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QUASI-LIKELIHOOD APPROACH TO RATER AGREEMENT PLUS LINEAR BY LINEAR ASSOCIATION MODEL FOR ORDINAL CONTINGENCY TABLES

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Abstract

Quasi-likelihood estimation methods, an alternative to maximum likelihood estimation of the regression parameters in marginal models, are mainly used to deal with non-normal multivariate data. The goal of this paper is to use quasi-likelihood approach as a method for the analysis of rater agreement plus linear by linear association model, which is a type of log-linear models of rater agreement in $R \times R$ contingency tables with ordinal variables. We discuss the differences between maximum likelihood and quasi-likelihood approaches and compared the standard errors.

1. Introduction

The analysis of contingency tables plays an important role in many fields of applied statistics. $R \times R$ square contingency tables are the special cases of two-way contingency tables (Bishop et al. [6]) and

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generally arise for dependent row and column variables. The dependency of the row and column variables in square contingency tables needs some special solutions.

Classical log-linear models are extensively used for modelling two-way contingency tables (Agresti [2]). In recent years, more attention has been paid to the assessment of rater agreement in contingency tables, especially in medical or behavioural sciences (Osius [14]; Valet et al. [16]; Bagheban and Zayeri [5]). Some types of log-linear models, such as diagonal parameter, linear by linear association and agreement plus linear by linear association models have been frequently suggested to describe the association and structure of rater agreement (Agresti [3]).

The classical technique for estimating the unknown parameters of a log-linear model is maximum likelihood (ML) method. In this paper, quasi-likelihood (QL) estimation method introduced by Wedderburn [18] is adopted to analyze rater agreement in $R \times R$ square contingency tables, since it is becoming very popular method as an alternative to ML for statistical modelling which uses an approximate likelihood function rather than a fully likelihood (Liang and Zeger [12]; Zeger and Liang [20]; Lipsitz et al. [13]; Fitzmaurice et al. [8]). Quasi-likelihood equations are applicable to avoid distributional assumptions for marginal models when the responses are count, dichotomous, categorical, ordinal or continuous data (Wedderburn [18]; Liang and Zeger [12]).

This paper only focuses on the linear by linear association plus agreement model $R \times R$ for square contingency tables. Classical approach is to fit log linear model to data and estimate the expected frequencies and estimation of model parameters. This paper considers quasi-likelihood equations to analyze $R \times R$ tables rather than ML and compares parameter estimates of quasi-likelihood method to those of the commonly used log-linear analysis with ML method.

In Section 2, the agreement plus linear by linear association model is described. In Section 3, quasi-likelihood approach for linear by linear association plus agreement model in the analysis of $R \times R$ tables is introduced. We provide a simulation study in Section 4 to give details on proposed methods with small, moderate, and large sample sizes. The numerical results are illustrated by an example in Section 5. The conclusions are discussed in Section 6.

2. Association Models

Association models for contingency tables are designed to describe association between row and column variables. In an $R \times C$ contingency table, let X denote the row variable with categories $i = 1, \ldots, R$ and Y denote the column variable with categories $j = 1, \ldots, C$. The cell frequencies n_{ij} have expected frequencies μ_{ij}. Goodman [9] proposed the association model as the form:

$$\log \mu_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \sum_{k=1}^{p} \beta_k u_{ik} v_{jk}, \quad i = 1, \ldots, R;\ j = 1, \ldots, C, \quad (1)$$

where λ is the grand mean, λ_i^X and λ_j^Y represent the row and the column marginal parameters and should satisfy the constraints $\sum_{i=1}^{R} \lambda_i^X = \sum_{j=1}^{C} \lambda_j^Y = 0.$

The Model (1) in Equation (1) is referred as linear by linear association model. The row and column scores $\{u_i\}$ and $\{v_j\}$ are assigned to reflect category orderings. The independence model is the special case $\beta = 0.$ Since the model has one more parameter (β) than the independence model, the residual *df* of the model is equal to $(R - 1)(C - 1) - 1.$ The parameter β in Model (1) specifies the direction and strength of association. When $\beta > 0,$ X tends to increase as Y increases (Agresti [3]). In this model, the equal interval scores are the common choice for the levels of the ordinal scales.

