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ON GENERAL HIRATA-AZUMAYA GALOIS EXTENSIONS

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Abstract

Let B be a general Hirata-Azumaya Galois extension of B^G with Galois group G. Then several characterizations of such a B are shown, and an equivalent condition is given for such a B satisfying the fundamental theory.

1. Introduction

Let B be a ring with 1, G be a finite automorphism group of B, C be the center of B, and $B^G = \{b \in B \mid g(b) = b \text{ for each } g \in G\}$. In [3, 6, 11], the class of central Galois extensions B over C with Galois group G was studied. In [5, 8, 14], the class of Hirata separable and Galois extensions B of B^G with Galois group G was investigated. In [1, 2, 15], the class of Galois extensions B of B^G with Galois group G such that B^G is an Azumaya C^G-algebra was explored, such a B is called an Azumaya

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Galois extension. Then in [17], the author studied the class of Hirata separable and Azumaya Galois extensions B of B^G with Galois group G. This is a broader class of Galois extensions than the class of central Galois extensions. Noting that a Galois extension B of B^G with Galois group G such that B^G is a separable C^G-algebra is not necessarily an Azumaya Galois extension Galois group G. In [13, 16, 18], a Galois extension B of B^G with Galois group G such that B^G is a separable C^G-algebra is called a general Azumaya Galois extension of B^G with Galois group G. The purpose of the present paper is to study the class of Hirata separable and general Azumaya Galois extensions B of B^G with Galois group G. We shall show several characterizations of such a Galois extension B and give an equivalent condition for such a B satisfying the fundamental theory which generalizes Theorem 3.4 in [12] and Theorem 3.6 in [17].

2. Basic Definitions and Notations

Let B be a ring with 1, C be the center of B, G be a finite automorphism group of B, B^G be the set of elements in B fixed under each element in G, and $B * G$ be a skew group ring over B in which the multiplication is given by $gb = g(b)g$ for $b \in B$ and $g \in G$, and $\overline{G}$ be the inner automorphism group of $B * G$ induced by G, that is, $\overline{g}(x) = gxg^{-1}$ for each $x \in B * G$ and $g \in G$. We note that $\overline{G}$ restricted to B is G.

Let A be a subring of B with the same identity 1. We call B a separable extension of A if there exist $\{a_i, b_i$ in B, $i = 1, 2, \dots, m$ for some integer $m\}$ such that $\sum a_i b_i = 1$, and $\sum ba_i \otimes b_i = \sum a_i \otimes b_i b$ for all b in B, where $\otimes$ is over A, and an Azumaya algebra is a separable extension of its center ([4]). A ring B is called a Hirata separable extension of A if $B \otimes_A B$ is isomorphic to a direct summand of a finite direct sum of B as a B-bimodule ([8]).

We call B a Galois extension of B^G with Galois group G if there exist elements $\{a_i, b_i$ in B, $i = 1, 2, \ldots, m\}$ for some integer m such that $\sum_{i=1}^{m} a_i g(b_i) = \delta_{1,g}$ for each $g \in G$ ([3]). A ring B is called a Galois algebra over R if B is a Galois extension of R such that R is contained in the center C of B, and a central Galois algebra over R if $R = C$ ([6]). We call B a Hirata Galois extension of B^G with Galois group G if it is a Hirata separable and Galois extension of B^G with Galois group G ([8]). A Galois extension B with Galois group G is called an Azumaya Galois extension of B^G if B^G is an Azumaya C^G-algebra ([1, 2]) and a general Azumaya Galois extension of B^G if B^G is separable over C^G ([16]).

As in [17], B is called a Hirata-Azumaya Galois extension of B^G with Galois group G if B is a Hirata separable and an Azumaya Galois extension of B^G with Galois group G. In this paper, we call B a general Hirata-Azumaya Galois extension of B^G with Galois group G if B is a Hirata separable and a general Azumaya Galois extension of B^G with Galois group G. A Galois extension B of B^G with Galois group G is called satisfying the fundamental theory if $\alpha : H \to B^H$ is a one-to-one correspondence between the set of subgroups of G and the set of separable extensions A of B^G in B with the inverse map $\alpha^{-1} : A \to G(A)$, where $G(A) = \{g \in G \mid g(a) = a$ for all $a \in A\}$.

