Journal of Statistics: Advances in Theory and Applications
Volume 15, Number 1, 2016, Pages 1-16
Available at http://scientificadvances.co.in
DOI: http://dx.doi.org/10.18642/jsata_7100121593

STRUCTURAL CHANGES ON SIEFORES' PRICE YIELDS BASED ON INVESTMENT PORTFOLIOS

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Abstract

The aim of this work is to perform a statistical analysis of the Siefores' (investment companies specialized in retirement savings funds in Mexico) historical prices with emphasis on determining whether there have been structural changes on yields through the time. The methodology followed in order to find the results is to perform a quantitative analysis using statistical tools, based on an analysis of historical data of the Siefores' prices from 1997 to 2015. Hence, the analysis consists of (1) to identify extreme values on a total of 4 Siefores' yields and per Afore (retirement savings fund managers), (2) to identify the dates where these extreme values occurred and to analyse coincidences on dates for all the Siefores and Afores, (3) to determine whether changes on the Siefores' investment portfolios occurred on these dates, (4) to define segments based on the dates defined previously, (5) to perform the so called "Chow hypothesis testing" in order to conclude whether there is a structural change on the Siefores' yields. The results are that there is a structural change on the Siefores' yields and that these have contributed to the decrease on the portfolios returns.

2010 Mathematics Subject Classification: 62, 62J05.
Keywords and phrases: pensions, yields, linear regression, Chow test.
Received December 3, 2015

1. Literature Review

1.1. Pensions in Mexico

Since 1925, the compulsory pension system in Mexico remained as defined benefit type; however, several factors led, in 1997, to change from defined benefit to a defined contribution scheme. Such factors included financial crisis by the government; which was caused, among other things, by the increase in life expectancy of pensioners because of medical advances causing a disproportionate growth among the population of pensioners and the economically active population.

In the unfunded defined benefit pension system, the workers received at the end of his working life a proportion of their last salary; however, due to the increase of the life expectancy, this model became untenable. Other factors influencing the change of the system were the decrease of the birth rate, together with population growth and mismanagement of financial resources by the government.

The defined contribution pension system in Mexico was created in 1997 with the so called Afores (pension funds management companies). This Afores open individual accounts for the workers to manage their savings charging an administrative fee. Each of the Afores invest the savings through the so called Siefores (investment companies specialized in retirement savings funds), which invest the savings according to the worker's age being the most conservative Siefore 1 targeted to workers aged 60 or more and the most aggressive to Siefore 4 for workers aged 36 or less.

As mentioned before, this pension system in Mexico consists on each worker having an individual account where throughout his or her working life a final amount will be available to obtain a pension. This amount is the total of the worker contribution plus the sponsor and the government contribution plus any yields that are gained from the investment of the savings, less any deduction made (such as agent's

commission). At retirement age, the worker has the option of buying a life annuity from an insurance company or programmed withdrawals from the Afore.

As the life annuity is an irrevocable contract between the parties, by which the worker transfers funds from his or her individual account to the insurance company in exchange for a fixed monthly income for life, this is usually chosen by the workers with high life expectancy, as well as by those being conservative when choosing the risk, as they prefer a secure pension. However, to compensate the risk incurred by the insurance company, this option means higher fees. On the other hand, the programmed withdrawals are chosen by those workers with low life expectancy or by those risk lovers.

1.2. The Chow test: Structural or parameter stability test of regression models

According to Gujarati [2], when using the regression model, a structural change could exist between the dependent variable and the independent variables X_i. That is, that the values of the parameters of the regression model does not remain the same during the period of analysis. This structural change may be due to external factors, policy changes, etc. and no all data is exempt from these changes. Hence, if a regression model is used with the incorrect parameters, the projections could be also incorrect. In order to find if there is a structural change, the Chow test is used. The method consists on the following steps:

(1) Estimate the regression model taking the whole period of data and obtain the residual sum of squares, which is called the restricted residual sum of squares (RSS_R).

(2) Define periods of time to segment the data, according to where we believe there is a structural change (i.e., policy changes, stock market crisis, etc).

(3) Estimate regression models for each of the periods defined and obtain the residual sum of squares for each of them.

(4) Assuming the periods in previous step are independent, obtain the sum of the residual sum of squares for all of the periods, which is called unrestricted residual sum of squares (RSS_{UR}).

