Then weakly $Q3$ = ($Q3$ or ((weakly $Q3$) and "not-T_0")) is a decomposition of weakly $Q3$ into two topological properties neither of which are weakly $Q3$ properties.*

Theorem 2.4. *Let Q be a topological property for which weakly $Q3$ exists. Then (weakly $Q3)3 = Q3$.*

Proof. Since (weakly $Q3$)3 = ((weakly $Q3$) and T_3) = ((weakly $Q3$) and (T_0 and T_3)) = (((weakly $Q3$) and T_0) and T_3) = ((weakly $Q3$)o and T_3) = ((weakly $Q3$)o)3, (weakly $Q3$)o = $Q3$, and ($Q3$)3 = $Q3$, then (weakly $Q3$)3 = ((weakly $Q3$)o)3 = ($Q3$)3 = $Q3$.

Theorem 2.5. *Let Q be a topological property for which weakly $Q3$ exists. Then weakly $Q3$ = ($Q3$ or (weakly $Q3$ and "not-T_3")).*

Proof. Since weakly $Q3$ = ((weakly $Q3$) and L) = ((weakly $Q3$) and (T_3 or "not-T_3")), then weakly $Q3$ = ((weakly $Q3$)3 or ((weakly $Q3$) and "not-T_3")) = ($Q3$ or ((weakly $Q3$) and "not-T_3")).

Theorem 2.6. *Let Q be a topological property for which weakly $Q3$ exists. Then, in a weakly $Q3$ space, "not-T_0" and "not-T_3" are equivalent.*

Proof. Let (X, T) be weakly $Q3$. Since weakly $Q3$ = ($Q3$ or ((weakly $Q3$) and "not-T_0")) = ($Q3$ or ((weakly $Q3$) and "not-T_3")), where each of ($Q3$ and ((weakly $Q3$) and "not-T_0")), and ($Q3$ and ((weakly $Q3$) and "not - T_3")) do not exist, then ((weakly $Q3$) and "not-T_0") = (weakly $Q3$) $\setminus Q3$ = ((weakly $Q3$) and "not-T_3"). Thus, since (X, T) is weakly $Q3$, then (X, T) is "not-T_0" iff (X, T) is "not-T_3".

Theorem 2.7. *Let Q be a topological property for which weakly $Q3$ exists and let (X, T) be weakly $Q3$. Then (X, T) is simultaneously T_0, T_3, Urysohn, T_2, and T_1.*

Proof. By the contrapositive of Theorem 2.6, (X, T) is T_0 iff (X, T) is T_3. Since T_3 implies Urysohn, which implies T_2, which implies T_1, which implies T_0, the proof is complete.

Thus, for weakly $Q3$ spaces, all of T_0, T_1, T_2, Urysohn, and T_3 are equivalent raising the new question: For each weakly $Q3$ space, are there related spaces in which all of the five separation axioms are equivalent?

Within a 2016 paper [9], the following result was proven: Let Q be a topological property for which weakly Qo exists and let $\mathcal{S} = \{So \mid S$ is a topological property, So exists, and So implies $Qo\}$. Then (weakly Qo)$o \in \mathcal{S}$, for each weakly Po property W such that Wo implies Qo, (weakly Wo)$o \in \mathcal{S}$, each element of $\mathcal{S}$ implies weakly Qo, and there exists the topological property $Q_{\min}$ = ((weakly Qo) or "not-T_0"), where "not-T_0" is the negation of T_0, weaker than weakly Qo such that $(Q_{\min})o \in \mathcal{S}$. $Q_{\min}$, given above, is unique in that it is the least topological property, which together with T_0, is in $\mathcal{S}$ [9].

Below the new question given above is tied to the known results above and then used to obtain solutions to the new question.