Throughout this paper, we assume that B is a Galois extension of B^G with Galois group G, C the center of B, $J_g = \{b \in B \mid bx = g(x)b$ for each $x \in B\}$ for a $g \in G$, and for a subring A of B with the same identity 1, $G(A) = \{g \in G \mid g(a) = a$ for all $a \in A\}$ and $V_B(A) = \{b \in B \mid ab = ba$ for each $a \in A\}$ the commutator (also called centralizer) subring of A in B.

3. Main Results

Keeping the definitions and notations in Section 2, in this section, we shall show several characterizations of a general Hirata-Azumaya Galois extension B of B^G with Galois group G, and give an equivalent condition for such a B satisfying the fundamental theory. We begin with a very useful property of an Azumaya algebra given by Ikehata.

Lemma 3.1 ([5], Theorem 1). *Let B be an Azumaya algebra and A be a subalgebra of B. If B is projective as a left A-module, then B is a Hirata separable extension of A.*

Lemma 3.2 ([4], Theorem 3.8, page 55). *Let A be a separable algebra and D be a subalgebra of A. If A is a left projective D-module and D is a direct summand of A as a D-bimodule, then D is a separable subalgebra of A.*

Theorem 3.3. *The following statements are equivalent*:

(1) *B is a general Hirata-Azumaya Galois extension of B^G with Galois group G.*

(2) *B is a general Azumaya Galois extension of B^G with Galois group G such that $C = C^G$.*

(3) *B is a Galois extension of B^G with Galois group G such that B is an Azumaya C^G-algebra and the order of G is invertible in B.*

(4) *$B * G$ is a Galois extension of $(B * G)^{\overline{G}}$ with an inner Galois group $\overline{G}$ induced by G such that $(B * G)^{\overline{G}}$ is a separable C^G-algebra and $C = C^G$.*

Proof. (1) $\Rightarrow$ (2) By hypothesis, B is a general Azumaya Galois extension of B^G with Galois group G. So it suffices to show $C = C^G$. In fact, since B is a Hirata separable Galois extension of B^G, $V_B(V_B(B^G)) = B^G$ ([8], Proposition 4-(1)). Hence $C \subset B^G$, and so $C = C^G$.

(2) $\Rightarrow$ (1) Since B is a Galois extension of B^G with Galois group G, B is a separable extension of B^G and a left projective B^G-module. Also, since B is a general Azumaya Galois extension of B^G with Galois group G by hypothesis, B^G is a separable C^G-algebra. Hence B is a separable C^G-algebra by the transitivity property of separable extensions. Noting that $C = C^G$ by hypothesis, we conclude that B is an Azumaya C^G-algebra. Thus, B^G is a subalgebra of the Azumaya C^G-algebra B such that B is left projective over B^G. Therefore, B is a Hirata separable extension of B^G by Lemma 3.1. Hence B is a general Hirata-Azumaya Galois extension of B^G with Galois group G.

(1) $\Rightarrow$ (3) By the proof of (1) $\Rightarrow$ (2), we have that $C = C^G$ and B is an Azumaya C^G-algebra. Hence, it suffices to show that the order of G is invertible in B. In fact, since B is a general Galois extension of B^G with Galois group G, B^G is a separable C^G-algebra. Hence B^G is a separable subalgebra of the Azumaya algebra B. Therefore $V_B(B^G)$ is a separable subalgebra of B by the commutator theorem for Azumaya algebras ([4], Theorem 4.3, page 57). Noting that B is a Hirata Galois extension of B^G with Galois group G, we conclude that the order of G is invertible in B ([8], Proposition 4-(3)).

(3) $\Rightarrow$ (1) Since B is a Galois extension of B^G with Galois group G, so B is a left projective B^G-module. Also B is an Azumaya C^G-algebra by hypotheses, so B is a Hirata separable extension of B^G by Lemma 3.1. Thus B is a Hirata Galois extension of B^G. Now it suffices to show that B^G is a separable C^G-algebra. In fact, since the order of G is invertible in B, B^G is a direct summand of B as a B^G-bimodule. Noting that B is a separable C^G-algebra and a left projective B^G-module, we conclude that B^G is a separable C^G-algebra by Lemma 3.2. Thus (1) holds.