Agresti [1] introduced a model having the structure of uniform association plus an agreement parameter over the main diagonal for ordinal categorical scales,

$$\log \mu_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \beta u_i v_j + \delta_{ij}, \quad i = 1, \dots, R;\ j = 1, \dots, C, \tag{2}$$

where β is the association parameter between the row and the column variables, u_i and v_j are the row and the column scores, respectively, as indicated in Model (1). Scores are assigned as equal intervals for the categories of the row variable, $u_1 \le u_2 \le \dots \le u_R$, and for the categories of the column variable, $v_1 \le v_2 \le \dots \le v_C$. In Model (2) defined as Equation (2), $\delta_{iR} = 0$ for all i and $\delta_{Rj} = 0$ for all j.

The δ_{ij} in Equation (2) denotes the agreement parameter as,

$$\delta_{ij} = \begin{cases} \delta, & \text{if } i = j, \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

The Model (2) has residual degrees of freedom $df = (R-1)^2 - 2$ and can be also expressed in terms of the odds ratios θ_{ij} as follows:

$$\log \theta_{ij} = \begin{cases} \beta + 2\delta, & i = j, \\ \beta - 2\delta, & |i - j| = 1, \\ \beta, & |i - j| > 1. \end{cases} \tag{4}$$

Agresti [1] has referred to Model (2) as *an agreement plus linear by linear association* model. The special case $\beta = 0$ is the Tanner and Young's [15] model for nominal scale agreement; the special case $\delta = 0$ is the linear by linear association model, and the special case $\beta = \delta = 0$ is the independence model. This model permits the linear by linear association of the main diagonal.

We can define the 3×3 design matrix for Model (2) as the following:

$$\begin{bmatrix} \log \mu_{11} \\ \log \mu_{12} \\ \log \mu_{13} \\ \log \mu_{21} \\ \log \mu_{22} \\ \log \mu_{23} \\ \log \mu_{31} \\ \log \mu_{32} \\ \log \mu_{33} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 2 & 0 \\ 1 & 1 & 1 & -1 & -1 & 3 & 0 \\ 1 & 0 & 1 & 1 & 0 & 2 & 0 \\ 1 & 0 & 1 & 0 & 1 & 4 & 1 \\ 1 & 0 & 1 & -1 & -1 & 6 & 0 \\ 1 & -1 & -1 & 1 & 0 & 3 & 0 \\ 1 & -1 & -1 & 0 & 1 & 6 & 0 \\ 1 & -1 & -1 & -1 & -1 & 9 & 1 \end{bmatrix} \begin{bmatrix} \lambda \\ \lambda_1^X \\ \lambda_2^X \\ \lambda_1^Y \\ \lambda_2^Y \\ \beta \\ \delta \end{bmatrix}.$$

The solution of β parameter satisfies the Equation (4). The log-linear model is one of the specialized cases of generalized linear models for Poisson-distributed data. Therefore, the iterative proportional fitting process generates ML estimates of the expected cell frequencies (Agresti [2]).

3. Quasi-likelihood Approach

Quasi-likelihood (QL) estimating equations introduced by Wedderburn [18] for generalized linear models are a useful framework for statistical modelling without a fully specified likelihood for non-normal multivariate data.

Let Y_i be a response variable having N independent observations $(Y_1, Y_2, \ldots, Y_N)$ and let $X_{i1}, X_{i2}, \ldots, X_{ip}$ be p covariates. In generalized linear models, the mean response μ_i is linearly related to the covariates via a link function $g^{-1}(.)$ as the following form:

$$g^{-1}(\mu_i) = \beta_0 + \beta_1 X_{il} + \ldots + \beta_p X_{ip}.$$

The QL estimate of $\beta = (\beta_0, \beta_1, \ldots, \beta_p)^T$ is obtained as a solution of the estimating equation in Equation (5).

$$U(\beta) = \sum_{i=1}^{N} (\partial \mu_i / \partial \beta)^T (V(Y_i))^{-1} (Y_i - \mu_i(\beta)) = 0, \tag{5}$$

where the response Y_i, $i = 1, \ldots, N$ have mean μ_i and variance $V(Y_i) = \phi v(\mu_i)$, $v(\mu_i)$ is a known variance function and ϕ is a scale parameter for the overdispersion $(\phi > 0)$ (Wedderburn [18]). The QL estimator of β is consistent and asymptotically normal even when the variance of the response has been misspecified (Fitzmaurice et al. [8]).