Corollary 2.4. *Let Q be a topological property for which weakly $Q3$ exists and let $\mathcal{S}(Q3) = \{So \mid S$ is a topological property, So exists, and So implies $(Q3)o\}$. Then $\mathcal{S}(Q3) = \{So \mid S$ is a topological property, So exists, and So implies $(Q3)\}$.*

Corollary 2.5. *Let Q be a topological property for which weakly $Q3$ exists and let $\mathcal{S}(Q3) = \{So \mid S$ is a topological property, So exists, and So implies $Q3\}$. Then (weakly $Q3$)$o \in \mathcal{S}(Q3)$, for each weakly Po property W such that Wo implies $Q3$, (weakly $W3$)$o \in \mathcal{S}(Q3)$, each element of $\mathcal{S}(Q3)$ implies weakly $Q3$, and there exists the topological property $((Q3)_{\min})$ = ((weakly $Q3$) or "not-T_0") weaker than weakly $Q3$ such that $((Q3)_{\min})o \in \mathcal{S}(Q3)$.*

In the 2016 paper [9], it was established that for a topological property for which weakly Qo exists, $Q_{\min}$ is the least topological property for which a space has property Qo iff it has property ($Q_{\min}$ and T_0), which is combined with the results above to give the following result.

Corollary 2.6. *Let Q be a topological property for which weakly $Q3$ exists. Then $(Q3)_{\min}$ is the least topological property for which a space has property $Q3$ iff it has property $((Q3)_{\min}$ and $T_0)$.*

Below, for each topological property Q for which weakly $Q3$ exists, the results above are used to give infinitely many topological characterizations of $Q3$.

Let m and n represent natural number greater than or equal to 2.

Definition 2.3. Let $A(n)$ represent a set with n distinct elements, let X be a set containing the elements of $A(n)$, and let $T(A(n))$ be the topology on X defined by $T(A(n)) = \{B \subseteq X \mid A(n) \subseteq B$ or $B = \phi\}$ [10].

Definition 2.4. A space (X, T) has property $T(n)$ iff there exists a subset $A(n)$ of X such that $T = T(A(n))$ [10].

In the 2016 paper [10], it was shown that each $T(n)$ space is "not-T_0" and not a weakly Po property, that $Q(n)$ = (weakly Qo or $T(n)$) is a topological property weaker than weakly Qo and stronger than $Q_{\min}$ such that a space has property Qo iff it has property $(Q(n)$ and $T_0)$, and that for $m < n$, $Q(m)$ and $Q(n)$ are distinct topological properties, which is combined with the results above to give the next result.

Corollary 2.7. *For each $n \geq 2$ and each weakly $P3$ property $Q3$, $Q(n)3$ = ((weakly $Q3$) or $T(n)$) is a non-weakly $P3$ topological property weaker than weakly $Q3$ and stronger than $((Q3)_{\min})$ such that a space has property $Q3$ iff it has property $(((Q(n)3))$ and $T_0)$.*

Thus, for each topological property Q for which weakly $Q3$ exists, there are infinitely many non-weakly $P3$ topological properties W weaker than weakly $Q3$ and stronger than $(Q3_{\min})$ such that a space has property $Q3$ iff it has property $(W$ and $T_0)$, as given above.

In the paper [9], it was shown that for a weakly *Po* property *Qo*, $Q_{(\min,\max)}$ = ((weakly *Qo*) and "not-T_0") also plays a special role: $Q_{(\min,\max)}$ is the least topological property weaker than *Qo* and stronger than weakly *Qo* such that a space is *Qo* iff it is ($Q_{(\min,\max)}$ and T_0). Combining this result with those above give the following result.

Corollary 2.8. *Let Q be a topological property for which weakly Q*3 *exists. Then* $(Q3)_{(\min,\max)}$ = ((*weakly Q*3) *and "not-*T_0") *is the least topological property weaker than Q*3 *and stronger than weakly Q*3 *such that a space is Q*3 *iff it is* (($Q3)_{(\min,\max)}$) *and* T_0).

Within the 2016 paper [10], for each weakly *Po* property *Qo*, $Q(1, n)$ was defined and used to give infinitely many more topological properties, which together with T_0, are equivalent to *Qo*.

Definition 2.5. Let Q be a topological property for which weakly *Qo* exists. A space (X, T) is $Q(1, n)$ iff it is weakly *Qo*, there exist n distinct elements $a_1, \cdots, a_n$ all of whose closures are equal, and, for all other $x \in X$, $Cl(\{x\}) = Cl(\{y\})$ iff $x = y$ [10].

Also, in the 2016 paper [10], for a weakly *Po* property *Qo*, it was shown that $Q(1, n)$ exists, $Q(1, n)$ is weaker than *Qo* and stronger than $Q_{(\min,\max)}$, *Qo* = ($Q(1, n)$ and T_0), and for each *Qo* space (Y, S), there are infinitely many topologically spaces all with topologically distinct topological properties that are weaker than *Qo* and stronger than $Q_{(\min,\max)}$, which together with T_0, equals *Qo* and all having a T_0-identification space homeomorphic to (Y, S).

