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VALIDITY OF SCHAFFERNAK AND CASAGRANDE'S ANALYTICAL SOLUTIONS FOR SEEPAGE THROUGH A HOMOGENEOUS EARTH DAM

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Abstract

Study of seepage from earth dam is important due to its applications in reservoir management. The loss of water due to seepage constitutes a substantial. The seepage loss results not only in depleted fresh water resources but also causes water logging, salinization, groundwater contamination, and health hazards. This study presents both a numerical and analytical solutions for seepage from a homogeneous earth dam resting on an impervious base. For this purpose, a steady-state two-dimensional flow through a homogeneous earth dam has been analyzed. Phreatic line location is computed by numerical simulation, using Seep/w software based on finite elements. Numerical simulation results are compared with Schaffernak and Casagrande's analytical solutions. Results showed that numerical solution gives more seepage rate than the two other solutions, i.e., Casagrande and Schaffernak's methods. In most cases, Schaffernak's solution has more than 20% error and Casagrande's solution has more than 30% error.

Keywords: Earth dam, phreatic line, homogeneous embankment, seepage.

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1. Introduction

Earth dams are among the world's oldest hydro-engineering structures. They are constructed to control flood and safeguard land, properties, and living beings. Several investigators have suggested various methods to determine the quantity of seepage and locus of the phreatic line. Kozeny [9] studied the seepage through an earth dam with a horizontal toe drain (under filter) resting on an impervious base assuming the earth dam to have a parabolic upstream face.

Kozeny [9] ignored the resistance of the portion bounded by the parabolic face and upstream straight slope surface conveniently assuming this portion to be comprised of rock-fill materials, though the ignored portion of the earth dam also controls the quantity of seepage and location of the phreatic line (Mishra and Singh [10]). Applying the method of fragments, Pavlovsky [12] determined the quantity of seepage and locus of phreatic line in an earth dam resting on an impervious base without a toe filter. The flow domain has been decomposed into three fragments and the hydraulic resistance of the soil in the upstream side has been considered for finding the flow characteristics. Casagrande [3] made a correction for the entrance condition at the upstream face and recommended the parabolic free surface to start at a point 0.3Δ upstream, where Δ is equal to base width of the upstream triangular part.

Numerov [11] has analyzed seepage through an earth dam having a straight upstream slope face and a toe drain. The problem has been identified as a Riemann-Hilbert problem. Using Numerov's solution, one can find the seepage quantity through a levee and location of the phreatic line. However, the solution to the Riemann-Hilbert problem is somewhat intractable for the computation of seepage characteristics (Mishra and Singh [10]).

In many cases, the seepage may result in excess hydrostatic pressures or uplift pressures beneath elements of the structure or landward strata. Relief wells are often installed to relieve these pressures which might otherwise endanger the safety of the structure (Gebhart [8]).

Several investigators (Cividini and Gioda [5]; Billstein et al. [2]; Bardet and Tobita [1]) have applied numerical techniques to determine the quantity of seepage and locus of the phreatic line. Determination of phreatic line by numerical techniques involves iteration and requires special formulation.

Several solutions have been proposed for determination of the quantity of seepage through a homogeneous earth dam resting on an impervious base. In this study, two of these solutions, i.e., Schaffernak [13] and Casagrande's [4] methods (as an analytical solution) will be considered. Then by using Seep/w software, based on finite elements numerical solution, comparison among numerical with analytical solutions will be carried out.

The goal of this study is to show the validity of two analytical methods, i.e., Schaffernak and Casagrande, respect to advanced method of finite elements solution. For this purpose, several homogeneous earth dam resting on an impervious base, are assumed and calculation of seepage with three mention method will be perform.

2. Material and Methods

Governing equations (Das [6])

Schaffernak's solution

For calculation of seepage through a homogeneous earth dam, Schaffernak [13] proposed that the phreatic surface will be like line ab in Figure 1, i.e., it will intersect the downstream slope at a distance l from the impervious base.