(5) Perform the Chow test which stablishes that if there is no structural change, RSS_R and RSS_{UR} are not statistically different. Then we define

$$F = \frac{(RSS_R - RSS_{UR})/k}{RSS_{UR} / (n_1 + n_2 + \ldots + n_i - 2k)} \sim F(k, (n_1 + n_2 + \ldots + n_i - 2k)),$$

where is the number of parameters estimated and the number of data in each period (i the number of periods). Accept or reject the null hypothesis that there is structural change.

Two assumptions for the Chow test hold as follows: (i) The error terms for the regression models in each period defined in Step 2, are normally distributed with mean 0 and same variance. (ii) The error terms are independently distributed.

2. Methodology

As mentioned previously, the objective of this study is to find if there is a structural change on the Siefores' investment portfolios, in order to determine possible causes of extreme changes on the yields values through the time. Hence, the methodology followed in this work is as follows:

(1) The stock prices of each of the Siefores per Afore are obtained from 1 of July 1997 to 19 of May 2015. Then, the Siefores yields through the time are easily calculated.

(2) To determine graphically if there are any extreme values on the Siefores' yields through the time. This is achieved by looking at the box plots of the data.

(3) To determine statistically if there are any extreme values on the Siefores' yields through the time. This is achieved by performing the Grubbs statistical hypothesis testing.

(4) In case the result in the previous two steps is positive for the existence of extreme values, to determine the dates in which these occurred, in order to identify and define a number of segments to evaluate.

(5) Based on the segments defined in the previous step, perform the Chow statistical test.

3. Results

The total data obtained for the prices of each Siefore and per Afore is 4,510 yields corresponding to the dates from 2 of July 1997[1] to 19 of May 2015. The yields for Siefore and per Afore were calculated and taken to present the analysis in this work. Then, the values for the Siefores's yields through the time are shown in Figure 1 to 4 and all values are shown in percentage points. It is worth to remind the reader that the most conservative portfolio belongs to Siefore 1 and the least to Siefore 4[2]. As a first attempt to show if these yields have presented atypical values through the time, Figures 1 to 4 plot these yields.

[1]The first date found is 1 of July 1997, however, when calculating the yields values the first data will become from 2 of July 1997.

[2]See Section 1.

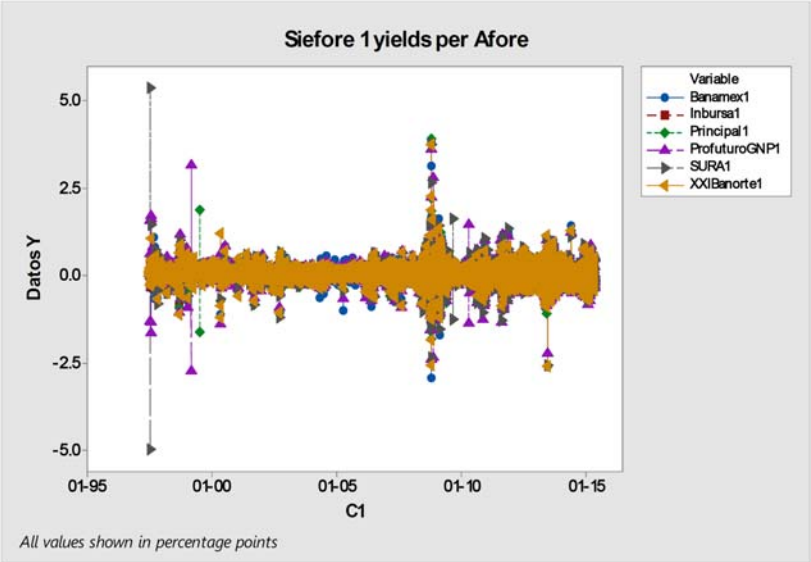


Figure 1.