(3) $\Rightarrow$ (4) Since B is a Galois extension of B^G with Galois group G, $B * G$ is a Galois extension of $(B * G)^{\overline{G}}$ with Galois group $\overline{G}$ with the same Galois system for B. Also, by hypotheses, B is an Azumaya C^G-algebra, that is, B is separable over C^G and $C = C^G$. Hence, it suffices to show that $(B * G)^{\overline{G}}$ is a separable C^G-algebra. In fact, since the order of G is invertible in B, $B * G$ is a separable extension of B. Hence $B * G$ is a separable C^G-algebra by the transitivity property of separable extensions. Moreover, since $B * G$ is a Galois extension of $(B * G)^{\overline{G}}$ with an inner Galois group $\overline{G}$ induced by G, $B * G$ is a left projective $(B * G)^{\overline{G}}$-module. Noting that $\overline{G}$ restricted to B is G, we have that the order of $\overline{G}$ is equal to the order of G. Hence the order of $\overline{G}$ is invertible in $B * G$. Thus $(B * G)^{\overline{G}}$ is a direct summand of $B * G$ as a $(B * G)^{\overline{G}}$-bimodule. Therefore $(B * G)^{\overline{G}}$ is a separable C^G-algebra by Lemma 3.2.

(4) $\Rightarrow$ (3) Since $B * G$ is a Galois extension of $(B * G)^{\overline{G}}$ with an inner Galois group $\overline{G}$ induced by G, B is a Galois extension of B^G with Galois group G ([9], Theorem 3.1). Also, since $B * G$ is a Galois extension of $(B * G)^{\overline{G}}$, $B * G$ is a separable extension of $(B * G)^{\overline{G}}$. Therefore, $B * G$ is a separable extension of C^G by the transitivity property of separable extensions because $(B * G)^{\overline{G}}$ is a separable C^G-algebra by hypotheses. Moreover, since $B * G$ is a left projective B-module and B is a direct summand of $B * G$ as a B-bimodule, B is a separable C^G-algebra by Lemma 3.2. But $C = C^G$ by hypotheses, so B is an Azumaya C^G-algebra. Next, we show that the order of G is invertible in B. Noting that $\overline{G}$ restricted to B is G, the order of $\overline{G}$ is equal to the order of G. Hence, it suffices to show the order of $\overline{G}$ is invertible in B. In fact, since

$B * G$ is a Galois extension of $(B * G)^{\overline{G}}$ with an inner Galois group $\overline{G}$, $B * G$ is a Hirata separable extension of $(B * G)^{\overline{G}}$ ([8], Corollary 3). Hence $B * G$ is a Hirata Galois extension of $(B * G)^{\overline{G}}$ with an inner Galois group $\overline{G}$. Moreover, since $(B * G)^{\overline{G}}$ is a separable C^G-algebra, $(B * G)^{\overline{G}}$ is a separable Z-algebra, where Z is the center of $B * G$ and is contained in $(B * G)^{\overline{G}}$. Noting that $B * G$ is separable over C^G, which is contained in the center Z of $B * G$, we have that $B * G$ is Azumaya Z-algebra. Thus $V_{B*G}((B * G)^{\overline{G}})$ is a separable Z-algebra by the commutator theorem for Azumaya algebras ([4], Theorem 4.3, page 57). Hence $B * G$ is a Hirata Galois extension of $(B * G)^{\overline{G}}$ such that $V_{B*G}((B * G)^{\overline{G}})$ is a separable subalgebra of $B * G$. Therefore, the order of $\overline{G}$ is invertible in $B * G$ ([8], Proposition 4). This completes the proof.

We remark that Theorem 3.3 (1) $\Rightarrow$ (4) shows that if B is a general Hirata-Azumaya Galois extension with Galois group G, then its skew group ring $B * G$ is also a general Hirata-Azumaya Galois extension with an inner Galois group $\overline{G}$ induced by G.