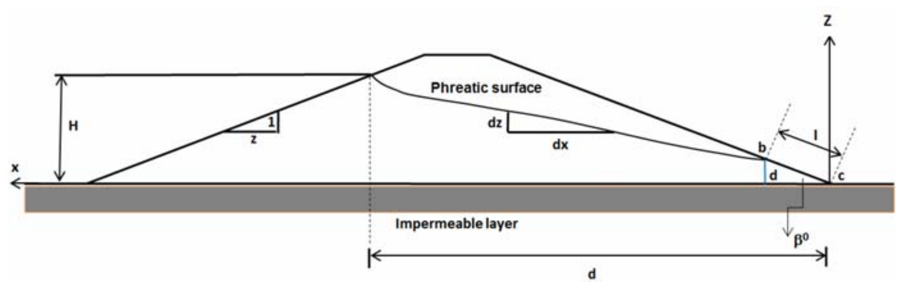

Figure 1. Schaffernak's solution for flow through an earth dam.

The seepage per unit length of the dam can now be determined by considering the triangle *bcd* in Figure 1:

$$q = -kAi,$$

$$A = \overline{bd}(1) = l \sin \beta.$$

From Dupuit's [7] assumption, the hydraulic gradient is given by $i = dz / dx$, so:

$$q = -kz\frac{dz}{dx} = (k)(l \sin \beta)(tg\beta), \tag{1}$$

$$\int_{\sin \beta}^{H} zdz = \int_{\cos \beta}^{l} (l \sin \beta)(tg\beta)dx,$$

$$\frac{1}{2}\left(H^2 - l^2 \sin^2 \beta\right) = (l \sin \beta)(tg\beta)(d - l \cos \beta),$$

$$\frac{H^2 \cos \beta}{2 \sin^2 \beta} - \frac{l^2 \cos \beta}{2} = ld - l^2 \cos \beta,$$

$$l^2 \cos \beta - 2ld + \frac{H^2 \cos \beta}{\sin^2 \beta} = 0,$$

$$l = \frac{2d \pm \sqrt{4d^2 - 4[(H^2 \cos^2 \beta) / \sin^2 \beta]}}{2 \cos \beta}.$$

So,

$$l = \frac{d}{\cos\beta} - \sqrt{\frac{d^2}{\cos^2\beta} - \frac{H^2}{\sin^2\beta}}. \tag{2}$$

Once the value of l is known, the rate of seepage can be calculated from the equation, $q = -kl\sin\beta . tg\beta$.

Casagrande's solution

Casagrande [3] showed experimentally that the parabola ab shown in Figure 1 should actually start from the point a' as shown in Figure 2.

Note that $aa' = 0.3\Delta$. So, with this modification, the value of d for use in Equation (2) will be the horizontal distance between points a' and c.

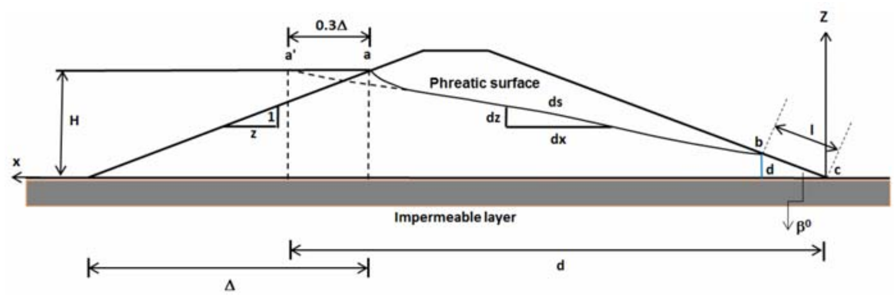

Figure 2. Casagrande's solution for flow through an earth dam.

(Note: length of the curve $a'bc = S$).

Equation (2) was obtained on the basis of Dupuit's assumption that the hydraulic gradient i is equal to dz/dx. Casagrande [3] suggested that this relation is an approximation to the actual condition. In reality (see Figure 2),

$$i = dz/ds. \tag{3}$$

For a downstream slope of $\beta > 30^\circ$, the deviations from Dupuit's assumption become more noticeable. Based on this assumption [Equation (3)], the rate of seepage is $q = kiA$. Considering the triangle bcd in Figure 2,

$$i = dz / ds = \sin\beta,$$

$$A = (bd)(1) = l\sin\beta,$$

$$q = k\frac{dz}{ds}z = kl\sin^2\beta, \tag{4}$$

$$\int_{\sin\beta}^{H} zdz = \int\left(l\sin^2\beta\right)ds,$$

where s is the length of the curve $a'bc$. Hence,

$$\frac{1}{2}\left(H^2 - l^2\sin^2\beta\right) = l\sin^2\beta(s - l),$$

$$l^2 - 2ls + \frac{H^2}{\sin^2\beta} = 0,$$

$$l = s - \sqrt{s^2 - \frac{H^2}{\sin^2\beta}}. \tag{5}$$

With about a 4-5% error, we can approximate s as the length of the straight line $a'c$. So,