We used quasi-likelihood estimating equation in Equation (5) to analyze the structure of association and agreement in the log-linear Model (2). Using the advantages of QL, we will adopt it to the Model (2). Based on QL approach in the assessment of rater agreement in $R \times R$ contingency tables, let Y_i be considered as n_{ij} which is the cell frequencies in i row and j column given for both X and Y raters and let μ_{ij} be the mean which is linearly related to the covariates via a log link function as Model (2):

$$\log \mu_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \beta u_i v_j + \delta_{ij}, \qquad i, j = 1, \ldots, R. \tag{6}$$

In $R \times R$ contingency tables, n_{ij} are Poisson responses, link function can be defined as "Poisson log". Solution of equations based on Model (2) gives QL estimates of parameters. We can obtain ML estimates for the cells under any log-linear model by iterative fitting algorithm. Parameter estimates and their standard errors can be expressed as a linear combination of the logarithms of the observed cell frequencies plus user-specified covariates. But the main purpose of this article is to compare between two competitive methods. In this paper, we apply the QL to random variables N_{rs} with N_{rs} the cell count for the ordinal outcomes $(Y_{i1} = r, Y_{i2} = s)$.

4. Simulation Study

A simulation study basically has been carried out to compare the standard errors of agreement and association parameters in Model (2). We used R-Project program to generate samples following multinomial distributions. Random samples with given fixed marginal totals are generated under multinomial sampling scheme.

In the data generation process, dimension of tables and sample sizes are set to $R = 3, 4, 5, 6$ and $N = 50, 150$, and 500, respectively. Simulation is replicated 1000 times. We often do not know the actual joint distribution of the row and column variables but we can use the sample as the estimate of the joint distribution. Detailed explanations of how to assign the marginal probabilities can be found in Yang [19]'s work. Multinomial probabilities and the joint distribution for 3×3 under multinomial distribution are shown in Table 2. Marginal probabilities for 4×4, 5×5, and 6×6 as follows:

For $R = 4$: $P_X(x) = (0.20, 0.10, 0.20, 0.50)$; $P_Y(y) = (0.20, 0.10, 0.20, 0.50)$.

For $R = 5$: $P_X(x) = (0.22, 0.16, 0.24, 0.25, 0.13)$; $P_Y(y) = (0.29, 0.21, 0.15, 0.27, 0.08)$.

For $R = 6$: $P_X(x) = (0.20, 0.14, 0.22, 0.23, 0.10, 0.11)$; $P_Y(y) = (0.27, 0.19, 0.13, 0.25, 0.10, 0.06)$.

For all simulated tables, only the tables that hold the linear by linear plus association model are selected. As the Kappa values greater than 80% represents very good agreement beyond chance (Altman [4]). All simulated tables are set to high agreement case by given high probabilities to main diagonals. The sum of the probabilities on the main diagonals is approximately 90%. This proportion generally corresponds to $\beta \approx 0$ and $2 \leq \delta \leq 3.5$. The δ values greater than 1 represent high agreement beyond chance. An example of 3×3 random samples is displayed in Table 1.

Table 1. A simulated table for $N = 500$

X/Y	1	2	3	Total
1	100	5	10	115
2	10	200	10	220
3	5	10	150	165
Total	115	215	170	500

Table 2. Marginal probabilities

X/Y	1	2	3	$P_X(x)$
1	0.20	0.01	0.02	0.23
2	0.02	0.40	0.02	0.44
3	0.01	0.02	0.30	0.33
$P_Y(y)$	0.23	0.43	0.34	1

For data in Table 1, weighted Kappa coefficient is calculated as 83.75% with standard error 0.0231. Table 3 displays the parameter estimates and their standard errors of 1000 replications. ML estimates are calculated by the classical log-linear model estimation.

All of the simulated tables are fitted to Model (2) at the level of 10% significance level. Due to the tables are generated under the multinomial model, parameter estimates are obtained as small but statistically significant at 10%. The coefficients β are all positive, this means the positive relation between the row and column variables. Some of the δ coefficients are close to zero.

Table 3. Parameter estimates and their standard errors under Model (2)