Corollary 3.4. *Let B be a general Hirata-Azumaya Galois extension of B^G with Galois group G. Then $B * G$ is a general Hirata-Azumaya Galois extension of $(B * G)^{\overline{G}}$ with an inner Galois group $\overline{G}$ induced by G.*

Proof. Since B is a general Hirata-Azumaya Galois extension of B^G with Galois group G, $B * G$ is a Galois extension of $(B * G)^{\overline{G}}$ with an inner Galois group $\overline{G}$ induced by G such that $(B * G)^{\overline{G}}$ is a separable C^G-algebra by Theorem 3.3 (1) $\Rightarrow$ (4). Hence $B * G$ is a Hirata separable extension of $(B * G)^{\overline{G}}$ ([8], Corollary 3). Thus $B * G$ is a Hirata Galois extension of $(B * G)^{\overline{G}}$ such that $(B * G)^{\overline{G}}$ is a separable

C^G-algebra. Clearly, the center Z of $B * G$ is contained in $(B * G)^{\overline{G}}$ and $C^G \subset Z^{\overline{G}}$. Therefore, $(B * G)^{\overline{G}}$ is a separable $Z^{\overline{G}}$-algebra. Hence $B * G$ is a general Azumaya Galois extension of $(B * G)^{\overline{G}}$ with inner Galois group $\overline{G}$, and so $B * G$ is a general Hirata-Azumaya Galois extension with Galois group $\overline{G}$.

Corollary 3.5. *Let B be a Hirata-Azumaya Galois extension of B^G with Galois group G. Then $B * G$ is a general Hirata-Azumaya Galois extension of $(B * G)^{\overline{G}}$ with an inner Galois group $\overline{G}$ induced by G.*

We note that in general for a Hirata-Azumaya Galois extension B with Galois group G, $B * G$ is not necessarily a Hirata-Azumaya Galois extension of $(B * G)^{\overline{G}}$ with an inner Galois group $\overline{G}$ induced by G.

Theorem 3.6. *Let B be a Hirata-Azumaya Galois extension of B^G with Galois group G. If G is commutative, then $B * G$ is not a Hirata-Azumaya Galois extension of $(B * G)^{\overline{G}}$ with an inner Galois group $\overline{G}$ induced by G.*

Proof. Since B is a Hirata-Azumaya Galois extension with Galois group G, B is an Azumaya Galois extension with Galois group G. Hence $B * G$ is an Azumaya C^G-algebra ([2], Theorem 3.1). But G is commutative by hypothesis, and so $C^G G$ is contained in the center of $(B * G)^{\overline{G}}$. Therefore $(B * G)^{\overline{G}}$ is not Azumaya over C^G. Thus $B * G$ is not an Azumaya Galois extension of $(B * G)^{\overline{G}}$ with an inner Galois group $\overline{G}$ induced by G, and so $B * G$ is not a Hirata-Azumaya Galois extension of $(B * G)^{\overline{G}}$ with an inner Galois group $\overline{G}$ induced by G.

Next, we show an equivalent condition for a general Hirata-Azumaya Galois extension B satisfying the fundamental theory. The proof is similar to the proof of Theorem 3.6 in [17] but more elaborate and without using the condition that B^G is an Azumaya C^G-algebra. We begin a lemma.

Lemma 3.7. *Let B be a general Azumaya Galois extension of B^G with Galois group G. Then B^H is a separable C^G-algebra for every subgroup H of G.*

Proof. Since B is a general Azumaya Galois extension of B^G with Galois group G, B^G is a separable C^G-algebra. Hence B is a Galois extension of a separable C^G-algebra with Galois group G. Therefore, B^H is a separable C^G-algebra for every subgroup H of G ([7], Proposition 3.1).

Theorem 3.8. *Let B be a general Hirata-Azumaya Galois extension of B^G with Galois group G. Then B satisfies the fundamental theorem if and only if for any separable extension A of B^G in B, $V_B(A) = \oplus \sum_{g \in G(A)} J_g$.*

Proof. $(\Rightarrow)$ Let A be a separable extension of B^G in B. Since B satisfies the fundamental theorem, we have that $A = B^{G(A)}$; and so $V_B(A) = V_B(B^{G(A)})$. But B is a Galois extension of $B^{G(A)}$ with Galois group $G(A)$, so $V_B(B^{G(A)}) = \oplus\sum_{g \in G(A)} J_g$ ([6], Proposition 1). Hence $V_B(A) = V_B(B^{G(A)}) = \oplus\sum_{g \in G(A)} J_g$.