$$s = \sqrt{d^2 + H^2}. \tag{6}$$

Combining Equations (5) and (6),

$$l = \sqrt{d^2 + H^2} - \sqrt{d^2 - H^2\cot g^2\beta}. \tag{7}$$

Once l is known, the rate of seepage can be calculated from the equation $q = kl\sin^2\beta$.

3. Numerical Simulation

The present seepage problem is solved by using the method of Schaffernak and Casagrande's analytical solutions as well as numerical method. Seep/w software (2007) was used for numerical method implementation.

For this purpose, 28 different assumed models were used with H = 12-18 meter and Z = 0.5-4 (β = 14.04-63.43 degree). At each model, location of intersection of phreatic line with downstream slope (l) was determined. Then percentage of error of two analytical solutions, i.e., Schaffernak and Casagrande's solution, respect to numerical method was calculated.

Figure 3 shows one of the homogeneous earth dams assumed in this study. In boundary condition, water level (total head) in upstream is 16 meter, water level in downstream was assumed zero meter. Also, the foundation's floor/base is impermeable (zero flow). Node at the toe of dam has atmospheric pressure (zero pressure). The upstream and downstream slope shell of dam has inclination 1V:2.5H. Used software for numerical simulation is Seep/w to calculate the location of phreatic intersection with downstream slope (l). Two dimensional simulation of homogeneous earth dam have 2597 elements (Figure 3).

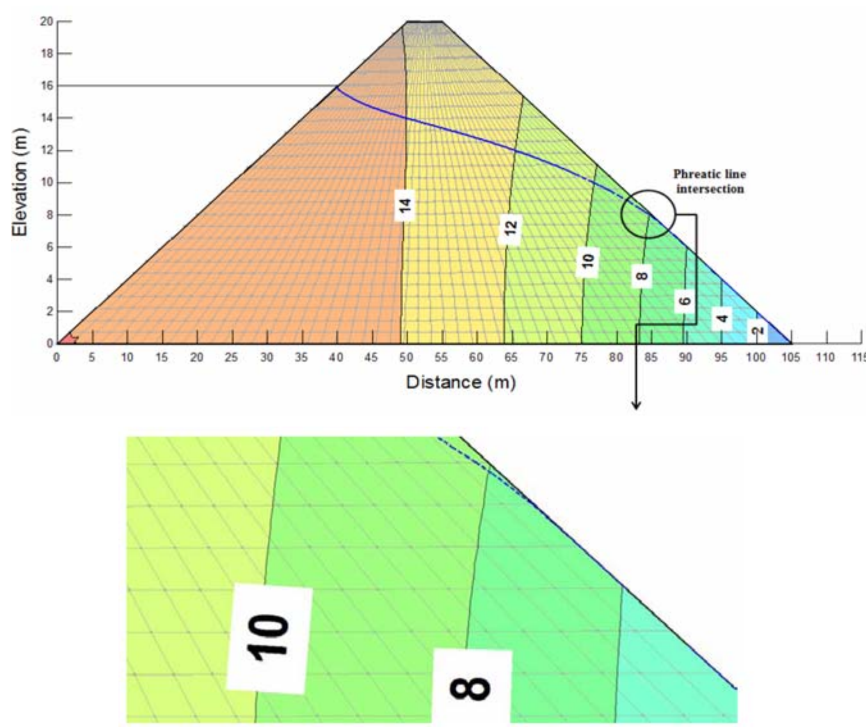

Figure 3. Cross section of homogeneous earth dam with H = 16 meter and Z = 2.5.

4. Results and Discussion

Tables 1-7 show the results of simulation. Figures 4 and 5 represent location of intersection of phreatic line with downstream slope (l) against Z in constant water level behind the dam, i.e., H = 18m and H = 12m, respectively.