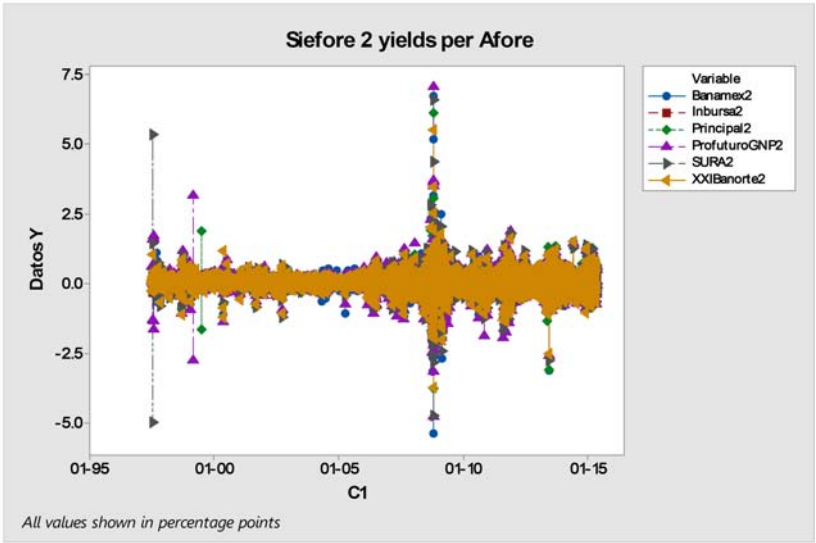


Figure 2.

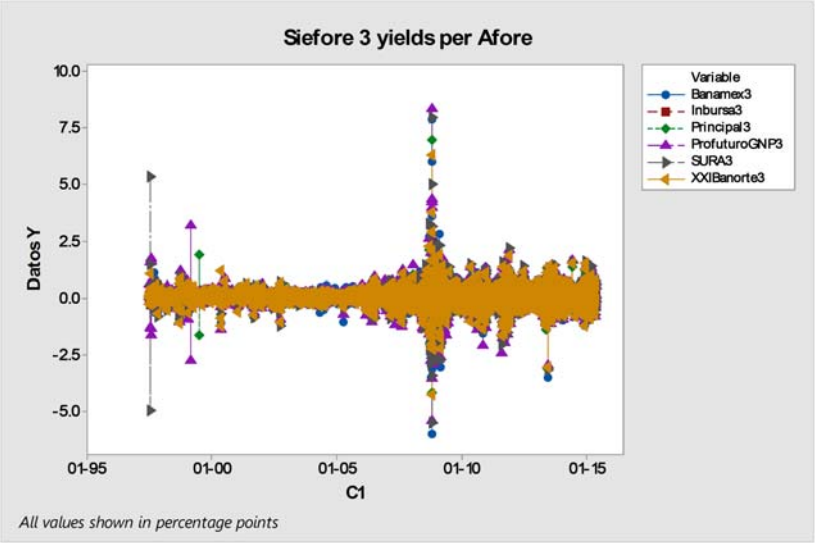


Figure 3.

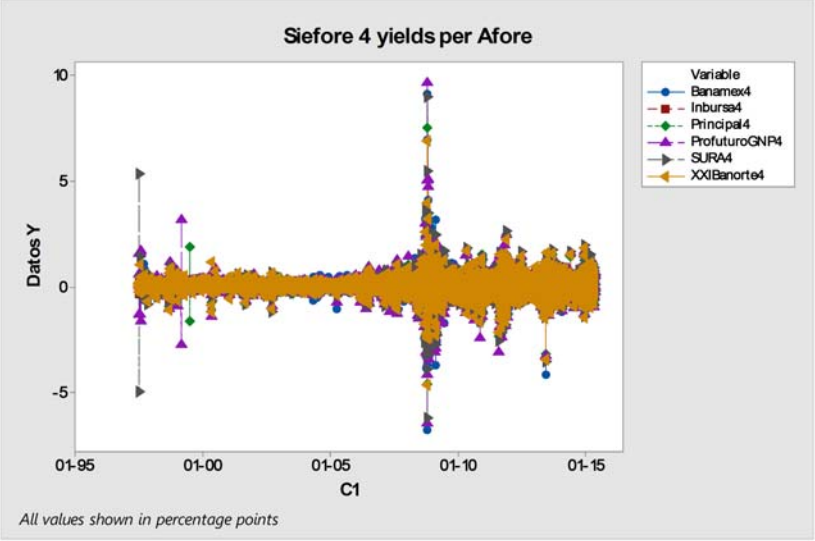


Figure 4.