Dimension	**Sample Size**	**QL**	**ML**
3×3	50	$\hat{\delta} = 3.726\ (0.212)$ $\hat{\beta} = 0.725\ (0.148)$	$\hat{\delta} = 3.726\ (0.735)$ $\hat{\beta} = 0.725\ (0.441)$
3×3	150	$\hat{\delta} = 3.034\ (0.046)$ $\hat{\beta} = 0.307\ (0.064)$	$\hat{\delta} = 3.034\ (0.324)$ $\hat{\beta} = 0.307\ (0.308)$
3×3	500	$\hat{\delta} = 2.197\ (0.049)$ $\hat{\beta} = 0.222\ (0.160)$	$\hat{\delta} = 2.197\ (0.185)$ $\hat{\beta} = 0.222\ (0.704)$
4×4	50	$\hat{\delta} = 3.698\ (0.180)$ $\hat{\beta} = 0.274\ (0.046)$	$\hat{\delta} = 3.698\ (0.395)$ $\hat{\beta} = 0.274\ (0.121)$
4×4	150	$\hat{\delta} = 3.342\ (0.103)$ $\hat{\beta} = 0.210\ (0.039)$	$\hat{\delta} = 3.342\ (0.243)$ $\hat{\beta} = 0.210\ (0.149)$
4×4	500	$\hat{\delta} = 2.211\ (0.054)$ $\hat{\beta} = 0.095\ (0.028)$	$\hat{\delta} = 2.211\ (0.121)$ $\hat{\beta} = 0.095\ (0.050)$
5×5	50	$\hat{\delta} = 3.728\ (0.240)$ $\hat{\beta} = 0.556\ (0.058)$	$\hat{\delta} = 3.728\ (0.364)$ $\hat{\beta} = 0.556\ (0.089)$
5×5	150	$\hat{\delta} = 3.008\ (0.085)$ $\hat{\beta} = 0.081\ (0.027)$	$\hat{\delta} = 3.008\ (0.214)$ $\hat{\beta} = 0.081\ (0.053)$
5×5	500	$\hat{\delta} = 2.977\ (0.080)$ $\hat{\beta} = 0.131\ (0.068)$	$\hat{\delta} = 2.977\ (0.191)$ $\hat{\beta} = 0.131\ (0.143)$
6×6	50	$\hat{\delta} = 2.671\ (0.045)$ $\hat{\beta} = 0.886\ (0.235)$	$\hat{\delta} = 2.671\ (0.312)$ $\hat{\beta} = 0.886\ (0.343)$
6×6	150	$\hat{\delta} = 2.159\ (0.077)$ $\hat{\beta} = 0.456\ (0.054)$	$\hat{\delta} = 2.159\ (0.111)$ $\hat{\beta} = 0.456\ (0.253)$
6×6	500	$\hat{\delta} = 1.928\ (0.075)$ $\hat{\beta} = 0.556\ (0.092)$	$\hat{\delta} = 1.928\ (0.098)$ $\hat{\beta} = 0.556\ (0.134)$

From the simulation results, note that model parameter estimates are exactly the same but with different standard errors. Standard errors calculated by QL give smaller values than for those calculated by ML. Standard errors of QL estimates are much smaller than ML estimates in each combination of simulation parameters. When the sample size is large, standard errors are expectedly becoming smaller. It can be seen that there is more disagreement between marginal probabilities as well as high probabilities for off-diagonal probabilities.

5. Numerical Example

The example in Table 4 gives frequencies from the study on the diagnosis of multiple sclerosis (MS) from Landis and Koch [11]. In this example, 149 Winnipeg patients were examined by two neurologists, one from New Orleans, and the other from Winnipeg. The two neurologists classified each patient into one of the following classes: (1 = Certain MS, 2 = Probable MS, 3 = Possible MS, 4 = Doubtful, Unlikely, or Definitely not MS).

Table 4. Diagnostic classification regarding multiple sclerosis for the Winnipeg patients

	Winnipeg neurologist				
New Orleans neurologist	**1**	**2**	**3**	**4**	**Total**
1	38	5	0	1	44
2	33	11	3	0	47
3	10	14	5	6	35
4	3	7	3	10	23
Total	84	37	11	17	149

Model (2) addresses the agreement and association between New Orleans neurologist and Winnipeg neurologist. Classical approach is to analyze data through general log-linear analysis. Likelihood ratio statistics for goodness of fit test is 9.416 with 7 *df*. This means that implies that the model is statistically significant at 5%. Model (2) fits the data well. Table 5 displays the parameter estimates under Model (2). Since constants are not parameters under the multinomial assumption, their standard errors are not calculated.