$(\Leftarrow)$ Since B is a Hirata separable Galois extension of B^G with Galois group G, $H \to B^H$ is a one-to-one map from the set of subgroups of G to the set of subextensions A of B^G in B ([14], Corollary 3.5). Since B a general Azumaya Galois extension of B^G with Galois group G, B^H is a separable C^G-algebra by Lemma 3.7; and so a separable extension of

B^G in B for every subgroup H of G. Hence $H \to B^H$ is a one-to-one map from the set of subgroups of G to the set of separable extensions A of B^G in B. Therefore, it suffices to show that for any separable extension A of B^G in B, $A = B^{G(A)}$ In fact, since A is a separable extension of B^G and B^G is a separable C^G-algebra, A is a separable C^G-algebra. But, by Theorem 3.3, $C = C^G$ and B is an Azumaya algebra, so A is a separable subalgebra of the Azumaya algebra B over C. Hence $A = V_B(V_B(A))$ by the commutator theorem for Azumaya algebras ([4], Theorem 4.3, page 57). Moreover, since B is a general Azumaya Galois extension with Galois group G, for a subgroup $G(A)$ of G, $B^{G(A)}$ is separable over C^G by Lemma 3.7, so $B^{G(A)}$ is a separable subalgebra of the Azumaya algebra B over C. Therefore $B^{G(A)} = V_B(V_B(B^{G(A)}))$ by the commutator theorem for Azumaya algebras. Now, since B is a Galois extension of $B^{G(A)}$ with Galois group $G(A)$, $V_B(B^{G(A)}) = \oplus\sum_{g\in G(A)} J_g$ ([6], Proposition 1), and by hypothesis, $V_B(A) = \oplus\sum_{g\in G(A)} J_g$. Thus $V_B(A) = \oplus\sum_{g\in G(A)} J_g = V_B(B^{G(A)})$. Hence $A = V_B(V_B(A)) = V_B(V_B\ (B^{G(A)})) = B^{G(A)}$. This completes the proof.

Noting that a Hirata-Azumaya Galois extension with Galois group G is a general Hirata-Azumaya Galois extension with Galois group G, and a central Galois algebra B with Galois group G is a Hirata-Azumaya Galois extension with Galois group G. Theorem 3.8 generalizes the characterizations for a Hirata-Azumaya Galois extension in [17] and for a central Galois algebra in [12] satisfying the fundamental theorem.

Corollary 3.9 ([17], Theorem 3.6)**.** *Let B be a Hirata-Azumaya Galois extension with Galois group G. Then B satisfies the fundamental theorem if and only if for any separable subalgebra A of B, $V_B(A) = \oplus \sum_{g\in G(A)} J_g$.*

Corollary 3.10 ([12], Theorem 3.4)**.** *Let B be a central Galois algebra with Galois group G. Then B satisfies the fundamental theorem if and only if for any separable subalgebra A of B,* $V_B(A) = \oplus\sum_{g \in G(A)} J_g$.

We conclude the present paper with two examples of a Galois extension B with Galois group G to illustrate that (1) B is a general Hirata-Azumaya Galois extension with Galois group G, but B is not a Hirata-Azumaya Galois extension with Galois group G; (2) B is a Hirata Galois extension of B^G with Galois group G, but B is not a general Hirata-Azumaya Galois extension with Galois group G.

Example 3.11. Let $B = \mathbb{Q}[i, j, k]$, the quaternion algebra over the rational field $\mathbb{Q}$, and $G = \{1, g_i\}$, where $g_i(x) = ixi^{-1}$ for all $x \in B$. Then

(1) B is a Galois extension with Galois group G with a Galois system $\{1, i, j, k; \frac{1}{4}, -\frac{1}{4}i, -\frac{1}{4}j, -\frac{1}{4}k\}$.

(2) Since G is inner induced by the elements $\{1, i\}$, B is a Hirata separable extension of B^G ([8], Corollary 3).

(3) $B^G = \mathbb{Q}[i]$ which is a separable $\mathbb{Q}$-algebra, where $\mathbb{Q}$ is the center of B and $\mathbb{Q}^G = \mathbb{Q}$. Hence B is a general Azumaya Galois extension with Galois group G.

(4) B is a general Hirata-Azumaya Galois extension with Galois group G by (1), (2), and (3).

(5) Since $B^G (= \mathbb{Q}[i])$ is not Azumaya over $\mathbb{Q}^G$, B is not a Hirata-Azumaya Galois extension with Galois group G.