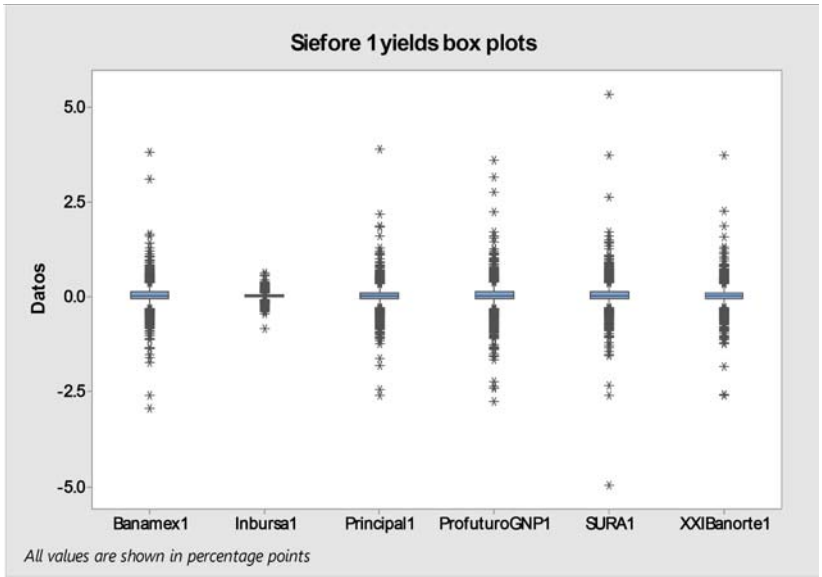
From the results shown in Figures 1 to 4, we can see that all Siefores show atypical values. This is what we expect as the portfolios on the Siefores have suffered of great changes through the time and also because these yields depend on the stock market which is very volatile. In order to determine whether there is a structural change on any of the Siefores' yields using the Chow statistical test, specific dates have to be defined which should be chosen according to where we believe there exist this structural change.

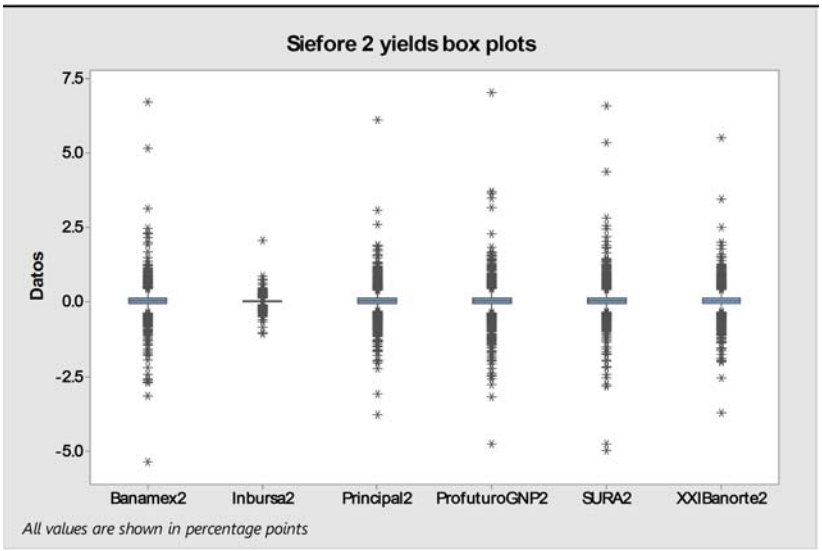
Hence, confident intervals at a 99% are constructed and plotted in Figure 5. When a match on the dates is made for all Siefores and Afores, it is found that atypical values exist during all months for all years. This is confirmed with the Grubbs test which consists in defining the following hypothesis:

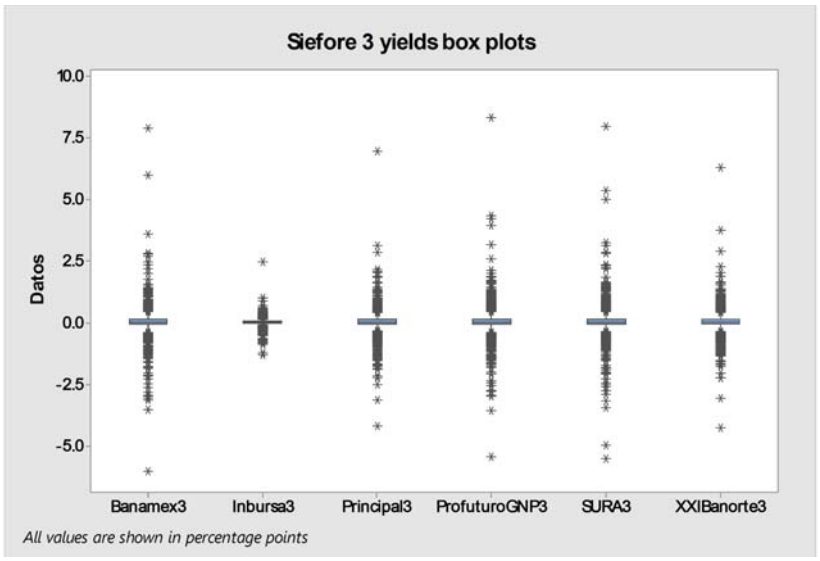
Null hypothesis: The values in the data belong to the same normal distribution.