Table 5. Parameter estimates for Model (2) using ML approach

Parameter	**Estimate**	**St. Error**	**95% CI**	**Z**	**Pr>Z**
Intercept	10.458	–	–	–	–
row = 1	5.071	1.003	(3.105, 7.036)	5.056	0.000
row = 2	4.088	0.766	(2.587, 5.589)	5.340	0.000
row = 3	2.435	0.522	(1.411, 3.458)	4.662	0.000
row = 4	0	0	0	–	–
col = 1	8.210	1.389	(5.488, 10.932)	5.912	0.000
col = 2	5.708	1.094	(3.564, 7.851)	5.218	0.000
col = 3	2.258	0.695	(0.896, 3.620)	3.250	0.001
col = 4	0	0	0	–	–
$\hat{\beta}$	0.804	0.155	(0.500, 1.108)	5.181	0.000
$\hat{\delta}$	– 0.028	0.243	(– 0.504, 0.448)	– 0.115	0.909

As mentioned in earlier sections, QL methodology is applied data for estimating the parameters (i.e., parameters in Table 5). Score statistics (Table 6) for QL analysis show that only column variable and beta parameters are the statistically significant model components.

Table 6. Score statistics based on Type 3 sums of squares in QL analysis

Source	**df**	**Chi-Square**	**Pr>ChiSq**
row	3	4.45	0.2172
column	3	8.73	0.0330
β	1	9.07	0.0026
δ	1	0.05	0.8218

Table 7 gives the QL parameter estimates under Model (2). Agreement parameter $\hat{\delta} = -0.0278$ indicates that there is no agreement between New Orleans neurologist and Winnipeg neurologist. $\hat{\beta} = 0.8038$ indicates that there is a positive association between two variables $(P < 0.001)$.

Table 7. Parameter estimates of Model (2) using QL approach

Parameter	**Estimate**	**St. Error**	**95% CI**	**Z**	**Pr>Z**
Intercept	10.458	2.122	(– 14.618, – 6.2985)	– 4.93	0.000
row = 1	5.071	0.747	(3.606, 6.535)	6.79	0.000
row = 2	4.088	0.575	(2.962, 5.214)	7.11	0.000
row = 3	2.435	0.369	(1.711, 3.159)	6.59	0.000
row = 4	0	0	0	–	–
col = 1	8.210	1.329	(5.604, 10.815)	6.18	0.000
col = 2	5.708	1.052	(3.646, 7.769)	5.43	0.000
col = 3	2.258	0.629	(1.026, 3.491)	3.59	0.000
col = 4	0	0	0	–	–
$\hat{\beta}$	0.804	0.137	(0.536, 1.072)	5.88	0.000
$\hat{\delta}$	– 0.028	0.122	(– 0.267, 0.211)	– 0.23	0.820

The methods ML and QL are difficult to compare (Hunger et al. [10]), however, it has been recognized that the parameters in the QL are estimated with more precision. Standard errors for model parameters are smaller than for those obtained from ML method. Expected frequencies under Model (2) from QL are presented in Table 8. Expected frequencies reflect the assumptions concerning the relationships between the variables under study.

Table 8. Expected frequencies and 95% CI for mean response under Model (2)

	Winnipeg neurologist				
New Orleans neurologist	**1**	**2**	**3**	**4**	**Total**
1	36.52 [33.32; 40.04]	6.87 [4.43; 10.66]	0.48 [0.19; 1.25]	0.11 [0.02; 0.66]	44
2	31.41 [27.93; 35.32]	12.49 [9.87; 15.81]	2.03 [1.21; 3.42]	1.06 [0.39; 2.93]	47
3	13.43 [9.53; 18.93]	12.27 [9.20; 16.37]	4.23 [2.93; 6.09]	5.06 [3.36; 7.64]	35
4	2.63 [1.44; 4.81]	5.37 [3.85; 7.49]	4.25 [2.66; 6.78]	10.76 [8.98; 12.88]	23
Total	84	37	11	17	149

6. Conclusion

In this paper, we advocate the use of quasi-likelihood (QL) approach over the more common log-linear analysis by using ML method in the analysis of $R \times R$ contingency tables. The main purpose is merely to focus on the agreement plus linear by linear association model which allows researchers to consider the ordinal nature of contingency tables. We consider QL approach, since it has several practical advantages that can be summarized as: (i) its computational simplicity rather than maximum likelihood (ML) for categorical data, (ii) QL does not require any multivariate distributional assumptions, (iii) consistent parameter estimates can be obtained even with misspecified variance. One can compare the empirical estimates with the model-based estimates. While constants are not parameters under the multinomial assumption, therefore their standard errors are not calculated, but QL treats constant term as a parameter and estimates it. There is no need for the model fit of the QL, because there is no likelihood function. The overall confidence intervals are much smaller in QL approach and parameter estimates from ML and QL methods are identical. Differences could be observed with respect to standard errors, regardless of sample size and dimension of the table.

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