Example 3.12. Let $A = \mathbb{Q}[i, j, k]$, the quaternion algebra over the rational field $\mathbb{Q}$, $B = \{\begin{pmatrix} a_1 & a_2 \\ 0 & a_3 \end{pmatrix} \mid a_1, a_2, a_3 \in A\}$, the ring of all 2 by 2 upper triangle matrices over A, and $G = \{1, g_i, g_j, g_k\}$, where $g_i(a) = iai^{-1}$, $g_j(a) = jaj^{-1}$, $g_k(a) = kak^{-1}$ for all a in A and $g\begin{pmatrix} a_1 & a_2 \\ 0 & a_3 \end{pmatrix} = \begin{pmatrix} g(a_1) & g(a_2) \\ 0 & g(a_3) \end{pmatrix}$ for $g \in G$. Then

(1) It is easy to check that the center of B is $\mathbb{Q}I$, where I is the identity 2 by 2 matrix and $B^G = \{\begin{pmatrix} q_1 & q_2 \\ 0 & q_3 \end{pmatrix} \mid q_1, q_2, q_3 \in \mathbb{Q}\}$, the ring of all 2 by 2 upper triangle matrices over $\mathbb{Q}$.

(2) B is a Galois extension with Galois group G with a Galois system $\{I, iI, jI, kI; \frac{1}{4}I, -\frac{1}{4}iI, -\frac{1}{4}jI, -\frac{1}{4}kI\}$.

(3) Since G is inner induced by the elements $\{I, iI, jI, kI\}, B$ is a Hirata separable extension of B^G ([8], Corollary 3).

(4) By (2) and (3) B is a Hirata Galois extension of B^G with Galois group G.

(5) Since $B^G = \{\begin{pmatrix} q_1 & q_2 \\ 0 & q_3 \end{pmatrix} \mid q_1, q_2, q_3 \in \mathbb{Q}\}$ which is not a separable $\mathbb{Q}I$-algebra ([10], Example 4.3-(8)), B is not general Azumaya Galois extension with Galois group G.

References

[1] R. Alfaro and G. Szeto, Skew group rings which are Azumaya, Comm. in Algebra 23(6) (1995), 2255-2261.

[2] R. Alfaro and G. Szeto, On Galois extensions of an Azumaya algebra, Comm. in Algebra 25(6) (1997), 1873-1882.

[3] F. R. DeMeyer, Some notes on the general Galois theory of rings, Osaka J. Math. 2 (1965), 117-127.

[4] F. R. DeMeyer and E. Ingraham, Separable Algebras over Commutative Rings, Volume 181, Springer Verlag, Berlin, Heidelberg, New York, 1971.

[5] S. Ikehata, Note on Azumaya algebras and H-separable extensions, Math. J. Okayama Univ. 23 (1981), 17-18.

[6] T. Kanzaki, On Galois algebra over a commutative ring, Osaka J. Math. 2 (1965), 309-317.

[7] T. Kanzaki, On Galois extension of rings, Nagoya J. Math. 27 (1966), 43-49.

[8] K. Sugano, On a special type of Galois extensions, Hokkaido J. Math. 9 (1980), 123-128.

[9] G. Szeto and L. Xue, Skew group rings which are Galois, International Journal of Mathematics and Mathematical Sciences 23(4) (1999), 279-283.

[10] G. Szeto and L. Xue, On central commutator Galois extensions of rings, International Journal of Mathematics and Mathematical Sciences 24(5) (2000), 289-294.

[11] G. Szeto and L. Xue, The structure of Galois algebras, Journal of Algebra 237(1) (2001), 238-246.

[12] G. Szeto and L. Xue, On Galois algebras satisfying the fundamental theorem, Communications in Algebra 35(12) (2007), 3979-3985.

[13] G. Szeto and L. Xue, On Galois extensions of a separable algebra, International Mathematical Forum 3(14) (2008), 677-683.

[14] G. Szeto and L. Xue, On Galois extensions with a one-to-one Galois map, International Journal of Algebra 5(17) (2011), 801-807.

[15] L. Xue, On Azumaya Galois extensions, Bull. Malays. Math. Sci. Soc. (2) 35(2A) (2012), 583-589.

[16] L. Xue, On Galois correspondence of a general Azumaya Galois extension, International Journal of Algebra and Statistics 1 (2012), 45-49.

[17] L. Xue, On Hirata-Azumaya Galois extensions, International Mathematical Forum 8(23) (2013), 1103-1110.

[18] L. Xue, On compositions of a Galois extension of a separable algebra, International Journal of Algebra 8(2) (2014), 67-73.

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