Alternative hypothesis: The minimum or maximum value in the data is atypical value.

Tested with a significance level of $\alpha = 0.05$, the results are shown in Figures 6 to 9 for each of the Siefores. These figures show the atypical values plot and the result for the test. In all Siefores, there is statistical evidence that atypical values exist.


Siefore 1 yields box plots
Datos
5.0
2.5
0.0
-2.5
-5.0
Banamex1
Inbursa1
Principal1
ProfuturoGNP1
SURA1
XXIBanorte1
All values are shown in percentage points


Siefore 2 yields box plots
Datos
7.5
5.0
2.5
0.0
-2.5
-5.0
Banamex2
Inbursa2
Principal2
ProfuturoGNP2
SURA2
XXIBanorte2
All values are shown in percentage points


Siefore 3 yields box plots
Datos
10.0
7.5
5.0
2.5
0.0
-2.5
-5.0
Banamex3
Inbursa3
Principal3
ProfuturoGNP3
SURA3
XXIBanorte3
All values are shown in percentage points

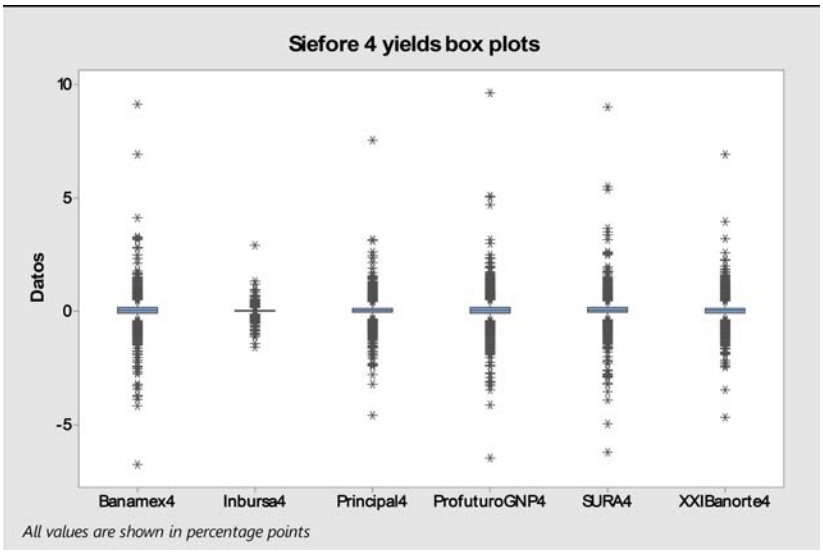

Siefore 4 yields box plots
Datos
10
5
0
-5
Banamex4
Inbursa4
Principal4
ProfuturoGNP4
SURA4
XXIBanorte4
All values are shown in percentage points

Figure 5.

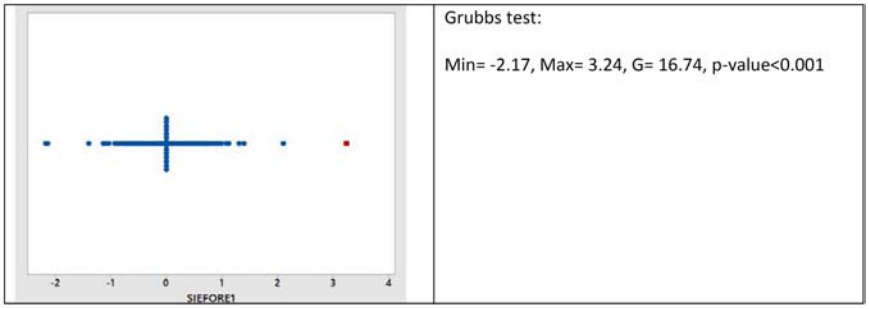


Figure 6.