Definition 2.6. Let Q be a topological property for which weakly $Q3$ exists. A space (X, T) is $((Q3)(1, n))$ iff it is weakly $Q3$, there exist n distinct elements $a_1, \cdots, a_n$ all of whose closures are equal, and for all other $x \in X$, $Cl(\{x\}) = Cl(\{y\})$ iff $x = y$.

Corollary 2.9. *Let Q be a topological property for which weakly $Q3$ exists. Then* $((Q3)\ (1,\ n))$ *exists*, $((Q3)\ (1,\ n))$ *is weaker than* $Q3$ *and stronger than* $((Q3)_{(\min, \max)})$, $Q3 = (((Q3)(1, n))$ *and* $T_0)$, *and for each* $Q3$ *space* (Y, S), *there are infinitely many topological spaces all with topologically distinct topological properties that are weaker than* $Q3$ *and stronger than* $((Q3)_{(\min, \max)})$, *which together with* T_0, *equals* $Q3$ *and all having a* T_0*-identification space homeomorphic to* (Y, S).

As indicated in the paper [10], if (Y, S) has property Qo, where Qo is a weakly Po property, with p or more elements, then $Q(1, n)$ can be extended to $Q(p, n_1, \cdots, n_p)$ that behaves in the same manner as $Q(1, n)$ and can be used to give many more topological properties weaker than Qo and stronger than $Q_{(\min, \max)}$, which together with T_0, is equivalent to Qo. In the same manner as above, each of these new topological properties can be used to give a topological property weaker than $Q3$ and stronger than $((Q3)_{(\min, \max)})$, which together with T_0, is equivalent to $Q3$.

Theorem 2.8. *Let P be a topological property such that* $(P$ *and* $T_0)$ *exists. Then* $(P$ *and* $T_0)$ *implies* $P3$ *iff* T_0, T_1, T_2, *Urysohn, and* T_3 *are equivalent in P spaces.*

Proof. Suppose $(P$ and $T_0)$ implies $P3$. Thus, if (X, T) is a P space with property T_0, then (X, T) has property $P3$, which implies (X, T) is T_3. Thus (X, T) is Urysohn, which implies T_2, which implies (X, T) is T_1, which implies (X, T) is T_0 and T_0, T_1, T_2, Urysohn, and T_3 are equivalent.

Clearly, the converse is true.

Corollary 2.10. *Let Q be a topological property for which weakly $Q3$ exists. Then for each topological property P given above for which $(P$ and $T_0) = Q3$, $(P$ and $T_0)$ is a characterization of $P3$ and within each P spaces, T_0, T_1, T_2, Urysohn, and T_3 are equivalent.*

3. Infinitely Many New Characterizations of the T_3 Separation Axiom

Using (weakly (weakly (regular)o))$o = T_3$ and the results above given the following infinite number of new topological characterizations of T_3.

Corollary 3.1. *Let (X, T) be a space. Then the following are equivalent*: (a) (X, T) *is* T_3, (b) (X, T) *is ((weakly (weakly regular)o) and* T_0), (c) (X, T) *is* $(((T_3)_{\min})$ *and* $T_0)$, (d) *for each natural number* $n \geq 2$, (X, T) *has property* $(((T_3)(n))$ *and* $T_0)$, *where* $((T_3)(n)$ *is as defined above*, (e) (X, T) *has property* $(((T_3))_{(\min, \max)})$ *and* $T_0)$, *and* (f) *for each* $n \geq 2$, (X, T) *has property* $(((T_3)(1, n))$ *and* $T_0)$, *where* $((T_3)(1, n))$ *is as defined above.*

Since $(T_3)(T_3) = T_3$, then by the equivalences above, the T_0 in Corollary 3.1 can be replaced by T_1 or T_2 or Urysohn giving many more new topological characterizations of the T_3 property. Also, as indicated above, if $Q(1, n)$ could be extended to $Q(p, n_1, \cdots, n_p)$, then infinitely many more topological characterizations can be given for the T_3 property.

Thus, as shown above, the introduction and investigation of weakly P properties continues to be a productive study revealing many long overlooked fundamental properties and providing greater understanding, insights, and knowledge into the study and expansion of topology.

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