Table 1. Locus of intersection of phreatic line with downstream slope (l) in meter for $Z = 4$

H (m)	Seep/w	Schaffernak	Casagrande	% error for Schaffernak	% error for Casagrande
18	38.62	35.19	26.85	8.87	30.48
16	28.49	23.57	19.52	17.26	31.50
14	20.67	15.96	13.93	22.75	32.60
12	14.57	10.62	9.63	27.12	33.90

Table 2. Locus of intersection of phreatic line with downstream slope (l) in meter for $Z = 3$

H (m)	Seep/w	Schaffernak	Casagrande	% error for Schaffernak	% error for Casagrande
18	29.71	26.30	20.57	11.49	30.76
16	21.82	17.73	15.01	18.73	31.20
14	15.96	12.05	10.74	24.50	32.70
12	11.97	8.03	7.44	32.93	37.87

Table 3. Locus of intersection of phreatic line with downstream slope (l) in meter for $Z = 2.5$

H (m)	Seep/w	Schaffernak	Casagrande	% error for Schaffernak	% error for Casagrande
18	25.93	21.88	17.55	15.58	32.30
16	18.01	14.83	12.85	17.64	28.63
14	14.00	10.10	9.22	27.85	34.19
12	10.00	6.75	6.39	32.56	36.11

Table 4. Locus of intersection of phreatic line with downstream slope (l) in meter for $Z = 2$

H (m)	Seep/w	Schaffernak	Casagrande	% error for Schaffernak	% error for Casagrande
18	20.53	17.63	14.70	14.14	28.39
16	16.44	12.03	10.81	26.85	34.25
14	12.39	8.22	7.78	33.60	37.21
12	8.31	5.50	5.41	33.77	34.89

Table 5. Locus of intersection of phreatic line with downstream slope (l) in meter for $Z = 1.5$

H (m)	Seep/w	Schaffernak	Casagrande	% error for Schaffernak	% error for Casagrande
18	18.03	13.53	12.12	24.94	32.76
16	14.03	9.32	8.99	33.51	35.93
14	10.02	6.41	6.51	36.03	35.10
12	8.01	4.31	4.55	46.26	43.28

Table 6. Locus of intersection of phreatic line with downstream slope (l) in meter for $Z = 1$

H (m)	Seep/w	Schaffernak	Casagrande	% error for Schaffernak	% error for Casagrande
18	14.14	9.74	10.12	31.15	28.41
16	11.11	6.82	7.62	38.66	31.40
14	8.16	4.73	5.59	42.03	31.56
12	6.12	3.20	3.94	47.74	35.64

Table 7. Locus of intersection of phreatic line with downstream slope (l) in meter for $Z = 0.5$

H (m)	Seep/w	Schaffernak	Casagrande	% error for Schaffernak	% error for Casagrande
18	13.21	6.20	9.56	53.10	27.62
16	10.16	4.47	7.47	55.99	26.47
14	8.13	3.17	5.65	61.04	30.52
12	6.10	2.17	4.10	64.35	32.82

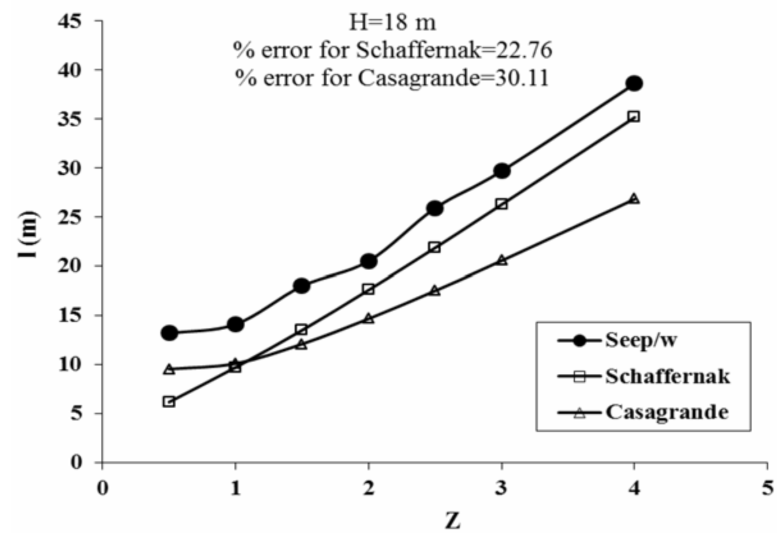

Figure 4. Locus of intersection of phreatic line with downstream slope (l) against Z for $H = 18$m.