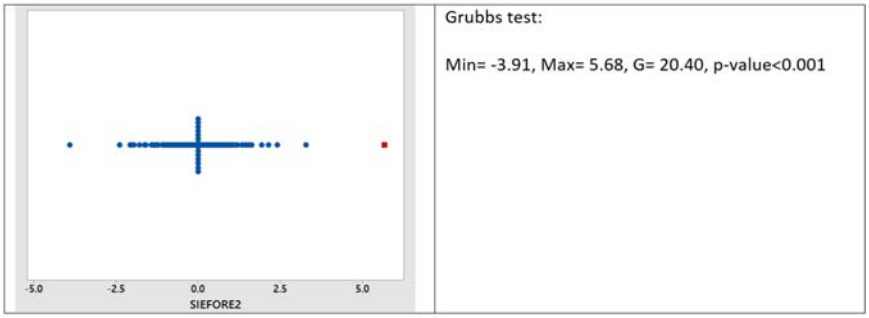


Figure 7.

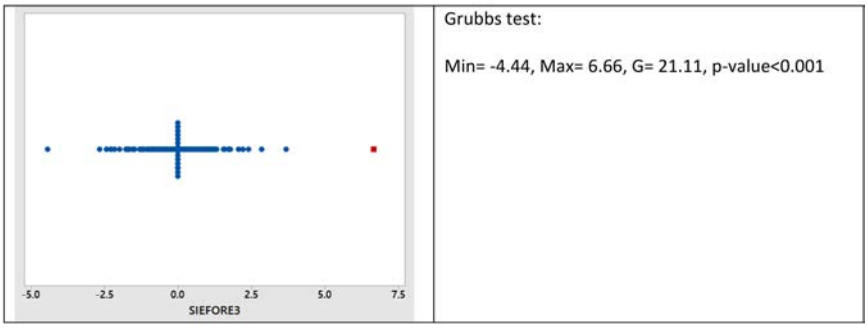


Figure 8.

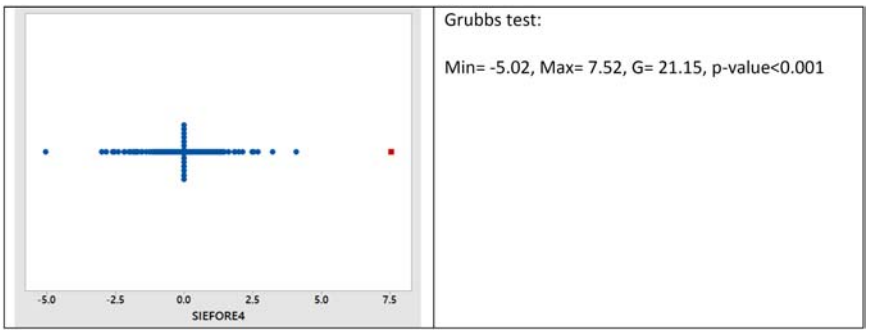


Figure 9.

As the Chow test requires to define periods of time where it is believed a structural change exists, the dates are selected from the historical changes on the Siefores' investment portfolios. These dates are shown in Table 2.

Table 2. Mexico-evolution in authorization in types of assets to pensions investments

INSTRUMENTS ALLOWED INSIDE SIEFORES'S INVESTMENT SYSTEM																	
Debt	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Foreign Exchange	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Equity	✗	✗	✗	✗	✗	✗	✗	✗	✓	✓	✓	✓	✓	✓	✓	✓	✓
CKDs and FIBRAS	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✓	✓	✓	✓	✓	✓	✓
Securities	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✓	✓	✓	✓	✓	✓	✓
Merchandise	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✓	✓	✓
Swaptions	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✓
Real estate	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✓
	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013

FIAP (2014), p. 122.

Notes:

- **CKDs:** Values issued by trusts that canalizes resources to sectors and activities with chance of long-term growth, as infrastructure.

- **Fibras:** Investment vehicle in real estate which offers recurring payments net tax outcome from the incomes and has the possibility of obtain gain (appreciation).

- **Securities:** Securities or values that represent credit rights issued through vehicles whose underlying assets has been those credit rights.

- **Swaptions:** Financial derivative consisting in an option whose underlying is a swap. Swap is a financial instrument consistent in a contract about a derivative product that allows interest rate coverage.

- **Real estate:** enterprises or funds issued listed share. Also at least the 90% of gain must be dispensed among its shareholders. This allocation is called dividends, and it is why this instrument is very attractive to invest because of the profit that provides.