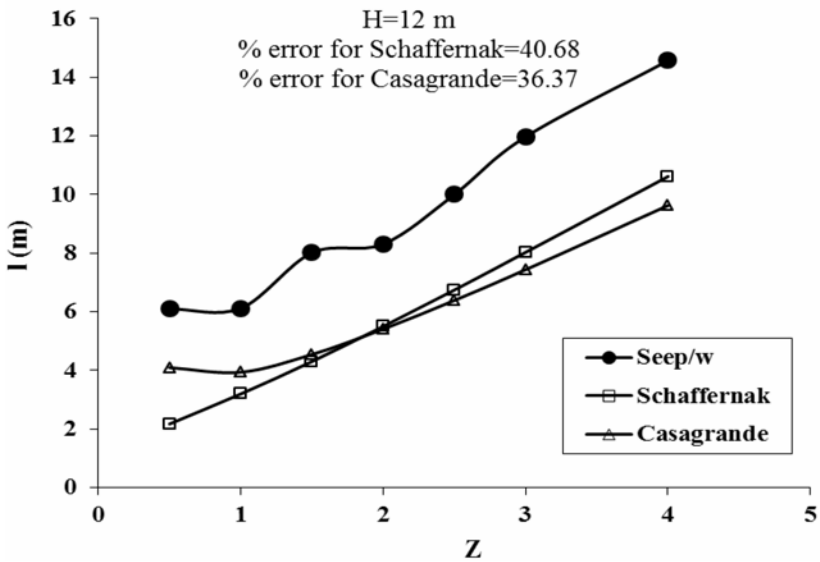

Figure 5. Locus of intersection of phreatic line with downstream slope (l) against Z for H = 12m.

As seen from Figure 5, for H = 12m, there is close agreement between Casagrande and Schaffernak's method, but numerical solution give much locus phreatic line (l) or seepage rate than Casagrande and Schaffernak's methods.

However, for higher values of β, i.e., for a steeper slope ($\beta > 30$ deg. or $Z < 1.5$), Schaffernak's solution is not enough. The approximate method of Schaffernak, provides a solution tractable for numerical computation, for $Z > 1.5$ (H = 18 meter) and the average error is 22.76%. For a flatter upstream slope, the Casagrande's method is not enough for finding the true seepage losses and/or locus of intersection of phreatic line with downstream slope *(l)*.

Figure 6 shows the variation of locus of intersection of phreatic line with downstream slope (l) against H, for Z = 2.5. In general, Schaffernak's method is more accurate from Casagrande's method.

Also numerical solution gives more seepage rate than the two other solutions, i.e., Casagrande and Schaffernak's methods. In most cases, Schaffernak's solution has more than 20% error and Casagrande solution has more than 30% error.

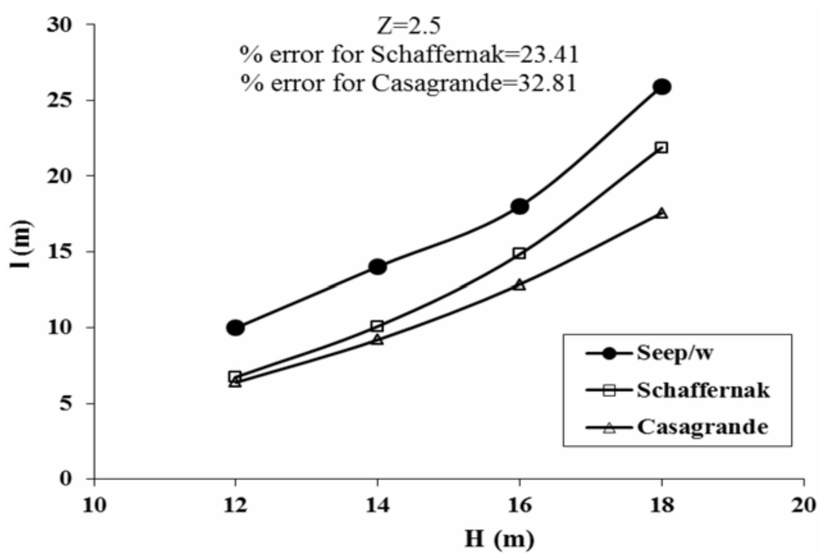

Figure 6. Locus of intersection of phreatic line with downstream slope (l) against H for Z = 2.5.

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