According to Table 2, the selected dates to perform the Chow hypothesis testing are 2005, 2007, 2011, and 2013, as these are the dates where changes to the Siefores' investment portfolios occurred. Therefore, the methodology described in Subsection 1.2 by Gujarati [2] will be followed.

The following regression model is defined: x = date, y_i = average of Siefore yield, where $i = 1, \ldots, 4$ to allow for the 4 Siefores. The average of Siefore i yield consists on the average per Siefore for all Afores. The following steps are performed for each of the Siefores, only the process for Siefore 1 is showed.

Step 1. Calculate $RSS_R = 162.713$.

Step 2. Define periods of time to test structural changes: 2005, 2007, 2011, and 2013.

Step 3. Calculate for each of the periods defined in Step 2. Therefore, 5 regression models are performed: $RSS_1 = 25.981$, $RSS_2 = 5.888$, $RSS_3 = 72.002$, $RSS_4 = 23.132$, and $RSS_5 = 35.064$.

Step 4. Calculate $RSS_{UR} = 25.981 + 5.888 + 72.002 + 23.132 + 35.064 = 162.067$.

Step 5. Calculate $F = 17.969$ which compared with the probability distribution $F_{(1,4508)} = 5.03$. The null hypothesis is rejected, which means that there is not structural change for Siefore 1 portfolio.

When performing the same procedure for Siefore 2 to Siefore 4, the result is to reject Ho. That is, there is no statistical evidence that a structural change exists, according to the Chow test. This result is surprising as statistical evidence is found that there are atypical values through the entire period of time analyzed. Also, as there have been important changes on the investment portfolios for all Siefores, a first guess was that structural changes are present on the Siefores' yields. However, when looking at the sum of squares of the residuals, there is no statistical evidence that these are different among the periods analyzed.

From the results found, it is concluded that historical changes within the Siefores' portfolio structure have resulted in important yields variations through the time, however it is "safe" to analyze the data as one period of time, due to there is no structural change. A possible reason for this result is that the dates when portfolio changes become into force for all Siefores, might not reflect an immediate change on the Siefores' yields as the Afores may take a while to actually make that changes. Then, in order to extend this analysis, the chow test is performed at different dates. The dates are now chosen with respect to the worldwide financial crisis occurred between 1997 and 2015. According to the website "digital freedom economics" (libertad digital economía) and "the world, business and economics" (el mundo, economía y negocios), the most important financial crisis worldwide have occurred in 2000, 2001, 2007, 2009, and 2010 (taking into account only from 1997 to 2015). The events occurred during these dates are shown in Table 3.

Table 3. Events that caused financial crisis worlwide

Date	Event
2000	.com crisis
2001	9/11 attempts and Argentinian crisis
2007	USA recession
2009	Greek financial crisis
2010	Foreign exchange crisis

Therefore, the chow test is performed according to these dates.

The results are that for all Siefores and all 6 periods (or segments) the observed statistics is smaller than the distribution value given before $(F_{(1,4508)} = 5.03)$. Then, the null hypothesis is rejected, meaning that there is no structural change within the periods chosen. This result is also surprising as theoretically when financial crisis occur, the portfolios which invest in risky assets, present atypical jumps on the value of the yields. A possible reason for this result is that when looking at the Siefores' yields, almost in all dates present atypical values. The Siefores' yields present a high volatility as seen previously when confidence intervals were analyzed. This makes rather complicated to divide the data into defined periods where it is obvious that atypical values are present.

4. Conclusion

Both results are surprising. When choosing dates where portfolio changes became into force, a possible reason is that these dates might not reflect an immediate change on the Siefores' yields as the Afores may take a while to actually make that changes. When choosing the dates where financial crisis have occurred worldwide, theoretically the portfolios which invest in risky assets present atypical jumps on the value of the yields. A possible reason for this result is that when looking at the Siefores' yields, almost in all dates present atypical values. The

mexican financial market is so volatile that the retirement pensions savings in Mexico present also lots of volatility or the managers of the retirement mexican system are really inefficient.

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[1] FIAP, Fortaleciendo los Cimientos del Sistema de Capitalización Individualpara Asegurar su Sostenibilidad, Primeraedición, 2014.

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[3] http://www.libertaddigital.com/economia/cronologia-de-las-crisis-financieras-del-ultimo-siglo-1276406696/.

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